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VIII INTERNATIONAL SYMPOSIUM ON MATHEMATICAL PROGRAMMING

STANFORD UNIVERSITY

AUGUST 27-31, 1973

ABSTRACTS

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CODE SYSTEM

Above each author's name in the abstracts is a code number indicating the day, time, subject matter, and lecture room. This code can be interpreted as follows:

DAY:	M = Monday;	W = Wednesday;	F = Friday;
	T = Tuesday;	TH = Thursday;	
TIME:	AM = Morning	PM = Afternoon	

SUBJECT:

1 Integer Programming: Enumerative and Branch and Bound Methods
Group Theory and Cutting Plane Methods
2 Optimization in Networks
Convex Polytopes and Linear Programming
Graphs and Combinatorics
3 Large Scale Systems
4 PANEL: Implementation of Mathematical Programming Algorithms
Computer Software and Mathematical Programming
SIGMAP: Computer Hardware and Mathematical Programming
5 SIGMAP: Computational Aspects of Nonlinear Programming
Nonlinear Programming: Algorithms
6 Quadratic Programming
Least Squares and Curve Fitting
Numerical Methods
7 Complementarity and Fixed Points
8 Nonlinear Programming: Theory
Unconstrained Optimization
9 Mathematical Programming: General
10 Game Theory
11 Stochastic Programming
12 Dynamic Programming and Control Theory
13 Applications: Engineering and Natural Sciences
Economics
Urban and Educational Planning
14 PLENARY SESSION

LECTURE ROOMS:

U Dinkelspiel Auditorium
V Physics Tank 100
W Skilling Auditorium
X Physics Tank 101
Y McCullough 134
Z 550A (Engineering)

CODE EXAMPLE: W-PM-10-Y = Wednesday -Afternoon - Game Theory - McCullough 134

JOINT AUTHORS: Jointly-authored papers will be presented by the person whose name is underlined. Only that person's affiliation is given.

EXAMPLE: K.J. ARROW, F.J. GOULD, S.M. HOWE, Harvard University

* = Invited speaker   P = Plenary speaker

T-AM-6-Y

N. N. ABDELMALEK, National Research Council, Ottawa, Canada

On the Discrete Linear L_1 Approximation and L_1 Solutions of Overdetermined Linear Equations

Usow's algorithm for solving the discrete linear L_1 approximation problem is generalized so that it can also solve an overdetermined system of linear equations in the L_1 norm. It is then shown that this algorithm is completely equivalent to a dual simplex algorithm applied to a linear programming problem in non-negative bounded variables. However, one iteration in the former is equivalent to one or more iterations in the latter.

A dual simplex algorithm is described which seems to be the most efficient and capable method for solving these two problems. Its efficiency is due to not using artificial variables and to its simplicity. Its capability is due to the fact that the Haar condition attached to Usow's method is completely relaxed. Numerical results are given.

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M-PM-8-V

R. A. ABRAMS, Northwestern University, Evanston

Projections of Convex Programs

Convex programs with closed objective function and closed feasible region are classified as degenerate if the objective function and the feasible region have a common direction of recession. For each degenerate program, a reduced form is defined by projecting the feasible region and the objective function epigraph on the orthogonal complement of the recession directions. A finite sequence of such reductions yields a non-degenerate problem for which the infimum is attained on a bounded set. Under a very mild condition the infimum of the reduced problem is equal to that of the original problem. It is shown that the objective and constraint functions of the "projected" problem may be obtained by calculating limits of the objective and constraint functions in the directions of recession. These results generalize the concept of degeneracy and reduction to canonical form which was originally developed for posynomial geometric programming.

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TH-PM-5-Z

J. ABRHAM, L. S. LUBOOBI, University of Toronto, Canada

A Numerical Method for a Class of Continuous Concave Programming Problems

The problem under consideration consists in maximizing

$$U(x) = \sum_{i=1}^{p} \int_0^T \alpha_i(t)\, g(x_i(t))\, dt \quad \text{subject to} \quad \sum_{i=1}^{p} a_{ki} \int_0^T x_i(t)\, dt = \mu_k,$$

$k = 1, \ldots, m$ where $x_1(t), \ldots, x_n(t)$ are functions bounded, measurable, and nonnegative on $[0,T]$. The quantities $\mu_k > 0$, $k = 1, \ldots, m$ and $a_{ki} \geq 0$, $k = 1, \ldots, m$, $i = 1, \ldots, p$ are given. The α_i's are nonnegative, nonincreasing and bounded functions on $[0,T]$ and the g_i's are nondecreasing concave functions on $[0,+\infty]$.

The above problem is approximated by two sequences of problems in which the α_i's are replaced by step functions forming sequences converging to the respective α_i's from above and from below. The approximating problems can then be solved by standard techniques. Both the convergence of the optimal solutions to approximating problems and the convergence of the corresponding optimal values are investigated. Some estimates of the accuracy of approximating solutions are also obtained.

* * *

T-AM-2-X

I. ADLER, University of California, Berkeley

Enumeration of All Vertices of a Polyhedral Set

The problem of enumerating all vertices (extreme points) of a given polyhedral set has attracted quite wide interest in the mathematical programming literature. Such enumeration is instrumental in some suggested optimization techniques.

In this presentation we review the main ideas behind the existing enumeration algorithms. Specifically, we discuss in some detail the cutting planes and vertex following procedures, and answer some open questions (which have appeared in the literature) concerning their efficiency.

Finally, we present two new enumeration algorithms.

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F-PM-2-W

D. ADOLPHSON, T. C. HU, University of Washington, Seattle

Optimal Linear Ordering

A problem of many applications is the quadratic assignment problem. A closely related one is the module placement problem (also called the blackboard wiring problem) in which a set of pins connected by wires are placed into a set of holes, one pin in each hole. The configurations of the holes are fixed. Usually, the holes occupy the lattice points in the plane, and the numbers of wires connecting each pair of pins are given in advance. The problem is to put the pins into the holes such that the total wire length is minimal.

We can represent the n pins connected by wires as a graph G with n nodes. The numbers c_{ij} are associated with the arc connecting the nodes N_i and N_j. The c_{ij} is the number of wires connecting the ith and the jth pins. We shall use the word "node" interchangably with "pin" from now on.

Most algorithms proposed for solving this problem have been heuristic, or branch-and-bound algorithms which can only handle problems when $n \leq 20$. An excellent review with many references can be found in Hanan and Kurtzberg. In this paper, we shall consider a special case of the above problem; namely, when the holes are all in a line with adjacent holes at a unit distance apart. We shall call this problem, "the optimal linear ordering problem" (abbreviated as O.L.O.). The O.L.O. problem was solved by Harper in the case where the graph G is a deBruijn graph of order four. It was also considered by Morse who used a different criteria of optimization. Since there are $n!$ possible configurations in the linear ordering of n pins, a straight-forward enumeration approach is not feasible.

In section 2, we will prove some lemmas about the O.L.O. problem and also its relationship with the theory of network flows, which is usually considered an entirely different subject.

In section 3, we will restrict the graph to be a tree and interpret the O.L.O. problem as a scheduling problem. We will then present an algorithm which will give the global optimum solution in $O(n \log n)$ steps.

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W-PM-11-Z

S. M. AHSAN, McMaster University, Hamilton, Canada

Chance-Constrained Programming, Lognormal Distribution and Portfolio Selection Theory

Naslund (1968) was the first to use chance-constrained programming to analyse the theory of asset choice. In the tradition of Charnes, Cooper et al., Naslund assumed that asset returns are normally distributed and confirmed the results obtained earlier by Tobin (1958) and others. However, normally distributed asset prices imply a non-zero probability that they will be negative. In a typical economic context, therefore, the interpretation of such results are somewhat ambiguous. This point has been stressed by Sengupta (1970), among others. Indeed, Sengupta has discussed the possibilities of using chi-square distribution. But he suspected that "the decision rules under chance-constrained programming may be sensitive to departures from normality unless some prior analysis shows the contrary".

In this paper, we approach the theory of portfolio selection within the framework of a chance-constrained programme, assuming a lognormal securities market. The empirical evidence that stock market prices are lognormally distributed has been emphasized by Lintner (1972), among others. Therefore, assuming, that the investor's choice can essentially be described as choices among the logarithmic moments of alternative investment returns, we attempt to show that one still can obtain the main results provided by Tobin, Naslund et al.

The "main results" are

(a) In the case when there is a safe asset

(i) (SEPARATION THEOREM): if the investor holds a safe asset, then he holds the risky assets in certain fixed proportions, which are independent of the size of his disposable wealth and his risk-aversion function

(ii) a proportional tax stimulates risk-taking.

(b) In the case where all assets are risky, we show that

(i) the necessary and sufficient condition for there to be a movement towards the high-risk asset from the low-risk asset, due to lump sum taxation, is that the former's expected yield is greater

(ii) the effects of a proportional tax depends on the constraint.

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T-PM-2-X

G. APPA, Middlesex Polytechnic, Queensway, Enfield, England

Some New Approaches for Problems Arising From Degeneracy in Linear Programming

Problems caused by the presence of degeneracy are different for different algorithms. For the simplex type algorithms the main problem is that of 'wasteful' iterations, i.e., of making a number of pivot changes without improving the value of the objective function. A problem may be highly degenerate from the start, and/or degeneracy may be encountered en route (to a solution) and/or an optimal extreme point may be highly degenerate. In either of these cases there are many reduced cost-coefficient vectors corresponding to the same geometrical extreme point. The usual lexicographic method orders these completely and makes pivot changes leading to higher and higher order vectors, for any of these problems. This is because it is designed to deal with a theoretical problem common to all the three situations, viz., cycling. We develop two other finite methods: (1) A Sequential Primal Method and (2) A Sequential Primal Dual Method. These attack directly the twin problems of reducing wasteful iterations and of generating as few additional rows and columns as possible. Another approach developed here introduces a cutting plane which eliminates the degenerate point from the feasible set, in such a way that all the feasible edges go through the cutting plane. As the edges are less degenerate than the extreme point from which they emanate, the cutting plane cuts down the number of iterations required in certain cases. Moreover the cutting plane also serves as a useful theoretical tool with implications for finding all extreme rays of a convex cone, Balas' intersection cuts etc. These are discussed.

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T-AM-1-W

R. D. ARMSTRONG, P. SINHA, University of Massachusetts, Amherst

Improved Penalty Calculations for a Mixed Integer Branch and Bound Algorithm

This paper presents an extension of Tomlin's penalties for the branch and bound mixed integer algorithm of Beale and Small. It is shown that the integer requirements on non-basic variables can be used in a way different from Tomlin's method by considering a future dichotomy on a non-basic integer variable that may come into the basis at the first dual simplex iteration. The additional penalty for enforcing this integer requirement can be easily computed, with little extra arithmetic and bookkeeping. The improvement is easy to incorporate for the normal case as well as when the variables are grouped into ordered sets with generalized upper bounds. The stronger penalties help reduce computational time and also provide a more powerful criterion for rejecting solution sets as suboptimal. Computational experience is provided.

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write him
Prof Dear Steve:

T-PM-8-V

K. J. ARROW, F. J. GOULD, S. M. HOWE, Harvard University, Cambridge

* A General Saddle-Point Result for Constrained Maximization

The paper aims to establish saddle-point results for constrained maximization in the case where the maximand or the constraints or both need not have the usual convexity properties. By necessity, only local results are possible. The essence of the device is to modify the Lagrangian so that violations of the constraints enter nonlinearly. Then the value of the primal variables in the constrained maximum will be a local saddle-point of the modified Lagrangian; more strongly, the modified Lagrangian will be strictly concave in the primal variables and convex in the dual variables, so that the saddle-point can be determined by a gradient method or by complementary pivoting.

* * *

Dear Subramani:

T-AM-1-W

S. ARUNKUMAR, University of California, Los Angeles

Optimal Synthesis of Computer-Communcation Networks

The problem of locating communication/computer centers in a given set of grid points and the synthesis of the network connecting the centers to the other points in the grid is considered in this paper. The problem is formulated as a mixed-integer minimal cost flow problem and is solved using Benders' decomposition. The relaxed master problem is solved by implicit enumeration. Bounds on the objective function are generated at each step and the routine terminates in a finite number of steps.

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M-PM-1-W

E. BALAS, Carnegie-Mellon University, Pittsburgh

* On the Use of Intersection Cuts and Outer Polars in Branch and Bound

In this paper we discuss the use of various intersection cuts, and more generally the use of outer polars, in the context of branch and bound-type emunerative procedures. In particular, we identify a class of cuts that have certain desirable properties, and describe a cutting plane-branch and bound algorithm which uses these properties to decide when to cut and when to branch. Also, we use outer polars to generate strong bounds on the objective function value, and get a relevant measure of the "integer infeasibility" of a given solution.

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T-AM-2-X

M. L. BALINSKI, CUNY, New York and Ecole Polytechnique Federale de Lausanne

* The Hirsch Conjecture for Some Transportation Polytopes

The general class of transportation polytopes is defined by
$P\{(a_1,\ldots,a_m;\ b_1,\ldots,b_n),(m,n)\} = \{x = (x_{ij}) : \sum_{j=1}^{n} x_{ij} = a_i,\ \sum_{i=1}^{m} x_{ij} = b_j,\ x_{ij} \geq 0\}$
where the a_i, b_j are any positive values with $\sum_i a_i = \sum_j b_j$. The Hirsch conjecture states that at most $m+n-1$ feasible basis changes are necessary to go from any one feasible basis to any other feasible basis. Consider $P\{(k_1m+1,\ldots,k_mm+1;\ m,\ldots,m),\ (m, \sum_i k_i+1)\}$ where $k_i \geq 0$ and integer. Its extreme points number $[(\sum_i k_i)!\ (1+\sum_i k_i)^{m-1}]/\pi_i(k_i!)$. The Hirsch conjecture holds and is established constructively. Consider $P\{(k_1m-1,\ldots,k_m-1;m,\ldots,m),\ (m, \sum_i k_i-1)\}$ where $k_i \geq 1$ and integer. Its extreme points number $[m^{m-2}(\sum_i k_i-1)!]/\pi_i((k_i-1)!)$. The Hirsch conjecture holds and is established constructively. Both of these classes include the class of transportation polytopes $P\{(n,\ldots,n;\ m,\ldots,m),\ (m,n)\}$ with $(m,n) = 1$, which has been shown by Bolker to contain the maximum number of extreme points over the class of transportation polytopes for fixed m and n.

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W-AM-13-Y

J. L. BALINTFY, University of Massachusetts, Amherst

* Non-Linear Programming and the Food Price Index

There is experimental evidence concerning the time-dependent character of food preferences which indicates that measurable non-linear utility functions can be associated with food consumption rates. This finding is utilized in the formulation of a diet model with non-linear objective function and a budget constraint involving food prices. It is shown that this model represents improvement over the fixed basket or linear programming approximations of the true food price index. Computational results with simulated food prices indicate that food price indexes obtained by non-linear programming techniques are fairly close to Fisher's ideal price indexes. The conclusion supports the argument that foods may differ from other commodities in their contributions to the true cost of living index.

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F-AM-13-Y

Deepak BAMMI, Dalip BAMMI, University of Illinois at Chicago Circle

Comprehensive Land Use Planning Via Mathematical Programming and Game Theory: Results

A mathematical programming and game theory model has been used to determine optimal land use allocations to form a basis for the comprehensive land use plan for 1990 at DuPage County, Illinois. Multiple objective functions considered are (1) maximization of tax income, (2) minimization of travel time of trips generated by the new allocations, and (3) minimization of "conflict" between different land uses existing next to each other. Game theory is used to determine a solution that maximizes the "satisfaction" of each objective function. Constraints are set on total acreage by land use, ratio of one type of land use to another, soil characteristics, population densities, and political restrictions. The model was also used to evaluate the existing zoning pattern and a land use plan synthesizing individual comprehensive plans of communities within the county. Evaluation was done to test if constraints were met and to see how well the objective functions are satisfied.

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T-PM-6-X

D. B. BANDY, Standard Oil Company, Chicago, Illinois

A Comparison of Cycling Algorithms

A comparison is made of two cycling algorithms that have been implemented in mathematical programming systems in recent years. One cycling algorithm is based on specifying the number of potential candidates to be considered in a given major iteration (a potential candidate being defined as a vector whose d_j value is of the correct sign). The other cycling algorithm is based on specifying the number of placements of more favorable potential candidates into the pricing region, which leads to a probability distribution for the number of potential candiates considered in a given major iteration.

In this paper equations are derived for this probability distribution, as a function of cycling and pricing values. These equations are then used to generate a set of tables that characterize the probability distributions.

Comparisons are then made of the theoretical results with actual results using a large LP model, in which the order of the columns had been randomized, and with a very large transportation-type LP model.

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F-AM-3-X

J. A. BATTILEGA, Martin Marietta Corporation, Denver

Relaxed Benders' Decomposition and Generalized Fixed Charge Problems

A computationally efficient algorithm has been developed for obtaining approximate or exact solutions for a class of large scale generalized fixed charge problems. The algorithm is based on a relaxation of the Benders' Decomposition Procedure, combined with a linear mixed integer algorithm specifically designed to solve the MIP associated with Benders Decomposition and a computationally improved generalized upper bounding (GUB) algorithm which solves a convex separable programming problem by generalized linear programming. A dynamic partitioning technique is defined and used to tailor the entire procedure for the class of fixed charge problems. The research and implementation was directed toward the solution of a class of weapon allocation problems, and computational results for problems of this type are given. The procedure for solving the associated MIP is applicable to any problem for Bender form. Additionally, several computational improvements have been developed which are applicable to any GUB implementation.

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W-AM-2-X

M. S. BAZARAA, R. W. LANGLEY, Georgia Institute of Technology, Atlanta

An Infeasibility Pricing Algorithm for the Multicommodity Minimum Cost Flow Problem

In this study we develop a price-directive algorithm for solving the minimum cost multicommodity flow problem. The algorithm is a specialization of Balas' infeasibility pricing method. The subproblems are first solved and the infeasibility in the common resources is used to generate a new direction for the price vector. By making use of the network structure, the process of finding the new direction is simplified. Updating the solution of the subproblems involves few pivots, which are directly made on the network. The procedure is illustrated by a numerical example.

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M-AM-14-U

E. M. L. BEALE, Scientific Control Systems Ltd, London, England

P The Current Algorithmic Scope of Mathematical Programming Systems

The one common feature of all practical large-scale mathematical programming problems is sparseness. This applies whether the constraints are linear or nonlinear and whether the variables are continuous or discrete. The simplex method retains its central role in mathematical programming systems because in addition to being a device for changing the set of independent variables as appropriate it is now also an efficient method of exploiting sparseness. Supporting facilities that are now widely available are generalized upper bounds, branch and bound methods for integer variables and special ordered sets.

The paper reviews these facilities, and comments on the solution of large-scale nonlinear programming problems using general mathematical programming systems.

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M-PM-8-V

C. R. BECTOR, University of Manitoba, Winnipeg, Canada

Weaker Convex Programming

Concept of the class of weaker convex functions (both non-differentiable and differentiable) on an appropriately restricted convex set is introduced. It is shown that, (i) the class of non-differentiable weaker convex functions is included between the class of convex functions and the class of strictly quasi-convex functions, (ii) the class of differentiable weaker convex functions is included between the class of differentiable convex functions and the class of pseudo-convex functions. An interesting property of the class of weaker convex functions is that it is always closed with respect to non-negative addition of its elements--a property which in general does not hold for the class of strictly quasi-convex or pseudo-convex functions. This property of the present class of functions makes it possible to extend some of the results and computational techniques of convex programs to weaker convex programs without effecting significant changes. For Wolfe type duality, weak and direct duality theorems, which in general (as shown by Mangasarian) do not hold for a pseudo-convex program, are shown to hold for a weaker convex program.

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W-PM-11-Z

C. N. BEER, University of Oklahoma, Norman

A Procedure to Rank Bases by Probability of Being Optimal Using Imbedded Hyperspheres

For a class of multivariate density functions two methods for ranking bases of a probabalistic linear program according to their probability of being optimal are given. These methods do not require probability calculations. Conditions for which the method is exact are given.

One method is based on an imbedded hypersphere concept. It can be used in an approximate fashion for a wide range probability density function. An example is given.

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W-PM-11-Z

B. BEREANU, Academy of the Socialist Republic of Romania, Romania

* Stable Stochastic Linear Programs and Applications

A linear program with random coefficients is said stable if $\xi^n \xrightarrow{P} \xi$ implies $\gamma(\xi^n) \xrightarrow{P} \gamma(\xi)$, where ξ^n $(n = 1,2,\dots)$ is the vector of random coefficients, $\gamma(\xi^n)$ is the corresponding optimal value and $\xrightarrow{P}$ stands for convergence in probability.

A sufficient regularity condition for such stability is given, slightly stronger then the necessary and sufficient condition that a stochastic linear program have optimal value altogether. This condition is fulfilled by most linear programs important in applications and is essential in establishing the convergence of two numerical methods, proposed for computing the probability distribution function and moments of the optimal value of a stochastic linear program. One of the methods is based on an extension of the Polya-Steklov convergence theorem for quadratic formulas and the other is a Monte-Carlo type method.

Ingredients of computer programs and numerical illustrations are provided. The first method seems also fitting when alternative probability distributions of coefficients are considered, because the amount of computation for updating this distribution is relatively small.

The relation between the distribution problem and two-stage programming under uncertainty is also investigated.

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M-PM-12-Y

H. G. BERGENDORFF, C. R. BLITZER, H. K. KIM, International Bank for Reconstruction and Development, Washington

An Algorithm for Solving Linear-Quadratic Control Problems with Linear Inequality Constraints

The problem which the algorithm solves arose from attempts to formulate multi-sectoral planning models with nonlinear utility function in control theory format.

The transformation equations become linear and a second order Taylor series expansion of the utility function is used to give a quadratic objective function. The linear inequality constraint can now be included in the objective function by forming the Lagrangian. The so redefined problem turns out to be a quadratic programming problem with the same dimensions as the number of inequality constraints. Computational experience on a small example is reported.

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F-PM-5-V

D. P. BERTSEKAS, Stanford University, Stanford

Combined Primal-Dual and Penalty Methods for Constrained Minimization

In this paper we consider a class of combined primal-dual and penalty methods often called methods of multipliers. The analysis focuses mainly on the rate of convergence of these methods. It is shown that this rate is considerably more favorable than the corresponding rate for penalty function methods. Some efficient versions of multiplier methods are also considered whereby the intermediate unconstrained minimizations involved are approximate and only asymptotically exact. It is shown that such approximation schemes may lead to a substantial deterioration of the convergence rate, and a special approximation scheme is proposed which exhibits the same rate as the method with exact minimization.

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W-PM-5-V

M. J. BEST, K. RITTER, University of Waterloo, Canada

An Accelerated Conjugate Direction Method To Solve Linearly Constrained Minimization Problems

An iterative method is described for the minimization of a continuously differentiable function $F(x)$ of n variables subject to linear inequality constraints. Without any convexity or second order derivative assumptions it is shown that every cluster point of the sequence $\{x_j\}$ constructed by the method is a stationary point. The method constructs sets of directions which are approximately conjugate. At appropriate points a special step is performed which utilizes the second order information of the previous conjugate directions to accelerate the rate of convergence. If z is a cluster point of $\{x_j\}$ and $F(x)$ is twice continuously differentiable in some neighborhood of z and the Hessian matrix of $F(x)$ has certain properties then $\{x_j\}$ converges to z and the rate of convergence is $(n-p)$-step superlinear where p is the number of constraints which are active at z. Furthermore, if the Hessian matrix of $F(x)$ satisfies a Lipschitz condition in a neighborhood of z then as a result of the accelerated step, the rate of convergence of $\{x_j\}$ will be $(n-p+1)$-step cubic.

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W-PM-10-Y

L. J. BILLERA, R. E. BIXBY, Cornell University, Ithaca

* Characterizing Market Games Without Side Payments

The class of games without side payments obtainable from finite trader markets having possibly infinite dimensional commodity spaces, individual compact, convex consumption and production sets, and concave upper-semi-continuous utility functions is considered. It is shown that these market games are precisely the totally balanced games. In fact, each totally balanced game can be realized by both a finite commodity market with production and an infinite commodity market without production.

The latter approach requires as a first step the characterization of those polyhedral games which can arise from simple m-commodity markets without production in which each trader's consumption set is the unit m-cube.

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W-PM-10-Y

L. J. BILLERA, R. E. BIXBY, University of Kentucky, Lexington

A Characterization of Pareto Surfaces

Given n concave continuous functions u_i defined over the unit m-cube I^m, the corresponding attainable set V and Pareto surface P are defined. In the economic interpretation, V corresponds to the set of attainable utility outcomes realized through trading, and P the set of such outcomes for which no trader can attain more, without another getting less. Sets of the form of V and P are characterized among all subsets of R^n. The notion of complexity (the smallest m for which a given V can be realized) is briefly discussed, as is the idea of a "market game".

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T-PM-5-Z

G. E. BLAU, The Dow Chemical Company, Midland, Michigan

Model Building and Parameter Evaluation by Nonlinear Optimization with an Application to the Distribution of Chemicals in an Ecosystem

In modelling chemical reaction systems, it is necessary to have techniques for distinguishing among viable alternative models and identifying the parameters in these models. Suitable statistical criteria can be postulated for model rejection and date generation. Then optimization theory is necessary to optimize these criteria to locate successive designed experimental points in an iterative experimentation-analysis program. This paper will present the methodology of chemical reaction modelling and identify some optimization problems which must be resolved before the technique can reach its full potential in an industrial environment. The method will be demonstrated by building a model which describes the fate and distribution of an insecticide added to an ecosystem. The broad applicability of the methodology to general modelling problems will be readily apparent.

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TH-PM-8-X

P. T. BOGGS, J.E. DENNIS, JR., Rensselaer Polytechnic Institute, Troy, New York

An Analysis for the Discretized Analogue of Steepest Descent

The steepest descent method for minimizing functionals is analyzed in the case where the partial derivatives involved are approximated by difference quotients utilizing a constant mesh size u. In this case the method may not converge to a minimum, but rather generates a sequence of iterates which eventually become boundedly close to the true solution. The bound, obtained by the application of stability theory to the continuous analog differential equation of the steepest descent method, is shown to be a natural function of u and the condition of the problem. (The condition of the problem is expressed here as the condition number of the Hessian of the functional evaluated at the solution.)

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N-AM-1-W

V. J. BOWMAN, JR., J. STARR, Carnegie-Mellon University, Pittsburgh

Ordinal Cuts and Zero-One Programming

This paper presents a new method for solving the general zero-one integer programming problem. A partial ordering of integer solutions to the problem is induced by the objective function. This partial ordering is used to define a set Z of unordered solutions, one of which must be optimal. In fact, a solution is in Z if and only if it is optimal for some objective function yielding the same partial ordering. The problem can be solved, therefore, by finding the set Z.

It is shown that an element of Z can be easily constructed for any problem. Using this solution and information provided by the partial ordering an ordinal cut can be constructed. This cut is a necessary condition for any feasible solution to be unordered with respect to the one previously constructed.

The method of solution involves enumerating the elements of Z by successively constructing solutions, generating cuts, and adding the cuts to the problem. This method is a combination of enumeration and cutting-plane techniques, but has several properties that set it apart from both.

1) The enumeration process involves no complicated construction decisions.

2) Ordinal cuts are cuts on the partial ordering and do not involve linear programming pivoting operations.
3) The method can be used when the objective function is not completely specified but its induced ordering is known.

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W-PM-1-W

G. H. BRADLEY, P. L. HAMMER, L. WOLSEY, Yale University, New Haven

* Coefficient Reduction for Inequalities in 0-1 Variables

For a given inequality with 0-1 variables there are many other "equivalent" inequalities with exactly the same 0-1 feasible solutions. The set of all inequalities that are equivalent to a given inequality is characterized and methods to construct the equivalent inequality with smallest coefficients are described. Results are developed for a special class of inequalities called "self-dual". The results are of interest for integer programming algorithms where the amount of computation is a function of coefficient size (e.g., Gomory's asymptotic algorithm), for approaches where coefficient size is a major problem (e.g., reducing integer programs to knapsack problems), and for obtaining "strongest" or "deepest" cutting planes. Computational results show that the reduction in coefficient size can be significant.

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T-PM-1-W

E. R. BROCKLEHURST, National Physical Laboratory, Teddington, England

A Heuristic Algorithm for Integer Linear Programming Problems

An 'approximate solution' to the Integer Linear Programming Problem is one that is integer, satisfies the problem's constraints, and has an objective function value close to that of the optimal solution. A procedure which attempts to find approximate solutions in a small fraction of the time required to solve the associated continuous problem is described. It may be used either by itself, or to provide a starting point for 'optimal algorithms'. A companion paper presents a new algorithm, designed specifically to take advantage of an approximate solution, that may be used in conjunction with this procedure to solve integer linear programs. The extension of the method to the mixed integer linear programming problem is discussed, and computational results reported.

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M-PM-12-Y

J. T. BUCHANAN, University of Manchester, England

Path Restriction and Functional Representation in Dynamic Programming

Dynamic programming may be used on those 'conformable' constrained optimisation problems (e.g., linear programming) to change the problem from one of 'point' optimisation in a subset of the appropriate Euclidean space to one of 'path' optimisation in the corresponding 'path space', i.e., feasible solutions to the point optimisation problem correspond to feasible paths joining points in state space, the link between two points in state space at successive stages representing the transformation which results from a specific choice of the appropriate component of the decision variable vector.

Given an initial feasible solution the path restriction algorithm associates a neighborhood with each of the points in state space on the corresponding path. Using the dynamic programming recursive equation we then seek a feasible path from the resultant 'tube' with best path value. The process of associating neighbourhoods and finding most improved paths within the tubes is repeated using at each cycle a tube constructed around the path which gave the best path value in the previous cycle.

Conditions are given which are sufficient to guarantee convergence of this sequence of paths to an optimal path (corresponding to an optimal point solution of the constrained optimisation problem).

To obviate the necessity of calculating functional values over a set of grid points in the neighbourhood the notion of functional representation is introduced, e.g., if the functional equation is of the usual form $f_n^T(\underline{b}_{.n}) = \max_{x_n \in X_n^T(\underline{b}_{.n})} \{g_n(x_n) \oplus f_{n-1}^T(t_n(\underline{b}_{.n}, x_n))\}$ where suffix T means that only 'tube restricted' paths are being considered then we derive a representation of the form $f_n^T(\underline{b}_{.n}) = F_n(K_n, \underline{b}_{.n})$ for some appropriate index set K_n and function F_n. The representation and neighbourhood N_n over which it is exact is derived recursively from the representation and appropriate neighbourhood at the previous stage. Starting with a representation at the first (or zeroth) stage we can then recursively generate all the necessary representations. The procedure in fact gives us optimal (tube restricted) decision rules for the 'improved' decision variable value at each stage and these rules mean that we need neither calculate nor store values over grid points since we now have analytic expressions for both state values and optimal decision rules.

Detailed results are presented for the linear programming problem and in this case the problem reduces to a particularly simple form. Applications to other constrained optimisation problems are shown and the computational implication of the scheme discussed.

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F-PM-3-X

C-A. BURDET, R. BREU, Carnegie-Mellon University, Pittsburgh

Experimental Results in Large Scale Integer Programming: Some Branching and Bounding Strategies

A series of numerical experiments has been conducted to determine the ability of a mixed-integer branch and bound code to solve all integer problems with many zero-one variables. Many of the recently proposed ideas in integer programming are tested and compared from a point of view of overall efficiency and their potential use in speeding up the search; among them are, Gomory cuts and related branching and bounding criteria, pseudo-cost, minimal preferred inequalities, convexity, intersection (including outer-polar) and diamond cuts, polaroids.

The study deals with large problems (more than 50 zero-one variables) essentially illustrating how some unsolved problems of the literature were solved.

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M-PM-6-Z

R. E. BURKARD, Universität Graz, Austria

A Perturbation Method for Solving Quadratic Assignment Problems

We consider the general quadratic assignment problem: Find a permutation matrix $X = (x_{ij})$ which minimizes the sum $\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{p=1}^{n} \sum_{q=1}^{n} d_{ijpq} x_{ij} x_{pq}$. It will be shown that by specially arranging the coefficients d_{ijpq} in a matrix D the cost coefficients in the main diagonal of D can be interpreted as cost coefficients of a linear assignment problem whereas the other coefficients form the perturbations. Now a reduction of the matrix D is considered which leaves the objective function invariant and minimizes the perturbations. Thereafter the unperturbed (linear) problem is solved and upper and lower bounds for the objective function of the quadratic problem are derived. A branch and bound procedure yields optimal (or if wanted: suboptimal) solutions. The necessary storage space is minimized too. Numerous numerical test problems show the efficiency of the proposed method.

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M-PM-12-Y

J. L. BURROUGHS, J. L. GETSCHMAN, Computer Sciences Corporation, Falls Church, Virginia

Sequencing Targets of a MIRV Missile With A Discrete Dynamic Programming Algorithm

The MIRV-bus propellant that is required to execute MIRV-deployment maneuvers depends on the order in which the MIRV's are separated from the MIRV-bus. The dynamic programming algorithm presented in this paper determines a sequence of maneuvers which require a minimum amount of propellant to deploy MIRV's to a given set of targets. The algorithm determines if there are any feasible target sequences; and if there are, the algorithm determines at least one feasible sequence and the propellant required to execute that sequence. The algorithm is effective for solving problems in which there are from 4 to 14 targets to be hit by the MIRV's. Impulsive maneuvers and constant flight path angles are assumed.

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TH-PM-7-Y

R. CARVAJAL, A. V. LEVY, Ciudad Universitaria, Mexico

Measures of a Cone

Four measures of the "width" of a convex cone are presented. They are based on the angle of the smallest spherical cone containing the cone, and the largest spherical cone contained within a cone. The relationship between the four measures are given. It is shown that the measures can be determined using the principal pivot method.

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TH-PM-7-Y

A. TAMIR, R. CHANDRASEKARAN, Case Western Reserve University, Cleveland

* On the Nonlinear Complementarity Problem

The problem focused on is the following nonlinear complementarity problem.

Given a mapping $F(x)$ from the n-dimensional Euclidean space R^n into itself such that $F(0) = 0$, and a vector $q \in R^n$, find $x \geq 0$, $y \geq 0$, $y = F(x) + q$ and $y'x = 0$. The problem is feasible if there exists $x \geq 0$ and $y = F(x) + q \geq 0$. $F(x)$ is a Q-function if the complementarity problem has a solution for each $q \in R^n$ and it is a P-function if there is a unique solution for each $q \in R^n$. If for each $q \in R^n$ feasibility of the problem implies the existence of a complementary solution, then $F(x)$ is a K-function.

Sufficient conditions which guarantee that a function belongs to one of these classes are given. In some cases the proofs introduced are constructive and can be used as algorithms to find a complementary solution. In particular, a nonlinear generalization of Z-matrices (i.e., $A = (a_{ij})$ and $a_{ij} \leq 0$, $\forall\, i \neq j$), and an algorithm that finds a complementary solution or indicates that none exists for a function in this class, are presented. Under some assumptions this algorithm can be viewed as a pivoting scheme.

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TH-AM-8-X

R. W. CHANEY, Chalmers University of Technology and University of Goteborg, Sweden

A Modified Conjugate-Gradient Method for the Minimization of Exterior Penalty Functions

We shall consider the nonlinear programming problem in which the constraints are equations. We wish to consider the solution of such problems by the use of classical exterior penalty functions. When exterior penalty functions are employed, the resulting unconstrained problems have a rather special character. This special character is determined by the eigenvalues of the Hessian of the penalty function, and it becomes more pronounced as the penalty coefficient is increased. In particular, as the penalty coefficient is increased, solution by steepest descent encounters problems of ill-conditioning and the rate of convergence slows badly. We present here a method for minimizing the exterior penalty function which takes into account and exploits the special structure of the function. Our study is theoretical, although there is some related experimental evidence that is encouraging. The main virtues of the method presented here seem to be these: No information about second derivatives is required and no large square matrix must be stored; the rate of convergence is linear (at least in a certain asymptotic sense) and the rate is "nearly" independent of the size of the penalty coefficient.

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W-PM-10-Y

* A. CHARNES, University of Texas, Austin

Coalitional and Chance-Constrained Solutions to n-Person Games

New concepts of solution are put forward for n-person games in which the values of coalitions are defined in an implicit or in a probabilistic fashion. The properties of these as extensions of the core-stem or union results (Charnes, Littlechild and Sorensen) and of the convex nucleus notions (Charnes, Kortanek and Keane) are developed.

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T-AM-9-Z

To-yat CHEUNG, Carleton University, Ottawa, Canada

Approximate Solution for Systems of Nonlinear Volterra Integral Equations

For a system of nonlinear Volterra integral equations satisfying some monotonicity property, a constrained minimization problem (CMP) is formulated. Approximate solutions and error bounds of the integral equations can be obtained by solving this CMP, using numerical approximation techniques and linear programming methods. The CMP is an iterative process. Each iterative cycle consists of two steps. Starting with some guessed approximation, Step 1 determines a better approximate solution by solving a linear program whose constraints are obtained by quasilinearizing the given integral equations. Step 2 is also a linear program, using the defects of the approximate solutions obtained in Step 1 to form the coefficients of the linear constraints. The product of the minimal solutions of Step 2 and the defects obtained in Step 1 form the error bounds. In this sense, the approximate solution is obtained by minimizing the error bound. Step 2 then feeds back some results to Step 1 to be used as the coefficients of the objective function during the next cycle.

Numerical results are included.

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W-AM-13-Y

M. CHRISTEN, Regie Autonome des Transports Parisiens, Paris, France

The Derivation of Train Shunting Tables for the Paris Metro

The following unit maintenance for the Paris-Metro is required: each unit must be washed at least once in every three days. Since the cleaning equipment is in a siding area, only those units shunted into the sidings during the middle part of the day can be cleaned.

Thus, the problem to be solved is how to draw up a shunting time-table setting out a method for selecting or modifying the length of each train so that the necessary servicing can be carried out.

This approach uses integer linear programming and concentrates on the minimisation of the maneuvering cost.

The result obtained ensures that the units are washed every three days but excludes the possibility of finding a solution that does not involve some maneuvering of units.

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W-PM-2-X

N. CHRISTOFIDES, P. BROOKER, Imperial College of Science and Technology, London, England

Optimal Expansion of an Existing Network

Given an existing network, a list of arcs which could be added to the network, the arc costs and capacities and an available budget, the problem considered in this paper is one of choosing which arcs to add to the network in order to maximise the maximum flow from a source s to a sink t, subject to the budgetary constraint. This problem appears in a large number of practical situations which arise in connection with the expansion of electricity or gas supply, telephone, road or rail networks.

The paper describes an efficient tree-search algorithm using bounds calculated by a dynamic programming procedure which are very effective in limiting the solution space explicitly searched. Computational results for a number of medium sized problems are described and computing times are seen to be very reasonable.

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TH-AM-2-W

V. CHVATAL, P.L. HAMMER, University of Montreal, Montreal, Canada

Reducing Packing and Covering Problems to a Knapsack Problem

A good characterization of the class of set-packing problems reducible to a knapsack problem (involving the same set of variables and having the same 0-1 solution space) is given. The characterization is accompanied by a polynomial-time algorithm which, for each set-packing problem, either produces an equivalent knapsack problem or shows that there is none. The latter is accomplished by demonstrating that the associated Boolean function is

2-summable (and therefore not linearly separable). The reducibility of set-covering problems to a knapsack problem--generalizing the above--is also examined.

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F-AM-13-Y

C. COHEN, M. REAGAN, J. STEIN, J. YOZALLINAS, Northwestern University, Evanston, Illinois

Design of an Optimization System for University Research and Teaching

An optimization system for university use is quite different from MPSX or OPTIMA. This paper deals with the design and development of an optimization system for Northwestern University's CDC 6400. The library of mathematical programming algorithms has evolved in the following stages: a) the "stand-alone" library--which includes over twenty linear, integer and non-linear programming algorithms. b) the MP system--developed to permit the user to state his problem in English and standard mathematical programming notation. The system, coded in FORTRAN, consists of a language pre-processor and individual overlays corresponding to the various solution algorithms. c) the interactive system--is designed to be used interactively at a teletype. It has editing and tutorial capabilities as well as options to interrupt computation. The advantages of such a system are numerous: facilitate the learning of basic principles (e.g., pivoting, cuts), maximize the utility in real time between the user and the computational process, and permit the user to interpose decisions based on his experience and intuition.

Our design objectives have been to stress ease of use, reliable algorithms, and expandability. Usage statistics for the system permit us to better plan for future software development or improvement and interactive optimization has prompted us to study mixed-strategy solutions.

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F-PM-5-Y

A. R. CONN, T. PIETRZYKOWSKI, University of Waterloo, Canada

A Penalty Function Method Converging Directly to Constrained Optimum

Consider the following non-linear optimization problem: find a local minimum x_0 of the function f on the set $F = \{x \in R^n | \varphi_i(x) \geq 0, i = 1,\ldots, K\}$. A penalty function p is defined as follows: $p(x) = \mu f(x) - \sum_{i=1}^{K} \min(0, \varphi_i(x)), x \in R^n$. Under some natural assumptions p has an unconstrained minimum that coincides with x_0. This function is, however, not everywhere differentiable, even for differentiable f and φ_i's, so the standard unconstrained minimization methods are not directly applicable.

The major result of this paper is a method that constructs a single sequence which minimizes p and converges directly to x_0. This is the first result of this type that uses penalty function techniques. The method is a continuation of earlier results which were, however, only approximate in nature.

The paper also provides a proof of convergence of the method along with some very encouraging numerical results.

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W-PM-10-Y

W. D. COOK, C. A. FIELD, M. J. L. KIRBY, York University, Downsview, Canada

Infinite Linear Programming in Games with Partial Information

An area of considerable recent research interest has involved the extension and modification of the basic model for two-person zero-sum game theory. One particular type of extension found in the literature involves the introduction of risk and uncertainty into the model by allowing the $m \times n$ payoff matrix $A = (a_{ij})$ to be a discrete random matrix which can assume a finite set of values. This paper considers both one and two-person games and investigates the situation in which A is a discrete random matrix which can assume a countably infinite set of values $A(1), A(2), \ldots, A(k), \ldots$ with probabilities $p(1), p(2), \ldots, p(k), \ldots$ respectively. It is assumed in addition that the players possess only partial information involving $P = p(1), p(2), \ldots, p(k), \ldots$ in the form of a system of linear inequalities constraining P. In case this system of inequalities is finite, the game problems which players 1 and 2 must solve to determine their optimal mixed strategies can be transformed into dual semi-infinite programming problems. A minimax theorem and some properties of optimal mixed strategies for the players are established. For the case involving an infinite system of inequalities on P, the game problems can be transformed into dual infinite programs and minimax results analogous to those of the semi-infinite case are proved. In addition, a number of extensions of some results due to Haar and Carathéodory are established.

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E-PM-11-Z

L. COOPER, I. N. KATZ, Southern Methodist University, Dallas

Aspects of Stochastic Location-Allocation Models

Problems are considered involving optimal location of sources and simultaneous allocation of sinks to those sources, in which the sinks are no longer deterministically given but are random variables with given probability distributions.

Results are presented including mathematical characterization of the problem, convergence proofs for computational algorithms and computational results for various distributions.

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F-PM-4-U

G. B. DANTZIG, Stanford University, Stanford

P On Systems Optimization Laboratories

From its very inception around 1947, it was envisioned that linear programming would be applied to very large, detailed models of economic and logistical systems. In the intervening 25 or so years, electronic computers have become increasingly more powerful, permitting general techniques for solving linear programs to be applied to larger problems. However, the world today is faced with a number of crises in energy, population, urban decay, conservation of resources, pollution, destruction of the ecosystem--that call for the development of very large-scale system models.

Solving large-scale systems cannot be approached piecemeal or by publishing a few theoretical papers. It is a complex art requiring the development of a whole arsenal of special tools. It is my belief that now is the time to develop one or more system optimization laboratories where methods for solving representative very large models could be experimentally tested.

The activities of such laboratories could be: (1) Collection and study of typical systems optimization problems arising in government, science, and industry. (2) Collection of, the development of, and the comparison of computer routines, on various test problems. (3) Serving as a clearance house where one can obtain information about techniques, test results, test problems and software.

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F-AM-13-Y

T. K. KUMAR, J. M. DAVID, Florida Technological University, Orlando

Optimum Utilization of Resources in a University Under Uncertain Alternatives

The use of mathematical programming for the efficient utilization of university resources has been demonstrated by Fox, David, Kumar, Sanyal, Sengupta and others. The solutions of these problems depends upon the specification of the three main components of the models: the preference function of the university administrators; the input-output relations; and the amounts of resources available. The determination of the preference function, unlike that of the two other components, has so far not been given its due attention. This paper presents an operational procedure to determine the university administrators' future preference function from their revealed preferences. We also examine the nature of the future resource vector. Specifically, we look into the factors that affect resource availabilities through a regression model of past performance, and ascertain the probability of future predictions on the basis of the distribution of the regression disturbances.

The determination of the preference function and the stochastic programming solutions are illustrated through a numerical example relating to the allocation of resources in a specific university system.

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F-AM-3-X

R. DEMBO, University of Waterloo, Canada

Modular Design by Decomposition

Modular design is a standardization procedure that can be formulated as a nonlinear integer programming problem with a very special structure. An especially interesting feature of the problem is that by fixing certain variables at a constant value, the problem reduces to a linear integer program, having a block-diagonal form with no coupling constraints. An extension of the Benders decomposition algorithm by Geoffrion provides a framework within which this property of modular design can be successfully utilised. The algorithm when applied to modular design requires iteration between two integer linear programming problems; the block-diagonal sub-problem and the master problem. When the integer restriction on the sub-problem is relaxed, convergence of the algorithm to a mixed integer solution can be proved. Owing to the existence of a duality gap however, in general this method will only yield upper and lower bounds on the optimal all integer solution. The method is discussed in detail and a numerical example is solved.

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F-AM-8-V

M. A. H. DEMPSTER, University of Oxford, England

A Minimal Kuhn-Tucker Theorem in Convex Spaces

This paper defines a minimal condition called local regularity under which the Kuhn-Tucker first order necessary conditions (in terms of Gateaux derivatives) hold at a local optimum of a non-linear constrained programme defined in locally convex Hausdorff vector spaces. It is shown that local regularity is sufficient in such spaces to ensure the Kuhn-Tucker conditions hold. In Banach spaces it is necessary for the condition to hold for the Kuhn-Tucker conditions to be valid. Unlike the minimal condition on the constraint function recently studied by Gould and Tolle, local regularity is a minimal condition on both constraint and objective function, i.e., on the given programme, which considers the fundamental set of the programme in a manner suggested by work of Gale and Walkup-Wets. Some applications to problems in the calculus of variations and optimal control are indicated.

Dear Prof Dennis

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TH-PM-5-V

J. E. DENNIS, JR., J. J. MORÉ, Cornell University, Ithaca

* A Characterization of Superlinear Convergence and its Application to Quasi-Newton Methods

Let F be a mapping from real n-dimensional Euclidean space into itself. Most practical algorithms for finding a zero of F are of the form $x_{k+1} = x_k - B_k^{-1} F x_k$ where $\{B_k\}$ is a sequence of non-singular matrices. The main result of this paper is a characterization theorem for the superlinear convergence to a zero of F of sequences of the above form. This result is then used to give a unified treatment of the results on the superlinear convergence of the Davidon-Fletcher-Powell method obtained by Powell for the case in which exact line searches are used, and by Broyden, Dennis, and Moré for the case without line searches. As a by-product, several results on the asymptotic behavior of the sequence $\{B_k\}$ are obtained.

An interesting aspect of these results is that superlinear convergence is obtained without any consistency conditions; i.e., without requiring that the sequence $\{B_k\}$ converge to the Jacobian matrix of F at the zero. In fact, a modification of an example due to Powell shows that all the known quasi-Newton methods are not, in general, consistent. Finally, it is pointed out that the above-mentioned characterization theorem applies to other single and double rank quasi-Newton methods, and that the results of this paper can be used to obtain their superlinear convergence.

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W-PM-1-W

D. McDANIEL, M. DEVINE, University of Oklahoma, Norman

Alternative Relaxation Schemes for Benders' Partitioning Approach to Mixed Integer Programming

As applied to mixed-integer programming, Benders' original work made two primary contributions: (1) development of a "pure integer" problem (Problem P) that is equivalent to the original mixed-integer problem, and (2) a "relaxation" algorithm for solving Problem P that works iteratively on a LP problem and a "pure integer" problem. In this paper we examine several alternative relaxation schemes for solving Problem P, that appear to offer substantial computational savings over Benders' original algorithm. These alternative approaches are explained geometrically and computational results are given.

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F-AM-3-X

M. A. DIAMOND, Universidade Federal do Rio de Janeiro, Brasil

The Solution of a Quadratic Programming Problem Using Fast Methods to Solve Systems of Linear Equations

Let A be a real symmetric positive definite $n \times n$ matrix and b a real column n-vector. The problem of finding n-vectors x and y such that $Ax = b + y$, $x^T y = 0$, $x \geq 0$ and $y \geq 0$ occurs when the method of Christopherson is used to solve free boundary problems for journal bearings. In this case A is a "finite difference" matrix. We present a direct method for solving the above problem by solving a number of linear systems $A_k x = b_k$. Each system can be solved using one of the recently developed fast direct or iterative procedures.

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TH-AM-4-V Mr.

J. C. DICKSON, Bonner and Moore Associates, Inc., Houston, Texas

* On Keeping Both Storage and I/O Requirements Low in Linear Programming

In preparing a recent LP system the core and I/O requirements were kept low by using Kalan's (Naive Linear Programming) technique for recording the matrix and inverse together with the Forrest-Tomlin update to maintain a triangular factor for the basis matrix. A scaling algorithm that did not degrade the effectiveness of Kalan's packing scheme was developed and results from tests on the effect of scaling on the number of unique coefficients is presented. Subsequent experience using the Forrest-Tomlin updating technique supports previous reports of 90% reduction in the growth of non-zero elements in the inverse file.

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M-PM-12-Y

J. J. DINKEL, Pennsylvania State University, University Park

Dynamic and Geometric Programming

The theory of generalized geometric programming is used as a framework to examine a class of discrete deterministic dynamic programming problems. The linear transition functions associated with this class of problems are embedded in the vector space structure of geometric programming. As a result of this transformation the primal program exhibits a separability which allows the computation of the dual geometric program associated with this class of dynamic programs. The duality theory of generalized geometric programming is then applied to this primal-dual pair to exhibit a duality theory for dynamic programs. These results are shown to extend and generalize an earlier application of the "maximum transform" to certain resource allocation problems.

A recursive decomposition of such programs into a sequence of geometric programs is developed and related to the usual recursive techniques of dynamic programming.

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F-PM-1-Y

J. DORAN, S. POWELL, SRC Atlas Computer Laboratory, Didcot, England

Some Relationships between Heuristic Search over Directed Graphs, Branch-and-Bound Methods, and Integer Programming

In recent years substantial progress has been made in the development of a coherent theory of branch-and-bound methods especially as such methods are used to solve problems in integer linear programming. In parallel artificial intelligence researchers have been developing mathematical formulations of heuristic search especially over directed graphs. Although these two areas of investigation have much in common they have rarely been juxtaposed. In this paper the relationship between them is discussed and a unified theoretical framework sufficient to encompass them both presented. Topics considered in detail are problem formulation, the derivation and use of branching functions, and the derivation and use of heuristic estimates and bounds.

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W-PM-2-X

I. DRAGAN, IASI, Romania

A Primal Algorithm for Solving the Minimum-Cost Flow Problem in a Network with Gains

Let R be a network with gains defined by: a finite connected graph $G = (\mathcal{N},\mathcal{A})$, without loops, a demand function $d(x)$, $(\forall x \in \mathcal{N})$, a capacity function $r(a) \geq 0$, $(\forall a \in \mathcal{A})$, and a gain function $g(a) \geq 0$, $(\forall a \in \mathcal{A})$. Let $c(a) \geq 0$, $(\forall a \in \mathcal{A})$, be a cost function defined on the network. Then, we consider the following problem: minimize $C(f) = \sum_{a\in\mathcal{A}} c(a).f(a)$ subject to

$$\sum_{a\in A(x)} f(a) - \sum_{a\in B(x)} g(a).f(a) = d(x) , \quad (\forall x \in \mathcal{N}), (\forall a \in \mathcal{A}).$$
$$0 \leq f(a) \leq r(a)$$

One proves that a flow is optimal, if and only if each supplemental flow shipped along a certain kind of non-absorbing subnetwork gives a non-negative supplemental cost. This result provides a primal algorithm for solving the problem, when an initial flow is available; a labelling procedure is used for finding a feasible non-absorbing subnetwork with negative cost. The algorithm is a natural extension of the algorithm given by V. V. Menon for solving the ordinary minimum-cost flow problem.

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T-PM-1-W

R. R. TRIPPI, S. I. DROBNEIS, California State University, San Diego

A Branch and Bound Algorithm for Optimal Replacement with Fixed-Charge Investment Costs

A facility replacement problem in which investment costs exhibit scale economies given by a fixed-charge, linear relationship is discussed. A model of this problem, in which operating costs are explicitly considered along with investment costs, is formulated as an integer program which is similar in structure to that of a facility location problem in which relative costs change over time. Aging in this model is assumed to increase operating costs in some predictable fashion and hence, in conjunction with possible technological improvements of an embodied nature, provides a motivation for eventual replacement of production units. A branch and bound solution procedure applicable to this class of problems is developed.

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T-AM-2-X

E. C. DUESING, State University of New York, Brockport

Determining the Convex Hull of a Finite Set of Points

Given a finite set S of points in R^n, it is of interest to determine the convex hull conv(S) of the collection. Wets and Witzgall [1] discuss a procedure for determining the set of vertices S_V of conv(S), and conjecture that "efficient determination of the facets requires prior determination of the lower dimensional faces." This paper disproves the conjecture of Wets and Witzgall by exhibiting an algorithm, based on the simplex algorithm, which allows efficient computation of bounding hyperplanes containing facets of conv(S). The primal linear programming problems are formulated so as to determine the endpoints of a line segment contained in conv(S). The associated dual problems have as solutions the coefficients of the equations of bounding hyperplanes. The basis for the optimal program determines candidates for the vertices generating the facet. The efficiency of the algorithm results from two exchange theorems, which allow neighborly facets (i.e., those having n-1 vertices in common) to be determined by inspection of the solutions (facets containing this vertex) are then derived. A new vertex is then selected, and additional facets determined until all points have been found to be vertices or boundary and interior points and all facets have been cataloged.

References: [1] Wets, R.J.B. and C. Witzgall, "Algorithms for Frames and Lineality Spaces of Cones," Journal of Research of the National Bureau of Standards, Vol. 71B (1967), pp. 1-7. [2] Lemke, C.E., "The Dual Method of Solving the Linear Programming Problem," Naval Research Logistics Quarterly, Vol. 1 (1954), pp. 36-47.

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T-AM-9-Z

R. J. DUFFIN, Carnegie-Mellon University, Pittsburgh

* On Fourier's Analysis of Linear Inequality Systems

Fourier treated a system of linear inequalities by a method of elimination of variables. This method can be used to derive the duality theory of linear programming. Perhaps this furnishes the quickest proof both for finite and infinite linear programs. For numerical evaluation of a linear program, Fourier's procedure is very cumbersome because a variable is eliminated by adding all pairs of inequalities having coefficients of opposite sign. This introduces many redundant inequalities. However, modifications are possible which reduce the number of redundant inequalities generated. With these modifications the method of Fourier becomes a practical computational algorithm for a class of parametric linear programs.

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TH-PM-7-Y

B. C. EAVES, Stanford University, Stanford

* Solving Systems of Convex Equations

An algorithm is described for solving the fixed point problem $y = \max_{\delta} P_{\delta} y + R_{\delta}$ under new conditions. The algorithm resembles Lemke's complementary pivoting scheme for quadratic programming and bimatrix games, Howard's policy improvement scheme for dynamic programming, and Newton's method for solving systems of equations.

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M-PM-6-Z

J. G. ECKER, R. D. NIEMI, Rensselaer Polytechnic Institute, Troy, New York

A Dual Algorithm for Quadratically-Constrained Quadratic Programs via a Modified Penalty Function Technique

In this paper an algorithm is developed for minimizing a convex quadratic function of several variables subject to inequality constraints on the same type of function. The duality theory of extended geometric programming yields a concave maximization problem with constraints which are essentially linear. If the primal constraints are not all active, however, the dual objective function is seen to be nondifferentiable at the optimal point as well as at other points in the dual constraint region. This difficulty is circumvented by considering a sequence of dual programs via a modified penalty function technique which does not eliminate the dual constraints but does insure that they will all be active at optimality. The algorithm has been implemented in a Fortran program and numerical results will be included.

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TH-AM-2-W Pub soon Jack

J. EDMONDS, University of Waterloo, Ontario, Canada

* Facets of Combinatorial Polyhedra

Many combinatorial problems can be formulated as: maximize cx over $x \in H$, where H is a set of vectors $x = \{x_j : j \in E\}$ determined by some combinatorial description D. Such problems may be called loco for "linear objective combinatorial". Usually there exists a finite set L of linear-inequality constraints having as its set of solutions a polyhedron $P \supseteq H$ such that $\max\{cx : x \in H\} = \max\{cx : x \in P\}$ for any c for which either maximum exists. Where the affine rank of H equals $|E|+1$, the minimal such L is unique, and its members are called the facets of P. It is generally difficult to discover a very good description of an L, and even more difficult to discover a very good description of the minimal L, relative to the description D of H. The talk will survey some instances of success, and of failure, and discuss the relationship of such descriptions to solving the associated loco problems. In particular, it will briefly review some old results on L's of matching, node-packing, branching, matroid, and master-group, loco problems, and outline some interesting new results in these areas by W. Pulleyblank, V. Chvatal, R. Giles, and J. Aroaz.

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T-AM-1-W

K. B. HALEY, A. N. ELSHAFEI, University of Birmingham, England

On Solving the Capacitated Facilities Location Problem With Concave Cost Functions

The problem of locating central facilities on a network has recently received considerable interest. In this paper a search procedure which is designed for problems in which there are economies of scale and capacity restrictions on the various sites is given. For the static case, several algorithms were derived to handle two versions of the problem, i.e., when the concave cost function is replaced by (a) many linear segments and (b) one linear segment. For the dynamic case, the interaction of many economic, accounting and technological factors affects the problem formulation and hence the solution approach. The current paper is limited to the static problem. The procedure is also applicable to the Simple Plant Location Problem. One of the algorithms developed for this problem is briefly described and computational experience given to enable comparisons with the capacitated problem experience to be made. The algorithms were tested on over 1300 randomly generated problems and many practical problems both described in the literature and facing several British organisations. The computational experience is also reported.

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TH-AM-5-Y

J. ELZINGA, T. MOORE, The Johns Hopkins University, Baltimore

The Central Cutting Plane Algorithm

We consider a general formulation of the convex programming problem, that of maximizing a linear function over a compact convex set defined by differentiable functions. An algorithm is developed which iteratively proceeds to the optimum by successively constructing a cutting plane through the center of a polyhedral approximation to the optimum. The centers of the polyhedral approximations are obtained by solving a linear program in the manner of Nemhauser and Widhelm. The algorithm generates a sequence of feasible points whose convergence properties are established. We also show the existence of sequences whose limits are the optimal Kuhn-Tucker multipliers. Since successive subproblems are generated by the addition of one constraint to the linear programming subproblem, the computational effort required in each iteration is small (directional searches are not required). We develop a simple, easily implemented, yet powerful rule for dropping prior cuts, thus keeping small the size of the linear programming subproblems.

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M-PM-13-X

S. W. EMERY, JR., Stanford University, Stanford

Preliminary Deep Ocean Bulk Carrier Design by Geometric Programming

The optimal design parameters for deep sea dry bulk carriers are sought by means of geometric programming. An analytical model is constructed based upon architectural, operational, legal, and geographic restrictions and relationships. Parametric cost functions for construction and operation of bulkers are included as well as logistic requirements of the route(s) and commodities in question.

The problem is to design one or a fleet of dry bulk carriers that will operate within the navigational limitations of the given route and that will minimize some objective criterion such as total discounted present value. The model so constructed is phrased as a constrained optimization problem which may be viewed as a large-scale geometric program. The solution method to the geometric program is that of a converging sequence of approximating linear programs formed by the cutting plane technique.

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T-PM-5-Z

M. FLORIAN, J. FERLAND, L. NASTANSKY, G. GUÉRIN, G. BUSHELL, University of Montreal, Canada

The Engine Scheduling Problem in a Railway Network

In this paper we present the mathematical version of a scheduling problem that is faced by any railway company that employs several engine types to provide power for its trains. Usually, a railway employs engines of several types that differ in their tractive effort capability and horsepower rating. There are two distinct but related aims in reducing the engine costs to the railway. One is to select the mix of engine types that gives the lowest capital investment and operating cost for the trains operated by the railway. The other is to provide a scheduling method that assigns available engines to trains on a short time horizon (e.g., a week ahead).

In the following we shall give a mathematical formulation of the problem of selecting the mix of engine types that gives the lowest capital investment and operating cost for the trains operated by the railway and outline a method of solution based on the decomposition method of Benders.

In the problem we are treating, the use of Benders decomposition seems to be the only attractive solution method, since the application of a Mixed Integer Programming algorithm in not practical. For a real life problem with 200 to 300 trains and 5 engine types, the number of integer variables is of the order of 1000 to 1500, a number which is well beyond the capabilities of presently available integer codes.

* * *

TH-AM-5-Y

Dear John:

E. M. L. BEALE, J. J. H. FORREST, Scientific Control Systems Ltd, London

* Global Optimization Using Special Ordered Sets

The task of finding global optima to general classes of nonconvex optimization problem is attracting increasing attention. McCormick (1972) points out that many such problems can conveniently be expressed in separable form, when they can be tackled by the special methods of Falk and Soland (1969) or Soland (1971), or by Special Ordered Sets. Special Ordered Sets, introduced by Beale and Tomlin (1970), have lived up to their early promise of being useful for a wide range of practical problems. Forrest, Hirst and Tomlin (1973) show how they have benefited from the vast improvements in branch-and-bound integer programming capabilities over the last few years, as a result of being incorporated in a general mathematical programming system.

Nevertheless, Special Ordered Sets in their original form require that any continuous functions arising in the problem be approximated by piecewise linear functions at the start of the analysis. The motivation for the new work described in this paper is the relaxation of this requirement, by allowing automatic interpolation of additional relevant points in the course of the analysis. This is similar to an interpolation scheme as used in separable programming, but its incorporation in a branch-and-bound method for global optimization is not entirely straightforward. Two by-products of the work are of interest: one is an improved branching strategy for general special-ordered-set problems. The other is a method for finding a global minimum of a function of a scalar variable in a finite interval, assuming that one can calculate function values and first derivatives, and also bounds on the second derivative within any subinterval.

The paper describes these methods, their implementation in the UMPIRE system, and preliminary computational experience.

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W-PM-5-V

W. FORSTER, The University, Southampton, England

On Constrained Nonlinear Optimization Problems with the Fixed Point Property

We consider a nonlinear objective function subject to inequality and equality constraints. We stipulate that the objective function satisfies an additional condition, namely, that it has a fixed point under a certain linear transformation. This additional condition is satisfied for a wide class of applications. It will be shown that under the above conditions the problem can either be solved: (i) by direct methods (e.g., dynamic programming, etc.), or, more economically, (ii) by employing numerical schemes in conservation law form (e.g., a generalized Lax-Wendroff method), and that the two methods are equivalent.

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T-AM-6-Y

S. FROMOVITZ, C. KIM, R. HAUSMAN, University of Maryland, College Park

An Algorithm for Nonlinear Curve Fitting

A new nongradient algorithm for minimizing a sum of squared, nonlinear functions is proposed and computationally tested. Our work extends previously published methods by Powell and Spendley and is specifically designed to overcome certain problems inherent in these methods.

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TH-AM-14-U

D. R. FULKERSON, Cornell University, Ithaca

P Results on Blocking Pairs of Matrices

Some aspects of the theory of blocking pairs of nonnegative matrices (blocking pairs of polyhedra) will be reviewed, and certain classes of examples where the blocking matrix of a given matrix in the class has been determined will be discussed, e.g., (1) the blocking matrix of the incidence matrix of all spanning arborescences with fixed root in a directed graph, and (2) the blocking matrix of the matrix of all integer-valued supply-demand flows in a capacity constrained network, where supplies, demands, and capacities are integers. The latter class includes, for example, the determination of the blocking matrix of the incidence matrix of the family of all (0,1)-matrices having prescribed row and column sums.

* * *

T-AM-14-U

D. GALE, University of California, Berkeley

P Interest Rates and Efficient Production Programs

A production activity is a pair (x,y) of non-negative n-vectors called respectively the input and output of the activity. A technology T is a set of activities. In the applications to follow, T will be assumed to be convex. A program π consists of an initial vector y_0 and a sequence of activities (x_t, y_{t+1}) in T, $t = 0,1,\ldots$. A steady state with growth factor γ, written $(x,y;\gamma)$, is a program such that $(x_t, y_{t+1}) = \gamma^t(x,y)$ for some (x,y) in T. We suppose there is given an objective n-vector c and we call the program π efficient if there is no other program π' such that $c\cdot(y_t' - x_t') > c\cdot(y_t - x_t)$ for all t. Our problem is to determine when a given steady state $(x,y;\gamma)$ is efficient.

THEOREM I. If c is positive and there exists $\rho > \gamma$ such that $c\cdot(y-\rho x) \geq c\cdot(y'-\rho x')$ for all x', y' in T then $(x,y;\gamma)$ is efficient.

THEOREM II. If $(x,y,;\gamma)$ is efficient then there exists $\rho \geq \gamma$ such that $c\cdot(y-\rho x) \geq c\cdot(y'-\rho x')$ for all (x',y') in T. The number ρ is called an interest factor and $\rho-1$ an interest rate. We will show by examples how simple interest rate calculations can be used to solve rather complicated problems concerning efficiency.

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M-PM-6-Z

G. GALLO, A. ULKUCU, University of California, Berkeley

Bilinear Programming: A Vertex Following Algorithm

The "Bilinear Programming Problem" can be stated in the following general formulation

$$\text{(1)}\qquad \begin{array}{rl} \text{Minimize} & (c^t x + d^t y + x^y Q y) \\ \text{Subject to} & Ax \leq a \\ & By \leq b \\ & x \geq 0,\ y \geq 0, \end{array}$$

where A, B and Q are real matrices; a, b, c and d are real vectors; x and y are real variable vectors. An interesting application of Bilinear Programming arises from investment problems, in which components of x correspond to activity levels, while components of y denote the investments needed to change the technologies by which the activities are performed (and, as a consequence, the cost of such activities). T. M. Ridley ("An investment policy to reduce the travel time in a transportation network", Transpn. Res. 1968), for instance, considers a particular problem of this kind. To be able to solve the problem he imposes some restrictions, and gives a combinatorial algorithm.

H. Konno ("Bilinear Programming", Tech. Rep. Stanford University, Aug. 1971) mentions some other examples of "real world" problems that can be formulated as Bilinear Programs. He proposes an algorithm which is based on the cutting plane idea of K. Ritter, and generates an ϵ-optimal solution in a finite number of steps.

In this paper, by means of a dual formulation of (1), we derive an equivalent problem

$$\begin{aligned} &\text{Minimize} \quad d^t u \\ &\text{Subject to} \quad u \in \Lambda \,, \end{aligned} \tag{2}$$

where d and u are real vectors, and Λ is a simply connected union of a set of faces in a polytope. Although Λ is not necessarily convex, using its special structure, we develop a vertex following algorithm for problem (2). This algorithm converges in a finite number of steps to a global optimum.

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T-PM-1-W

L. F. ESCUDERO, E. GARBAYO, IBM Madrid Scientific Center, Spain

The Cutting Stock Problem: Application of Combinatorial Technique and Mixed Integer Programming

The purpose of this paper is to describe some execution trim techniques, analyzing a general purpose trim algorithm called TRIMLAJ. The basic characteristics of the TRIMLAJ are the following:

1. TRIMLAJ is a specially designed model for use in optimal one and two dimensional cutting.
2. The model becomes analytic (as opposed to heuristic) as soon as all the longitudinal cuttings have been already made for each of the patterns. Therefore what is obtained is not only a solution but the optimal one.

3. The mathematical aspect of TRIMLAJ relies upon combinatorial techniques and mixed integer programming.
4. The optimization problem may be of three different types:
   a) Minimization of TRIM waste from the raw material used.
   b) Maximization of PROFIT, depending upon the planning of the order ticket.
   c) Minimization of TRIM percentage wasted.
5. The most important conditions that this model takes into account are:
   A) Main general conditions for the "order ticket": 1. Minimum length of a cutting pattern; 2. Maximum number of permissible patterns in the order ticket.
   B) Main conditions for each different bobbin of raw material: 1. Minimum required margin; 2. Quantity (in meters or kilograms) available of each type of raw material bobbin; 3. Maximum quantity (in meters or kilograms) to be cut from each type of raw material bobbin for the whole set of its patterns; this maximum has to be either the total available quantity within that bobbin or a significantly lower bound ensuring that the remainder of the bobbin will not have to left as waste.
   C) Main conditions for each order: 1. Quantity (in kilograms or meters) requested; 2. Gross or net unit selling price (by the kilogram or meter) of each order; 3. Maximum number of patterns in which each order can be included.

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TH-AM-5-Y

U. M. GARCIA-PALOMARES, O. L. MANGASARIAN, Simon Bolivar University, Caracas

Superlinearly Convergent Quasi-Newton Algorithms for Nonlinearly Constrained Problems

We propose algorithms for nonlinearly constrained optimization problems which have the novel idea of using updating schemes to estimate the Hessian of the Lagrangian associated with the objective and constraint functions. Under suitable conditions, a local superlinear rate of convergence is established. The subproblems of the algorithms consist of a sequence of linearly constrained quadratic minimization problems, for which efficient methods of solution are available. This feature makes the algorithms computationally tractable. Numerical experience is very encouraging. Although in establishing the rates of convergence it is assumed that second derivatives of the functions involved are Lipschitz continuous,

fast convergence has also been obtained in a number of test problems where the functions involved are not even differentiable once. If, instead of updating the Hessian of the Lagrangian, second derivatives are explicitly evaluated in the subproblems, a local quadratic rate of convergence is obtained.

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W-PM-11-Z

S. J. GARSTKA, University of Chicago, Chicago

Misspecification of Distributions in Stochastic Programming Problems

The following variational problem is studied: Let $\varphi(u,\Sigma)$ denote the optimum value of a two-stage stochastic program with recourse as a function of the mean μ and covariance matrix Σ. Under what conditions is φ concave and Lipschitz? If μ_0 is the true mean and Σ_0 the true covariance matrix and $\varphi(u_0,\Sigma_0)$ is dualizable, solvable and stable, is φ dualizable, solvable and stable in some neighborhood of (u_0,Σ_0)? What inferences can be drawn about needs for estimation accuracy for different classes of distributions? What are the implications for programs whose second stage decision is a function of the random variable as well as the first stage decision?

* * *

T-PM-6-Y

S. GAUNT, National Research Council of Canada, Ottawa

Computational Experience with the Parallel Theory for Matrices and Network Systems

Reduction in computer time to less than 30 seconds, for resolving cyclic problems ranging to $N = 57$, is potentially evident.

The proposed algorithm, based on the Parallel Theory concept, equates column vectors of a matrix to equivalent electrical parallel resistors. By combining a partitioning technique with the developed algorithm significant time reductions, for resolving large scale matrix problems, appear feasible.

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TH-PM-5-Z

K. R. GEHNER, Western Electric Engineering Research Center, Princeton, N.J.

The Structure of Feasible Direction Algorithms

The general structure of feasible direction algorithms for solving minimize $F(x)$ over $x \in S$, where S is a subset of R^n, as developed by Topkis and Veinott is used to reexamine the problem of zigzagging or jamming in feasible direction algorithms. It is shown that jamming occurs only if the first local minimum in the chosen direction is not attained within the feasible region for infinitely many interactions. This leads us to consider a feasible direction algorithm which

(1) chooses a feasible direction which makes uniform progress in improving the function value, i.e., there is a $\delta > 0$ such that for all d^k $F(x^{k-1} + sd^k) < F(x^{k-1}) - s\delta$ for all $0 \leq s \leq \delta$ and

(2) chooses a step length in the chosen direction to guarantee uniform movement towards the minimum along this direction.

Conditions for convergence for both constrained and unconstrained problems are given. Special well-known examples of the general algorithm are also considered.

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F-AM-14-U

A. M. GEOFFRION, University of California, Los Angeles

P Integer Programming and Facility Location: Lessons on One I've Learned From the Other

Attempting to apply integer programming techniques to facility location problems has taught me some rather unexpected lessons having to do with the art of modeling, the design and implementation of specialized software, and the proper use of integer programming for solving real problems. This presentation will describe some of these lessons and reflect on their general significance.

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TH-PM-8-X

P. M. GHARE, W. C. TURNER, Virginia Polytechnic Institute and State University, Blacksburg

A Computational Comparison of Incomplete Relaxation-Multistep Algorithms for Unconstrained Optimization with Other Search Methods

In recent papers we have proposed the use of Incomplete Relaxation-Multistep algorithms for unconstrained optimization. In most cases the arguments have been that convergence is good and the methods are easily understood and applied. "Good" convergence properties are difficult to define so some comparison of performance with more famous or popular methods is in order.

This paper takes some of the more popular methods for unconstrained optimization and compares their performance on chosen test functions. The methods used for comparison are "Conjugate Gradients" discussed by Hestenes and Stiefel and others. Quasi-Newton algorithms, such as "Deflected Gradients" discussed by Fletcher and Powell, and "Parallel Tangents" discussed by Shah, Buehler and Kempthorne. Each of these is used to solve three test functions all starting at the same point. These performances are then compared with those of a set of Incomplete Relaxation-Multistep algorithms on the same functions.

The functions chosen range in complexity from a simple three variable convex problem to a complicated non-convex four variable problem. Two of these have been widely used in the literature. They are Rosenbrock's and Wood's functions.

Results indicate that the Incomplete Relaxation-Multistep Methods are very competitive with the more popular algorithms. Ranking of methods, of course, is virtually impossible but the performances show that these methods do deserve further consideration.

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M-PM-1-W

P. L. GHILARDOTTI, ENEL--Direzione Studi e Ricerche--Milano, Italy

Geoffrion's 0-1 Programming Method Revisited; An Extension to Integer Programming

The Balas 0-1 implicit enumeration method and Geoffrion's improvements are considered in order to take advantage for integer programming of this enumeration technique and of the use of surrogate constraints. A straight-forward extension, suggested by Geoffrion, of binary-infeasibility and conditional binary-infeasibility tests is analysed and applied to constraints with bounded integer-valued variables. Binary algorithms for 0-1 expansion of integer problems are demonstrated to be less efficient than the integer enumerative approach we present here. In addition, imbedded linear programming for the generation of surrogate constraints seems to be closer to integer-structured problems. Computational experience is reported.

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M-PM-6-Z

F. GIANNESSI, E. TOMASIN, University of Pisa, Italy

Global Optimization in Nonconvex Quadratic Programming and Integer Programs

The general nonconvex quadratic programming problem is considered:
$P{:}\min\ \varphi(x) = c^T x + \frac{1}{2} x^T Dx,\ x \in \{x{:}Ax \geq b;\ x \geq 0\}$.

A short review of the up-to-date, proposed methods is considered.

Because of the importance of problem P and as the existing methods are either inefficient or not finite, a new algorithm is proposed in this paper. Its main features are the following.

The complementarity problem Q (equivalent to P) is considered, which consists in minimizing the linear form $\frac{1}{2}(c^T x + b^T y)$, (x,y) satisfying the Kuhn-Tucker system.

Denote by Q_0 the problem obtained by deleting in Q the complementarity constraints; an ϵ-perturbation device is introduced to ensure that Q_0 always has finite optimal solutions.

In general, assume that a linear programming problem Q_r is given (at $r = 0$, this is true). An optimal solution of Q_r is found. If it satisfies the complementarity conditions, then its x-components are an optimal solution of P too. In the contrary case, an inequality (cutting plane) is determined, which is weakly satisfied by all the adjacent vertices of the optimal solution of Q_r, and such that the problem Q_{r+1}, obtained by adding to Q_r the above inequality, has the same basic solutions of Q_0, but the optimal one (if this is degenerate, the number of its adjacent vertices may be greater than the number of nonbasic variables; this case requires a lot of additional work, as we could have to determine more than one inequality as cutting plane).

The preceding algorithm is finite, as the vertices of the feasible region Q_{r+1} are those of the feasible region of Q_r, but one; and as the set of optimal solutions of Q is a face of the feabile region of Q_0.

A facial decomposition of the feasible region is used to obtain an implicit enumeration algorithm, which has the preceding one as a sub-algorithm.

The particular case of $\varphi(x)$ concave is of much interest. In fact, an integer linear programming can be formulated equivalent as a problem P, where $\varphi(x)$ is concave.

The algorithm proposed to solve Q enables us to solve a more general problem: to find, among the vertices of a convex polyhedron, those which satisfy a given condition, i.e., a complementarity condition, or an integer condition. Thus, it can be used (suitably specialized) to solve (0,1)-linear programs directly, i.e., without transforming them into a problem like Q (via a concave problem P).

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T-PM-6-Y

P. E. GILL, W. MURRAY, National Physical Laboratory, Teddington, England

* The Simplex Method Using the LQ Factorization in Product Form

At the Hague symposium an implementation of the simplex algorithm was described utilizing the LQ factorization of the basis. The algorithm was so arranged that the matrix Q was not required, hence need not be stored or updated. Subsequently, M. A. Saunders (1972) has described a product form of this algorithm based on storing L as the product of "special" lower triangular matrices. In this paper we describe a method that stores both L and Q in product form.

The main objection to retaining Q was the expense of updating it. Almost all the work involved was in multiplying out the product form and this is now no longer necessary. A number of advantages occur from having Q available; for instance, it is no longer necessary to store the current (and past) basis.

References

Gill, P.E. and W. Murray (1970). "A numerically stable form of the simplex algorithm," National Physical Laboratory, Math 87.

Saunders, M.A. (1972). "Product form of the Cholesky factorization for large scale linear programming," Computer Science Report CS 301, Stanford University, Stanford, California.

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F-AM-8-V

C. R. GLASSEY, University of California, Berkeley

* Explicit Duality for Convex Homogeneous Programs

When all the functions that define a convex program are positively homogeneous and nonnegative (or polyhedral) then a dual program can be constructed which is also a convex program, and is defined in terms of the primal data only (the primal variables do not appear). Furthermore, the dualizing process, carried out on the dual program, yields the primal. Several well-known examples of convex programs with explicit duals are shown to be special cases, but application of this theory to quadratic programs gives a new dual form.

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W-PM-2-X

F. GLOVER, D. KLINGMAN, J. STUTZ, University of Colorado, Denver

* New Basis Updating Procedures for Generalized Networks

Procedures are given for efficiently updating the basis representation, flows and node potentials in an adjacent extreme point (or "simplex" type) method for generalized network problems. These procedures employ a special representation that reduces 17 cases of reconstituting the basic "quasi-trees" to one, and are compatible with a variety of list structures.

* * *

Dear Jean-Louis:

T-AM-2-X

J.-L. GOFFIN, Ecole des Hautes Etudes Commerciales de Montreal, Canada

On the Finite Convergence of the Relaxation Method for Solving Systems of Linear Inequalities

The main concern of my work is to rejuvenate the relaxation method for solving systems of linear inequalities, which uses as primitive the notion of hyperplanes, instead of the more derived concept--computationally more intricate--of vertices or bases, as in the simplex method.

The relaxation method constructs a sequence of points $\{x^q\}$ in the following manner: at step q we find the constraint most violated by the point x^q of the sequence, and move on the interior normal of the half-space H^{iq} corresponding to this constraint by a distance which is a given multiple λ--the relaxation parameter--of the distance between x^q and H^{iq}.

My main result is that the range of values of the relaxation parameter which guarantees finite convergence is far wider than was previously known. The smooth enough property for polyhedra is defined, and it delineates a class of problems where the method works particularly well.

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W-PM-5-V

D. GOLDFARB, City University of New York, New York

Numerically Stable Variable Metric Methods for Linearly Constrained Optimization Problems

Two different numerically stable ways of implementing variable metric methods for problems with linear inequality and equality constraints are presented.

Let $G = LL'$ be a positive definite approximation to the Hessian matrix of the objective function given in factorized form where L is lower triangular and let $\bar{N}'Q = \bar{N}'[Q_1|Q_2] = [R'|0]$ where $\bar{N} = L^{-1}N$, N is the active constraint matrix, and Q and R are orthogonal and upper triangular matrices respectively.

The first implementation is based upon writing the projection $P = I - G^{-1}N(N'G^{-1}N)^{-1}N'$ as $(L')^{-1}Q_2Q_2'L^{-1}$ and computing the vector of Lagrange multipliers $u = (N'G^{-1}N)^{-1}N'G^{-1}g$ by solving $Ru = Q_1'L^{-1}g$, where g is the gradient of the objective function. The second approach is based upon using a factorized approximation $\bar{Q}_2'G\bar{Q}_2 = \bar{L}\bar{L}'$ to the Hessian matrix with respect to variations in the active constraint manifold, and the orthogonal triangularization, $N'\bar{Q} = N'[\bar{Q}_1|\bar{Q}_2] = [\bar{R}'|0]$, where $\bar{L}$, $\bar{R}$ and $\bar{Q}$ are lower triangular, upper triangular and orthogonal matrices, respectively. In this case the vector of Lagrange multipliers $u = (N'N)^{-1}N'g$ is obtained by solving $\bar{R}'\bar{R}u = N'g$. Formulas are given for modifying L, R, and Q, ($\bar{L}$, $\bar{R}$, and $\bar{Q}$), when a variable metric update is performed and when a basis change occurs.

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BANQUET SPEAKER: Thursday evening

R. E. GOMORY, IBM Watson Research Center, Yorktown Heights, New York

P Models and Technology

Technology is related to the ability to model aspects of the real world whether mathematically or otherwise. Some trends in this areas will be discussed.

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M-PM-1-W

A. S. GONCALVES, University of Coimbra, Portugal and University of Alberta, Edmonton, Canada

An Explicit Solution for the Integer Linear Programming Problem

An explicit (noniterative) representation of the set of optimal solutions for both pure and mixed integer linear programming problems is presented. The process turns to be also an efficient way of solving this class of programs.

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W-PM-1-W

M. GONDRAN, Électricité de France, Clamart, France

An Efficient Cutting-Plane Algorithm by "The Method of Decreasing Congruences"

In the case of the "pure" integer linear programming problems, the author gives a cutting-plane algorithm founded on the two following ideas:

1) The integrity conditions of the basic variables of the continuous solution are used to generate a sequence of cuts. After pivoting on this small number of cuts, we obtain a dual-feasible integer solution of the augmented problem.
2) This sequence of cuts is computed by "the method of decreasing congruences" (Gomory, Gondran).

These two ideas permit us to improve notably "the accelerated Euclidean Algorithm" of Martin. Numerical experiments are given for the generalized set covering type problems.

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TH-PM-2-W

S. GORENSTEIN, IBM Watson Research Center, Yorktown Heights, New York

Data Migration in a Distributed Data Base

This paper presents a network model for the optimal (least cost) data migration in a distributed data base. For a sequence of requests for a record from various locations in a computer network, a shortest path algorithm determines the optimal sequence of locations of the data set to which the record belongs. Among the costs considered are: transmitting the requested record (keeping the data set where it is); migrating the data set; duplicating the data set.

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TH-PM-7-Y

M. L. FISHER, F. J. GOULD, University of Chicago, Chicago, Illinois

* An Algorithm for the Nonlinear Complementarity Problem

In recent years there has been rapidly developing a body of knowledge concerning the application of a complementary pivoting algorithm to a broad spectrum of problems in optimization theory. The applications include linear complementarity, fixed point approximation, convex programming with subgradients and unconstrained optimization without derivatives. The present work adds to this information in the form of a combinatorial algorithm for the nonlinear complementarity problem. The algorithm is based on a triangulation of the non-negative orthant and a special labeling of the vertices. Derivatives are not required. Convergence is proved under coercive-like assumptions on the problem functions. Acceleration results are given and computational experience with various problems, primarily from the area of convex programming, is presented.

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F-PM-3-X

G. W. GRAVES, R. D. McBRIDE, University of California, Los Angeles

* Factorization in Large-Scale Linear Programming

A new approach for solving large-scale linear programming problems is developed. The approach involves factoring (partitioning) an LP tableau in a way that permits a large portion of the tableau to be carried implicitly and generated from the remaining explicit part. The benefits of this approach are two-fold:

1. A large reduction in high speed memory requirements is obtained.
2. The work per pivot is reduced.

Factorization can be either dynamic or static. The special structure of some of the LP constraints is utilized in the static case. This includes GUB, block diagonal, and network constraints. The classic GUB LP algorithm and its extension to block diagonal LP problems with coupling constraints are recovered as special cases.

In the dynamic case the size of the explicit part floats from pivot to pivot. As a direct extension of GUB, a dynamic factorization algorithm has been implemented to solve the set partitioning LP problem. Good computational results have been obtained with this algorithm.

A particularly successful factorization has been obtained for transportation and network problems. Here the whole principle part of the LP tableau is carried implicitly and reconstructed logically as needed to carry out execution of the algorithm. Excellent computational results have been obtained. For example, 100 × 100 dense transportation problems have been solved with a median time of 1.5 seconds on an IBM 360/91.

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F-AM-3-X

M. D. GRIGORIADIS, W. W. WHITE, IBM Corporation, New York

* A Framework for the Experimental Study of Partitioning Methods for Structured Linear Programs

Studying the computational characteristics of a proposed partitioning method for large scale structured linear programs requires a major systems design and programming effort. Resources for this type of effort have seldom been available to the experimenter. An approach, requiring minimal user programming, is proposed for performing this experimentation on commercially available large LP codes and systems.

Most partitioning methods known to the authors, correspond to particular computational strategies of the primal, dual or primal/dual simplex methods. The pivoting tasks required to perform an exchange of variables are common among partitioning methods and allow a unified classification. These methods further differ by the extent of their reliance upon three distinct classes of such pivot steps whose unit computational cost varies according to the assumed constraint matrix structure and data processing implementation.

The computational strategy of the applicable simplex method corresponding to a particular partitioning method is specified to the LP code and the overall large-scale problem is solved to obtain the number of pivot steps required for an optimal solution. These pivot steps are easily identified with the three pivoting classes employed by such a partitioning method. Having thus established an experimental rate of convergence, the total computational effort attributed to this partitioning method is estimated or bounded by a computational complexity analysis of each pivot step class for a particular matrix structure and for a particular proposed data processing implementation.

Examples of analyzing partitioning methods within a large-scale LP code and some experimental results will be presented.

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W-PM-1-W

M. M. GUIGNARD, IBM Philadelphia Scientific Center, Philadelphia and Wharton School, University of Pennsylvania

Systematic Combination of Inequalities in 0-1 Programming

In branch and bound schemes as well as in enumerative procedures it is important to get good bounds for the objective function as well as information about the constraints. One may often obtain relatively strong inequalities for the objective function by combining its expression in terms of the non-basic variables of a given linear programming tableau with suitably chosen derived constraints of especially simple form. Such constraints have been proposed by the author (constraintes additionnel les en variables bivalentes) and others (e.g., minimal preferred inequalities). The objective function inequalities, in turn, can be utilized to construct penalty tables for non-basic and basic variables.

The constraints are usually derived from single inequalities of the given system and fail to take the overall structure into account. This deficiency can be alleviated by a preprocessing of the system, in which certain linear combinations of the inequalities are generated. Two approaches have been used, one based upon the solution of linear systems of congruences (via the Smith normal form), the other being an attempt at eliminating variables by a process of diagonalization.

The above procedures havebeen programmed in APL and have been successfully incorporated within a state enumeration algorithm.

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T-PM-6-Y

R. K. BRAYTON, F. G. GUSTAVSON, E. L. JOHNSON, IBM Watson Research Center, Yorktown Heights, New York

LU Update of the GUB Simplex Algorithm

The "LU Update" version of Forrest and Tomlin of the simplex algorithm has proved to be of considerable importance to large-scale linear programming, and so has "GUB" (Generalized Upper Bounding); this paper shows how to incorporate both these features at one time into the simplex algorithm.

Our version does not alter the eta files for L and U. In fact, the only subsequent operations performed once an eta is formed are deletions or setting an element to zero. A small amount of additional incore boolean information, proportional to the number of key variable swaps and the number of GUB variables associated with the swaps is required. This information, the S matrices, is used with the U part of the eta file to form dyadic correction matrices which are applied at strategic times during the FTRAN and BTRAN iterations. In the ordinary update one skipped over the modified vectors of the U eta file; here, one applies the dyadic correction matrices.

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T-AM-13-V

P. HANSEN, M. PICAVET, Institut d'Economie Scientifique et de Gestion,
Lille, France

Capacity Expansion with Storable Output

Capacity expansion problems have been much studied recently, mostly by Manne, his co-workers and followers. According to Sengupta and Fox, "the major limitation of the capacity expansion models is the assumption of nonstorable output". Only one paper by Tapiero, using optimal control theory, does, to our knowledge, relax this assumption.

We consider a capacity expansion problem with a single area and a deterministic increasing demand for a single product which must be satisfied. Plants are to be constructed to satisfy this demand and the construction cost is subject to economies of scale. Storage of output is allowed. Let us call a policy, a sequence of decisions characterizing the sizes and times of construction of the plants and the level of inventory at all times. An optimal policy is a policy which minimizes the sum of discounted construction and inventory holding costs over an infinite future. In this paper we concentrate on policies for which the level of inventory is zero each time a new plant begins production. Necessary conditions for optimality are derived in the case of an arbitrary increasing time path of demand both when temporary imports are and are not allowed. These results are specialized to the case of linear growth of demand, in which a constant cycle time policy is optimal, generalizing results of Manne and Erlenkotter. Computational results for the case of the aluminum industry in India, based on these authors' data are obtained by a direct search algorithm and compared to those they obtained for the nonstorable output case.

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T-PM-8-V

W. A. FARR, M. A. HANSON, Florida State University, Tallahassee

Continuous-Time Programming

A class of continuous-time nonlinear programming problems relating to problems of acquisition, stockpiling, and distribution of materials will be described. Nonlinearity appears in both the objective function and the constraints. Time delays are also provided for. Necessary and sufficient conditions (Kuhn-Tucker conditions) are established and optimal solutions are characterized in terms of duality theorems.

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T-AM-9-Z

G. HATFIELD, Naval Personnel and Training Research Laboratory, San Diego

The Theory and Application of Linear Decision Programming

The investigation of mathematical forms that generalize the ordinary linear programming problem has led to the identification of a problem which we call the decision programming canonical form. The study of this canonical form indicates the possibility of unifying certain theories and methods of decision-making, i.e., (1) linear programming, (2) vector maximization, (3) goal programming, (4) two person zero-sum games, (5) the Chebyshev approximation problem, and (6) satisficing. It is shown that solving a certain linear decision programming problem under certainty is equivalent to solving a linear vector minimization problem for an efficient point. Also it is shown that a two person, zero-sum game is equivalent to a linear decision programming problem under risk where the payoff matrix is the set of goals. Satisficing follows directly from the canonical form by considering inequality goals. A general algorithm, called the minimum distance method, is developed for a class of decision programming problems.

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F-AM-3-X

R. F. HAUCK, United States Steel Corporation, Pittsburgh

The POLYPLEX Method: An All-Primal Extreme-Point Decomposition Method for Large-Scale Linear Programs of All Structures

This paper develops from first principles a new decomposition method for large-scale linear programming. The coefficient matrix need not have any special structure in order to be decomposed. This matrix is first partitioned horizontally into T partitions. T subproblems are then defined in terms of each partition which decomposes the problem into $\underline{T}$ separate parts. If any partition possesses a special structure it is further partitioned dynamically into two subpartitions. The basis-inverse of the subproblem is bi-factored and represented in its most compact form. The method is the same for all special structures, no variants or extensions are required. Bounded variables are handled implicitly with relative ease in every instance. The subproblem solutions are extreme-point solutions and the related solution to the entire problem is also an extreme-point solution, at every step. The algorithm is a primal one operating on primal problems and converges at the same rate as the simplex method.

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TH-AM-4-V

C. A. HAVERLY, Haverly Systems Inc., Denville, New Jersey

* New MAGEN/PDS

A new MaGen matrix and report generator system has been developed. It builds on the successful concepts of the older MaGen but incorporates extensive enhancements. The language has been broadened with many new features, including data base access, extensive file handling, etc. New compiler concepts were developed for the implementation to provide ease of use, versatility and fast execution. The use is oriented toward all types of mathematical programming models and their reports and provides for combining mathematical programming with other OR techniques such as simulation.

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M-PM-6-Z

D. HEARN, W. D. RANDOLPH, University of Florida, Gainesville

Dual Approaches to Quadratically Constrained Quadratic Programming

For a convex quadratically constrained quadratic program with n variables and m constraints it is shown that the Wolfe dual objective can be reduced to a concave function in the m dual variables. The only constraints are nonnegativity of the dual variables. Peterson and Ecker, and Rockafellar, have derived duals with $(n+1)(m+1)$ variables and n linear constraints plus $m+1$ non-negativity restrictions. It is shown that the Peterson-Ecker dual can be derived from the Wolfe dual. Comparisons are made between these duals from the viewpoint of computational efficiences.

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Dear Eli

T-PM-6-Y

E. HELLERMAN, D. RARICK, Bureau of the Census, Washington

* <u>The Partitioned Preassigned Pivot Procedure (P^4) for Sparse Matrix Inversion</u>

An algorithm which reorders the rows and columns of a sparse matrix so as to reveal a structure and pivot sequence that is amenable to fast matrix inversion is presented. Exposition is made via a sample problem. The basic technique involves the following steps: 1. column singleton assignments; 2. row singleton assignments; 3. tentative assignments via Ford-Fulkerson maximal assignment procedure; 4. partitions via predecessor-successor relationships; 5. spike selection; 6. lower-upper decomposition; 7. appending the tail. Some computational results are presented.

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M-PM-13-X

G. T. HERMAN, State University of New York at Buffalo, Amherst, New York

<u>A Relaxation Method for Reconstructing Objects from Noisy X-rays</u>

An algorithm is proposed for estimating the density distribution in a cross section of an object from X-ray data, which in practice is unavoidably noisy. The data gives rise to a large sparse system of overdetermined inconsistent equations, not untypically 10^5 equations with 10^4 unknowns, with only about 1% of the coefficients non-zero.

Using the physical interpretation of the equations, each equality can in principle be replaced by a pair of inequalities, giving us the limits within which we believe the sum on the left hand side must lie.

An algorithm is proposed for solving this set of inequalities. The algorithm is basically a relaxation method. A finite convergence result is proved.

In spite of the large size of the system, in the application area of interest practical solution on a computer is possible because of the simple geometry of the problem and the redundancy of equations obtained from nearby X-rays.

The algorithm has been implemented, and is demonstrated by actual reconstructions.

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T-PM-1-W

E. HEURGON, J. DELORME, Regie Autonome des Transports Parisiens, Paris, France

Set Covering Problems by Linear Programming and Branch and Bound Algorithm

The problem is to resolve more than 3000 times a year integer linear programs involving several thousands booleans variables subject to several hundred constraints of exact covering with a few exceptions.

It is resolved in two steps:

STEP 1. application of an algorithm of linear programming. If the solution (X^0) obtained is integer, the problem P is wholly resolved; if not the value f^0 of the economic function in X^0 is a lower bound of the optimal and integer solution.

STEP 2. resolution of the whole problem by means of a procedure of implicit enumeration inspired by that of Garfinkel and Nemhauser. The essential differences between the two are the choice of criteria employed in the construction of the hierarchies and the computation of the evaluation function. We have tried thus to employ to the maximum the results obtained from the continuous solution of the linear problem, in particular the variables which happen to have the value of 1 and the marginal costs of the variables.

In the last part of the paper, an interesting reduction of the problem is proposed.

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T-PM-8-V

R. J. HILLESTAD, University of California, Los Angeles

A Useful Characterization for Nonconvex Feasible Regions Defined by Concave Constraints

This paper presents a characterization of the solutions to the nonconvex problem

$$\min cx$$
$$\text{subject to: } g(x) \leq 0,$$

where g is a vector of concave functions.

First, necessary and sufficient conditions for local solutions in terms of a linearized problem are provided. Next, the concept of a "basic solution" to the above problem is introduced and the important properties of these solutions are given. These include the facts that the number of basic solutions is finite, that the extreme points of the feasible region are basic solutions, that one only need consider basic solutions in searching for the global optimum, and that the convex hull of the feasible region is polyhedral. Finally a finite method of finding all basic solutions through linear programming is presented for the sub-class of the above problem in which only one constraint is strictly concave. Extensions of this algorithm to the general case are discussed.

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TH-AM-5-Y

R. L. STAHA, D. M. HIMMELBLAU, University of Texas, Austin

Constrained Optimization Via Moving Exterior Truncations

A new algorithm has been developed to solve the general nonlinear programming problem

$$\begin{aligned} \text{minimize} \quad & f(\underline{x}),\ \underline{x} \in E^n \\ \text{subject to} \quad & h_i(\underline{x}) = 0,\ i = 1,\ldots, m \\ & g_i(\underline{x}) \geq 0,\ i = m+1,\ \ldots\ ,\ p \end{aligned}$$

where $f(\underline{x})$ is the objective function, $h_i(\underline{x})$ is an equality constraint, and $g_i(\underline{x})$ is an inequality constraint.

The new algorithm transforms the constrained problem into a series of unconstrained problems with the penalty function

$$P(\underline{x},t) = \text{Min}\{0,\ [t-f(\underline{x})]\}^2 + \sum_{i=1}^{m} h_i^2(\underline{x}) + \sum_{i=m+1}^{p} \text{Min}[0,\ g_i(\underline{x})]^2$$

where $p(\underline{x},t)$ is the penalty function and t is a truncation level. The procedure to utilize this penalty function is to minimize $P(\underline{x},t)$ for a monotonic increasing sequence of truncation levels converging to the value of the objective function at the constrained minimum. The sequence of unconstrained minima thus obtained approach the constrained minimum from outside the feasible domain.

A new technique for generating the sequence of truncation levels was developed which resulted in a faster rate of convergence for most problems than previous methods utilizing the same penalty function. The new algorithm is guaranteed to converge for convex programming problems with a nonempty, compact constraint set if the problem functions have continuous first-order partial derivatives in E^n.

The method of Fletcher was used for the unconstrained minimizations of the penalty function. Fletcher's method is a variable metric method modified to eliminate unidimensional searches. Fletcher's algorithm was modified to accept difference approximations for derivatives so that analytical derivatives need not be furnished in order to use the new algorithm.

The effectiveness of the proposed algorithm was determined by solving a number of nonlinear programming test problems and comparing the results with those of existing nonlinear programming codes. The conclusion was reached that the new algorithm is a very effective technique for solving the general nonlinear programming problem.

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F-PM-3-X

J. HO, A. S. MANNE, Stanford University, Stanford

Nested Decomposition for Dynamic Models

This nested decomposition algorithm is intended for solving linear programs with the staircase structure that is characteristic of dynamic multi-sector models for economic development. Staircase problems represent a special case of the discrete-time optimal control problem. Our method is based upon the same principles as that of Glassey (1971), but appears easier to describe and to relate to control theory.

Computational experience is reported for a series of test problems. The algorithm has been coded in MPL, an experimental language for mathematical programming. This translator has made it possible to obtain a more readable program--and with fewer instructions--than one written in a conventional language. However, because the present version of MPL does not permit the use of slow access memory, this has prevented us from exploring the full potential of nested decomposition for solving larger problems than can be handled by conventional simplex techniques.

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TH-AM-2-W

A. J. HOFFMAN, IBM Watson Research Center, Yorktown Heights, New York

* On Combinatorial Problems and Linear Inequalities

The following topics will be treated: (a) Balanced matrices in the sense of Berge ((0,1) matrices with no odd order submatrices having all row and column sums 2), for which we shall provide alternative proofs and generalizations of some of Berge's results (this is joint work with Rosa Oppenheim); (b) Generalizations of Dilworth's theorem and the max-flow min-cut theorem from the viewpoint of linear programming.

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TH-PM-2-W

W. W. HOGAN, United States Air Force Academy, Colorado

Single Commodity Network Problems with Resource Constraints

Single commodity minimal cost network flow problems with joint resource constraints on some arc flows are approached via decomposition schemes which attempt to exploit the efficiencies of network algorithms. A price directive approach results from the application of Dantzig-Wolfe decomposition in a manner analogous but not equivalent to proposed methods for the resource constrained multi-commodity case. A resource directive approach is also developed through a problem manipulation and an application of a Benders' type decomposition with tangential approximation using simplifications which arise from the special structure of the network formulation.

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W-PM-10-Y

J. T. HOWSON, Jr., Boston College, Chestnut Hill, Massachusetts

A Note on the Relationship Between Nondegeneracy and Some Properties of Nash Equilibria of Noncooperative Games

Nondegeneracy, as the term is understood for linear programming and related areas, is shown to be related to solvability and stability, defined in terms of Nash equilibria, for (finite) noncooperative games. The connection is made through the expression of the definition of an equilibrium in terms of a complementarity problem for which the notion of nondegeneracy arises naturally in the course of constructing solutions. For a nondegenerate problem, solvability of a game is equivalent to the game's possessing a unique equilibrium, and "strong" solvability is equivalent to this unique equilibrium consisting entirely of pure strategies. If the complementarity problem is linear, as it is for bimatrix and polymatrix games, an equilibrium is stable if and only if it is nondegenerate.

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TH-AM-2-W

T. C. HU, University of Wisconsin, Madison

* Combinatorial Optimization

Three problems of combinatorial optimization are described. The first problem is a conjecture on multi-terminal network flows. The second problem is to characterize the necessary and sufficient conditions for an heuristic algorithm of a knapsack problem to achieve the optimum solution. The third problem is on binary search trees.

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W-PM-2-X

P. P. BANSAL, S. E. JACOBSEN, University of California, Los Angeles

An Algorithm for Optimizing Network Flow Capacity Under Economies-of-Scale

The problem of optimally allocating a fixed budget to the various arcs of a single source single sink network for the purpose of maximizing network flow capacity is considered. The initial vector of arc capacities is given and the cost function, associated with each arc, for incrementing capacity is concave; therefore the feasible region is nonconvex. The problem is approached by Benders' decomposition procedure and a finite algorithm is developed for solving the nonconvex relaxed master problems. A numerical example of optimizing network flow capacity, under economies-of-scale, is included.

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T-PM-1-W

R. JAIKUMAR, Booth Fisheries, Chicago

A "Proximal Preference" Algorithm in Large Multi-Criteria Resource Constrained 0-1 Problems

An approach is suggested to maximize "satisfaction" obtained from a number of different criteria. The problem can be stated as

$$\text{Maximize } U[f_1(x), f_2(x), f_3(x), \ldots, f_r(x)], \quad x \in X$$

The decision vector x is constrained to satisfy certain resource constraints and every element in the vector is either 0 or 1. The criterion function f_i is assumed explicitly known; however, all that is known of the preference function is that it is monotonically increasing with increasing value of each criterion, all other criteria being the same.

Initially, the problem is modified to solve for each criteria separately. Lagrange multipliers are obtained to dualize the problem and the resource constraints appear in a Lagrangian function. Iteratively, preference information is obtained for each criteria around the region where an optimal solution is expected. The algorithm is structured to obtain rapid convergence to stable trade-off values between criteria. Lower bounds are explicitly used as constraints and the problem solved with a mathematical programming formulation with a single objective function.

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M-PM-8-V

L. S. JENNINGS, M. R. OSBORNE, Stanford University, Stanford

An Application of Penalty Functions in the Calculation of Singular Values

The problem of minimising the smallest singular value of a rectangular matrix with respect to a parameter is formulated as a constrained optimisation problem. It is shown that penalty functions can be given for this constrained problem which are exact in the sense that the solution to the constrained problem can be deduced from a single unconstrained minimisation of the penalty function. The results of some numerical experiments are also presented.

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W-AM-1-W

R. G. JEROSLOW, Carnegie-Mellon University, Pittsburgh

New Techniques and Algorithms in Zero-One Integer Programming

We shall discuss four new techniques which can be used as subroutines in zero-one integer programming. These techniques are: the linear search; the cube cut, which is an enumerative cut that is valid after the current best solution from the linear search is stored in memory; bisection, which creates two new asymptotic problems from a given asymptotic problem by the simple device of dividing the asymptotic region in two with a hyperplane (the two new asymptotic problems to be treated as new nodes in a branch-and-bound bookkeeping scheme); and reverse implicit enumeration, a procedure in which assigned variables are freed in a fathoming test (the reverse of implicit enumeration, in which free variables are assigned following the failure of a fathoming test), starting from a full assignment that represents a zero-one point generated by any of several heuristics.

Two finite algorithms which utilize these new techniques will be discussed. In the first algorithm, one obtains an L. P. optimal asymptotic region, performs the linear search, then the cube cut, and then re-optimizes; and if sufficient progress is not made by these procedures, the asymptotic program is bisected. In the second algorithm, the linear search is used to generate zero-one points for reverse implicit enumeration, and the enumerative tests are strengthened by the addition of the cube cut. Computational results on algorithms will be reported.

Many other algorithms, other than those mentioned here, can be based entirely on the four techniques, or can incorporate these techniques as subroutines.

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W-PM-11-Z

J. E. JEWETT, University of Rochester, Rochester

A Team Theoretic Approach to Organizational Data Base Design: Mathematical Programming Under Uncertainty

Mathematical descriptions of alternative organizational data bases are derived from the mathematical programming under uncertainty formulation of the team problems of Marschak and Radner. Utilizing the equivalency of constraint structures of the mathematical programming problems with data base descriptions, optimization over all possible organization data bases becomes possible while distinguishing between central data base collection and maintenance costs and the costs of individualized reports generated to decision makers. The optimal organization data bases are derived from feasible sets defined by permissible data base element aggregation and refinement rules. Also, preliminary work about multiple decision makers with different utility functions is given, where data base design determines the central data base and who should get what information in order for everyone to take actions optimizing the overall organization control (utility) function. Preliminary work is also formulated when response functions of decision makers to different demands for reports generated are met. In each case above, global organization analysis determines the central data, reports generated, decision maker responses, and organizational utility based on decision making under uncertainty.

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W-AM-1-W

C.-A. BURDET, E. L. JOHNSON, IBM Watson Research Center, Yorktown Heights, New York

A Subadditive Approach to the Group Problem of Integer Programming

Solving the linear program associated with an all-integer program gives the group problem $Nx \equiv b \pmod 1$, $x \geq 0$ and integer, $z = cx$ (minimize), upon relaxation of non-negativity of the basic variables, where $c \geq 0$ and N is the fractional part of the updated, non-basic columns. A method is given for solving this problem which does not require an explicit group representation and is not dependent on knowing the order of the group. From a diamond gauge function the algorithm constructs a continuous function π, which is shown to be subadditive on the unit hypercube. Such continuous functions yield valid inequalities and are used in solving the group problem.

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TH-PM-2-W

P. HANSEN, L. KAUFMAN, University of Brussels, Belgium

A Primal-Dual Algorithm for the Three-dimensional Assignment Problem

The three-dimensional assignment problem (TDAP) may be stated as follows:

Minimize $\sum_i \sum_j \sum_k c_{ijk} x_{ijk}$ under the constraints

$$\sum_i \sum_j x_{ijk} = \sum_j \sum_k x_{ijk} = \sum_i \sum_k x_{ijk} = 1$$

and

$$x_{ijk} \in \{0,1\}, \; (i = 1,2,\ldots, n; \; j = 1,2,\ldots, n; \; k = 1,2,\ldots, n)$$

where the c_{ijk} are real numbers.

In this paper, the hungarian method for the assignment problem (HM) is extended and adapted to solve the TDAP. It is well known that the HM is based on König's theorem on matching in graphs. For the TDAP an hypergraph is considered at each current iteration. The nodes and edges of this hypergraph are associated to the null elements and the planes of the three-dimensional coefficients matrix. Berge and Las Vergnas have shown that an extension of König's theorem holds for balanced hypergraphs. As, in general, the hypergraphs defined above are not balanced, the HM cannot be extended in a straightforward manner. However several important steps of the HM may be generalized and incorporated in a combined primal-dual and branch-and-bound algorithm. The algorithm has been coded and computational experience is reported.

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TH-PM-7-Y

R. N. KAUL, D. BHATIA, University of Delhi, India

Positive (Semi-) Definite Programming in Complex Space

Abstract not available at time of printing.

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W-PM-1-W

A. A. J. M. van den HOOVEN, D. KIJNE, Technological University, Eindhoven, The Netherlands

A Labeling Method to Find All Solutions of an Integer Programming Problem by Group Minimisation.

Let be given the integer programming problem

$$(1)\qquad \begin{aligned} &\max\ \underline{c}^T\underline{x} \\ &\text{subject to } A\underline{x} = \underline{b},\ \underline{x} \geq \underline{0},\ x \text{ integer,} \end{aligned}$$

where A is an (m,n)-matrix with integer elements containing an (m,m)-identity matrix, and $\underline{b} \in Z^m$. Let B be an optimal basis for the L.P.-problem connected to (1), with $|\det(B)| = D$; via the integer cone-problem with respect to B:

$$(2)\qquad \begin{aligned} &\min(\underline{c}_B^T B^{-1} N - \underline{c}_N^T)\underline{x}_N \\ &\text{subject to } \underline{x}_B = B^{-1}(\underline{b} - N\underline{x}_N),\ \underline{x}_B \text{ integer, } \underline{x}_N \geq \underline{0},\ \underline{x}_N \text{ integer} \end{aligned}$$

the group minimisation problem in a group $G(\underline{s})$ with elements $\underline{g}_\sigma$ $(\sigma = 1,2,\dots, D)$ is derived:

$$(3)\qquad \begin{aligned} &\min \sum_{\nu=1}^{n'} k_\nu t_\nu \\ &\text{subject to } \sum_{\nu=1}^{n'} t_\nu \underline{g}_\nu = \underline{g}_0,\quad t \geq 0,\quad \text{integers .} \end{aligned}$$

Here the $\underline{g}_\sigma$ are integer vectors modulo $\underline{s}$, i.e., $0 \leq (\underline{g}_\sigma)_i < s_i$, $(\underline{g}_\sigma)_i$ and s_i integers $(i = 1,2,\dots, m)$, while s_i divides s_{i+1} for $i = 1,2,\dots, m-1$, and $\prod_{i=1}^{m} s_i = D$.

For the solution (3) mostly a dynamic programming approach is used. To find all solutions of (1) it is necessary to find all those of (3), to derive all corresponding solutions of (2) and, finally, to investigate whether or not the latter also satisfy the conditions of (1). A labeling algorithm yielding these solutions has been evaluated which may be considered an extension of Hu's labeling method (1968). A considerable improvement of the process of adding and comparing group elements is obtained by introducing a unique representation of the elements g_σ by an integer $f(g_\sigma)$, $0 \leq f(g_\sigma) < D$.

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T-PM-14-U

V. KLEE, University of Washington, Seattle

P Convex Polytopes and Mathematical Programming

From Euler's time until the first decade of the present century, the study of the combinatorial structure of convex polytopes was a popular area of mathematical research, attracting the attention of many famous mathematicians (Cauchy, Cayley, Sylvester, Schläfli, Steiner, etc.). But then, due perhaps to the normative influence of Klein's Erlanger program, and to the fact that the most important remaining unsolved problems seemed hopelessly difficult, the subject languished for more than three decades. The current resurgence of interest in convex polytopes is due in large measure to the influence of mathematical programming and the realization that convex polytopes provide the proper geometric setting for many questions in mathematical programming. The past few years have witnessed a number of breakthroughs in the geometry of convex polytopes, some of them directly inspired by questions from mathematical programming and some of them casting new light on various computational procedures. The lecture will be devoted to a survey, at a widely accessible level, of some of these recent developments.

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W-AM-2-X

D. KLINGMAN, D. KARNEY, F. GLOVER, University of Texas, Austin

Implementation and Computational Studies on Start Procedures and Basis Change Criteria for a Primal Network Method

This paper presents extensive computational experience with a special purpose primal simplex algorithm using the augmented predecessor index method for solving pure network problems. The performance is compared to that of several "state of the art" out-of-kilter computer codes. The computational characteristics of several different primal feasible start procedures and pivot selection strategies are also examined.

The study discloses the advantages, in both computation time and memory requirements, of the primal approach over the out-of-kilter method. The test environment has the following distinguishing properties:

(1) all of the codes are tested on the same machine and the same problems,

(2) the test set includes capacitated and uncapacitated transshipment networks, transportation problems, and assignment problems, and

(3) problem sizes ranging from 100 to 8,000 nodes with up to 35,000 arcs are examined.

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W-AM-1-W

A. KORSAK, Stanford Research Institute, Menlo Park, California

Homological Integer Programming: Optimal Chains on Simplicial Complexes

A new approach to integer programming problems is presented here, generalizing methods employed for optimal network flows, in particular the "out-of-kilter" algorithm of Ford and Fulkerson. The new method uses concepts from algebraic topology and homological algebra to arrive at a "primal-dual" algorithm of the "out-of-kilter" type with polynomial time efficiency for a subclass of the three and higher order matroid intersection problems classified by recent work of Karp and Lawler. This subclass is characterized by the condition that the simplicial complex associated with the problem be orientable, that is, any (combinatorial) manifold as a subset of the complex (polyhedron) must be orientable. So far, it remains an open question whether the algorithm will always terminate successfully for non-orientable complexes, an example of which is the case of the "solid transportation problem" of rank 1. Applications to job and machine scheduling and other areas of operations research are indicated.

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F-PM-5-V

B. W. KORT, D. P. BERTSEKAS, Stanford University, Stanford

Combined Primal-Dual and Penalty Methods for Inequality Constraints

During recent years it has been shown that the performance of penalty function methods for constrained minimization can be improved significantly by introducing gradient type iterations for solving the dual problem. In this paper we present a new penalty function algorithm of this type which offers significant advantages over existing schemes for the case of the convex programming problem. The algorithm treats inequality constraints explicitly and can also be used for the solution of general mathematical programming problems.

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T-PM-8-V

K. O. KORTANEK, Carnegie-Mellon University, Pittsburgh

Classifying Convex Extremum Problems

It is shown that any convex or concave extremum problem possesses a subsidiary extremum problem which has certain homogeneous properties. Analogous to the given problem, the "homogenized" extremum problem seeks the minimum of a convex function or the maximum of a concave function over a convex domain. By using homogenized extremum problems new relationships are developed between any given convex extremum problem (P) and a concave extremum problem (P*) (also having a convex domain), called the "dual" problem of (P). This is achieved by combining all possibilities in tabular form of (1), the values of the extremum functions and (2), the nature of the convex domains including perturbations of all problems (P), (P*), and each of their respective homogenized extremum problems.

This detailed and refined classification is contrasted to the relationships obtainable by combining only the possible values of the extremum functions of the problems (P) and (P*) and the possible limiting values of these functions stemming from perturbations of the convex constraint domains of (P) and (P*) respectively.

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M-PM-6-Z

P. F. KOUGH, Washington University, St. Louis

Global Solution to the Indefinite Quadratic Programming Problem

The global solution to the indefinite quadratic problem will be obtained via a generalized Benders cut procedure.

The Benders cut method iteratively adds cuts to a master problem. Maximizing the master problem is shown to be equivalent to maximizing several convex quadratic subproblems. Each cut creates a subproblem. This decomposition provides for the solution of the master problem by an implicit enumeration algorithm combined with Tui cuts.

In order to accelerate convergence only a subset of the subproblems is solved. The formal method is then combined with the accelerated procedure to insure convergence to the global optimum.

These methods are ϵ finite. That is, given any ϵ, in a finite number of steps a point is located and recognized to have an objective value within ϵ of the global maximum.

Finite convergence is achieved by the introduction of exact cuts. Exact cuts correspond to the quadratic pieces of the reduced objective functions (obtained by eliminating the concave variables). The finite procedure is as follows, an ϵ-optimum is found. Exact cuts are then generated in the neighborhood of the ϵ-optimum. A piecewise strategy is employed. This leads to the determination of a global maximum for the ϵ-nbhd. Remaining regions can then be eliminated by the standard Benders cut method.

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F-AM-2-W

T. C. HU, P. J. KOUTAS, University of Wisconsin, Madison

Shortest String Containing All Permutations

In this paper, we consider the problem of constructing a shortest string of $\{1,2,\ldots,n\}$ containing all permutations on n elements as subsequences. For example, the string 1 2 1 3 1 2 1 contains the 6 $(= 3!)$ permutations of $\{1,2,3\}$. No string containing less than 7 digits can contain all six permutations. Note that a given permutation, such as 1 2 3, does not have to be consecutive but must run from left to right in the string.

For $n = 4$, a shortest string has 12 digits and for $n = 5$, Mr. M. Newey claims that a shortest string must have 19 digits. The problem of finding a shortest string of $\{1,2,\ldots,n\}$ was first proposed by Professor R. M. Karp. It is also listed as an open problem in Chvatal, Klarner and Knuth.

We shall first give a rule for constructing a string of $\{1,2,..,n\}$ of infinite length and then show that the leftmost n^2-2n+4 digits of the string contain all the $n!$ permutations (for $n \geq 3$). We conjecture that the number of digits $f(n) = n^2-2n+4$ (for $n \geq 3$) is the minimum.

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T-PM-5-Z

C. R. GAGNON, S. L. S. JACOBY, R. H. HICKS, J. S. KOWALIK, Boeing Computer Services, Inc. Seattle Dr.

A Nonlinear Programming Approach to a Large Scale Hydroelectric System Optimization

A large scale hydroelectric system optimization is consdered and solved by using a nonlinear programming method. The largest numerical case involves approximately 6,000 variables, 4,000 linear equations, 14,000 linear and nonlinear inequality constraints and a nonlinear objective function. The solution method is based on:

(i) partial elimination of independent variables by solving linear equations;

(ii) essentially unconstrained optimization of a compound function that consists of the objective function, nonlinear inequality constraints and part of the linear inequality constraints. The compound function is obtained via penalty formulation.

The algorithm takes full advantage of the problem's structure and provides useful solutions for real life problems that in general, are defined over empty feasible regions.

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T-PM-13-Z

J. KRARUP, Y. GOERTZ, M. LILHOLT, The University of Copenhagen, Denmark

Dimensioning of a Water Supply System

The design of a water supply system is usually carried out in three steps: 1) determination of the "shape" of the network subject to the location of the potential customers and to existing pipelines, 2) determination of the capacity of each section and the materials to be used (concrete, steel, etc.) and 3) the water distribution and loss of pressure.

In general, 1) and 2) are done manually and then tested (and perhaps adjusted) by means of 3). An iterative procedure due to Hardy Cross is applied for the solution of 3). However, the computing time is frequently exorbitant, even for networks of moderate size. It turns out that 3) can be formulated in terms of a network flow problem (circulation) with convex (here: cubic) arc flow costs. We have extended the out-of-kilter algorithm by Fulkerson to cover nonlinear cost functions and implement a version with real-valued dual variables on a computer. The computational experience so far is indeed promising.

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W-AM-5-V

J. L. KREUSER, University of Wisconsin, Madison

Some Quadratically Convergent Methods for the Nonlinearly Constrained Optimization Problem

A class of algorithms for the nonlinearly constrained optimization problem is presented. The algorithms consist of a sequence of major iterations generated by linearizing each nonlinear constraint about the current point, and modifying the objective function. In some cases the modified objective function corresponds to the Lagrangian. However, various other representations of the modified objective function are available and these representations are characterized. A Kantorovich-type theorem is given, showing quadratic convergence in terms of major iterations for the class of algorithms. This is essentially a local result, however, some globally convergent results are also given. The algorithms are compared computationally and theoretically with some other quadratically convergent methods including a Newton method and some updating methods.

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TH-PM-2-W

P. KROLAK, J. NELSON, Vanderbilt University, Nashville

A Multi-Terminal Truck Dispatching Algorithm

The author has previously shown that heuristics employing an information hierarchy are efficient and near-optimal for a wide class of combinatoric problems. This paper develops a new algorithm for the multi-terminal truck dispatching problem using the location/allocation problem as the basis of the information hierarchy. Computational experience on problems taken from the literature and industry is given. Advantages over previously developed assignment based information hierarchies will be described.

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TH-PM-7-Y

H. W. KUHN, J. G. MacKINNON, Princeton University, Princeton

* The Sandwich Method for Computing Fixed Points

Many equilibrium and optimization problems in economics and operations research can be put in the form: Find x such that $f(x) = x$, where x is a nonnegative vector with component sum one and f is a continuous function (not necessarily differentiable, convex, or concave). In 1967, Scarf proposed a combinatorial algorithm to solve such problems. Subsequently, improvements and extensions have been provided by Hansen, Kuhn, and Eaves. A comprehensive treatment is provided by Scarf and Hansen, which also establishes connections with earlier work of Lemke and Howson. All of these algorithms involve systematic search on a discrete set of points and have been shown to be subsumed by Sperner's Lemma on a pseudo-manifold.

The Sandwich Method appears to be more efficient and flexible than the earlier algorithms. Unlike them, it is capable of utilizing prior information on the nature of the solution, and of providing answers to any desired degree of accuracy. This is achieved by imbedding the problem in an artificial problem of one higher dimension. The following description uses notation and terminology to be found in Kuhn, 1969.

Let $f: S_n \to S_n = \{x \mid x = (x_1, \ldots, x_n) \geq 0,\ \Sigma_j x_j = 1\}$ be continuous. For any integer $D > 0$, choose a vector of integers $c = (c_1, \ldots, c_n) \geq 0$ such that $c/D \in S_n$ and is the best available approximation to a fixed point of f.

The data f, D, and c define a proper labeling of the regular subdivision of S_{n+1} of degree $D+1$ as follows:

$$\ell(v_1, \ldots, v_n, v_{n+1}) = \begin{cases} k & \text{if } v_{n+1} = 0 \text{ and } v_k - c_k = \max\limits_{j=1,\ldots,n} \{v_j - c_j\}; \\ k & \text{if } v_n + 1 = 1 \text{ and } f_k((v_1, \ldots, v_n)/D) \leq v_k/D \neq 0; \\ n+1 & \text{if } v_{n+1} \geq 2\ . \end{cases}$$

There is a unique start for a Sperner path in the regular subdivision of S_{n+1}, namely: $(c_1 + \delta_1^i, \ldots, c_n + \delta_n^i, \delta_{n+1}^i)$ for $i = 1, \ldots, n+1$. The path terminates with a completely labeled simplex with vertices: $(v_{i1}, \ldots, v_{in}, 1)$ for $i = 1, \ldots, n$ and $(v_1, \ldots, v_n, 2)$. Let $\hat{x}_j = \Sigma_i v_{ij}/nD$ for $j = 1, \ldots, n$.

THEOREM. Given $\epsilon > 0$ there exists $\delta > 0$ such that for all D with $1/d \leq \delta$, $|f(\hat{x}) - \hat{x}| < \epsilon$. Furthermore, $|\hat{x} - \bar{x}| < \epsilon$ for some $\bar{x}$ with $f(\bar{x}) = \bar{x}$, which may depend on D.

Informally, $\hat{x}$ is nearly a fixed point and is near a fixed point for fine subdivisions. Repeated application, increasing D and using $\hat{x}$ to choose c is very efficient.

* * *

TH-PM-5-Z

L. S. LASDON, A. D. WAREN, R. FOX, Case Western Reserve University, Cleveland

* One Dimensional Search and Penalty Method--Some Theoretical and Computational Results

This talk will describe a one dimensional search procedure based on quadratic and cubic interpolation, using function values only. Some results on the convergence rate of such algorithms will be discussed. Comparative computational results will be given, using the algorithm in interior and exterior penalty methods.

Also described will be an application of an interior penalty method using this one dimensional search procedure to a problem in Sonar Accoustic Array Design. The problem here is to specify the locations, phase, and amplitude shadings of a group of sonar transducers so that the resulting pattern is highly directive. The nonlinear programming formulation of this problem is nonconvex, with potentially hundreds of constraints. A number of representative array designs will be presented, showing the improvements achieved and the computation time required.

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TH-PM-8-X

M. L. LENARD, University of Toledo, Toledo

Practical Convergence Conditions for the Davidon-Fletcher-Powell Method

The convergence properties of the Davidon-Fletcher-Powell method when applied to the minimization of convex functions are considered for the case where the one-dimensional minimization required at each iteration is not solved exactly. Conditions on the error incurred at each iteration are given which are sufficient for the method to achieve the same order of convergence as the best known to apply when exact line searches are performed.

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TH-PM-4-V

J. LERMIT, University of Illinois, Urbana

A Linear Programming Implementation on ILLIAC IV

This paper discusses the implementation of the simpler algorithm for solving Linear Programming problems on a machine with unusual architecture, the array processor, ILLIAC IV.

The array consists of 64 processing elements each capable of an identical arithmetic or logical operation on different data. Since each processing element operates only on its own data, the storage scheme for the matrices and vectors involved must be carefully arranged to allow the greatest number of calculations to be performed at one time. The storage scheme adopted is to allocate each row of the matrix to a particular processing element, in this way any column may be processed in a parallel fashion and interchange of information among the processing elements is minimized.

The algorithm selected is the Cholesky factorization method developed by M. Saunders in which the basis is expressed as the product of a lower triangular and an orthogonal matrix. Only the former being stored. This gives comparable sparseness to the more usual product form of the inverse method and has proved simpler to implement. Better control of numerical error may also be experienced.

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M-PM-13-X

A. M. LESK, Fairleigh Dickinson University, Teaneck, New Jersey

Application of Interactive Computer Graphics to the Phase Problem of X-Ray Crystallography

The phase problem of X-Ray crystallography is a computational problem of structural chemistry that can be formulated as a mathematical programming problem. The pictorial interpretation of the objective function as an approximation to a molecular structure makes it possible to use an interactive computer graphics system to display the current stage of the optimization and to permit an observer to terminate or modify a computation upon the appearance of a significant structural feature. By stopping the computation as soon as a solution is found that is near enough to optimal to be satisfactory, it is possible to avoid the lengthy verification of optimality characteristic of certain algorithms.

In the mathematical programming formulation of the phase problem, the variables are the arguments of the Fourier coefficients of the electron density--the moduli are measurable--the objective function is the degree to which the chemical structure computed from current values of the variables matches an expected model, and the equations of constraint ensure that there is nowhere a negative density of matter. If the crystal possesses a center of symmetry, variables are restricted to one of two discrete values, producing a Boolean programming problem.

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TH-AM-5-Y

E. LEUENBERGER, D. J. WILDE, Stanford University, Stanford

Transcendental Programming

Geometric Programming, despite its wide applicability, cannot solve a transcendental primal problem where either objective function or constraints contain both an exponential function and a power function of the same variable. The transcendental programming theory presented here gives, without iteration, the solution for continuous, differentiable constrained optimization problems having a feasible optimum, N independent variables, K of them exponential, and a total number of terms equal to $N+1-K$. An iterative, locally convergent algorithm to a feasible stationary point is presented for the case where the number of terms is greater than $N+1-K$.

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T-PM-2-X

T. M. LIEBLING, Institut fur Operations Research der ETHZ, Switzerland

On the Number of Iterations of the Simplex Method

Rényi and Sulanke studied the properties of the convex hull of a set of points thrown at random independently of each other. Carnal generalized their results concerning the asymptotic behaviour of the number of faces of that convex hull as the number of points increases. The present work connects those results with the mean number of pivot steps used by the Simplex Method to solve certain Linear Programming Problems. This is made possible through the polar interpretation of the LP-Problems. Monte Carlo Simulation confirmed the results. Aside from statistical applications, it is felt that this kind of an approach will make it possible to characterize optimization problems with polyhedral feasible sets as to their benevolence.

T-AM-6-Y

C. K. LIEW, University of Oklahoma, Norman

The Stability Condition of the Inequality Constrained Least-Squares Estimation

When the parameters of a regression equation or a linear combination of them are constrained by inequalities, the estimation problem reduces to the fundamental problem (i.e., $v = W\lambda + q$, $v'\lambda = 0$, $v \geqq 0$, and $\lambda \geqq 0$ where W is a fixed matrix, v, λ, and q are vectors).

In the inequality constrained least-squares estimation (ICLS), the elements of the q vector become random variables. Consequently, the basic variables and the ICLS estimators become random variables with finite means and variances. The variance-covariance matrix of the ICLS estimators is meaningful for the samples of the regression residuals which would not cause the basic variables to change. The paper investigates the stability condition of the variance-covariance matrix of the ICLS estimators and shows that, in the case of a sufficiently large sample, the values of the q vector converge to the set which would not cause the basic variables to change. In this case, any choice of samples of the residuals would not make the basic variables to change. The variance-covariance matrix of the ICLS estimators exists for all possible samples of the residuals.

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T-AM-13-V P-8

D. LIGGINS, University of Birmingham, England

Applications of Integer Programming in National Economic Planning

The paper will report on information received via a questionnaire on the use of integer programming actually made by national and regional planning organizations in many countries, particularly underdeveloped ones. Information is presently being compiled from approximately 50 countries in Latin America, Africa, Asia and Europe. Visits to a number of these countries have already taken place or will take place later this year, and this will provide further information from which conclusions will be drawn. In view of the very rapid growth in the area of theoretical integer programming, it is desirable at the present time to evaluate the present needs for practical algorithms and to see in which areas these needs are likely to be greatest.

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T-PM-5-Z

S. C. LITTLECHILD, G. F. THOMPSON, University of Aston, Birmingham, England

Optimal Aircraft Landing Facilities and Fees

Aircraft landing facilities may be constructed to a variety of sizes depending upon the type of aircraft which it is desired to accommodate. One problem is to determine the optimal set of aircraft to accommodate, that is the set which maximizes benefits of landing less the cost of providing landing facilities. A second problem is to design a set of landing fees which efficiently decentralizes this problem in that they encourage just this set of aircraft to land. The landing fees must also be, in some sense, fair.

A mathematical programme model is proposed for handling the first problem. Associated with this is a game whose characteristic function represents the benefits less costs of providing facilities for each sub-set of aircraft types. Any imputation in the core of this game corresponds to an efficient set of landing fees. The Shapley value may be thought of as one particular fair solution.

This model is applied to Birmingham Airport (England) where 11 types of aircraft are under consideration. The special structure of the characteristic function enables the set of landing fees in the core to be characterized and also enables the Shapley value to be computed.

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W-AM-5-V

D. G. LUENBERGER, Stanford University, Stanford

* Some Results on the Convergence Rates of Nonlinear Programming Algorithms

The spirit of the basic convergence rate analysis of steepest descent, leading to the Kantorovich ratio, can be extended so as to apply to a wide variety of popular nonlinear programming algorithms. This leads to the identification of a canonical (or natural) rate of convergence associated with a nonlinear programming problem which governs the rate of convergence of many algorithms when applied to that problem.

Within this framework it is possible to obtain estimates for the rates of convergence of steepest descent, partial conjugate gradient methods, quasi-Newton methods, projected and reduced gradient methods, penalty methods, and dual methods. The associated analysis clearly points out areas of possible deficiency in some algorithms and often suggests alternative methods that are free of these deficiences.

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F-AM-13-Y

L. LUNDQVIST, The Royal Institute of Technology, Stockholm

Mathematical Models for Urban Planning

It is the aim of this paper to present a model framework for the long-term land-use planning of the Stockholm region. A principally sequential decomposition of the overall problem is proposed. The main emphasis is laid upon the structure and solution properties of appropriate submodels. Tentative computational results are given.

On the top level a coordinated expansion of the transportation system structure and the building stock pattern is studied. Due to indivisibilities of transportation projects and nonlinear welfare criteria the problem turns out to be of the nonlinear mixed integer type. An approximating model is solved by heuristic tree-searching.

Given the broad outline of the infrastructural capacity, the spatial allocation of a number of mutually dependent activities is performed in a second step. For this purpose nonlinear programming models have been constructed.

With a fixed location pattern for the main urban activities, a third level of more disaggregated spatial and sectoral studies is of practical relevance. Here, the interest is concentrated upon linear programming or linear assignment models.

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M-PM-13-X

G. MAIER, Technical University (Politecnico), Milan, Italy

* Quadratic Programming in Elastic-Plastic Analysis

The nonlinear analysis of discrete models of engineering structures is formulated in various ways as linear complementarity problems and as equivalent pairs of quadratic programs. Mechanical interpretations are given and solution techniques are discussed also by means of examples.

Methods for bounding plastic strains and displacements of elastoplastic structures in both the quasi-static and dynamic ranges are presented and reduced to mathematical programming procedures, in particular to quadratic and linear programming problems.

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F-PM-3-X

S. F. MAIER, Duke University, Durham

Decomposition of Linear Programs with a Staircase Structure - An Approach for Finding Near Optimal Primal Feasible Solutions

In a recent paper, Glassey proposed an algorithm for solving a linear programming problem with a staircase structure by a nested decomposition approach. Glassey's algorithm terminates when the values of the primal and dual objective functions are sufficiently close together, at which point a feasible dual solution is available. Using this dual solution, a primal solution is then reconstructed. Unfortunately, this primal solution need not be feasible.

Manne recognized the shortcomings of the Glassey approach and attempted to construct a feasible primal solution from the proposals of previous stages. The shortcomings of this method is that all proposals generated by all stages must be kept in storage, a very severe computational handicap.

In this paper we propose an algorithm that maintains a primal feasible solution at all times, and therefore, does not require a reconstruction phase, nor the storage of old proposal vectors. The proposed algorithm is essentially a variant of decomposition, but it still may be terminated when the primal and dual objective functions are sufficiently close together.

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W-PM-1-W

A. MAJTHAY, University of Florida, Gainesville

Facet "Stripping," Cuts and the Quasiconcave Minimization Problem

An algorithm is presented for finding the global minimum of a quasiconcave function subject to linear constraints. It is a combination of an enumerative scheme in which the convex polyhedron of the feasible solutions is "stripped" from its unwanted facets, and of conventional convexity cuts. We can give two alternative interpretations to the facet stripping procedure: a geometric one and a combinatorial one. With regard to the former interpretation we shall say that we separate the unwanted facets of the feasible region by cutting off infinitesimally thin layers of the boundary. This idea is based on a cutting procedure developed by Majthay and Whinston. By using the geometric interpretation we get an intuitive insight and we can prove that the procedure is finite. The combinatorial interpretation of the facet stripping procedure suggests the development of both a storage and retrieval system and an identification procedure for the implementation of the algorithm. As the facet stripping procedure relieves us from the burden of insuring finiteness, we can apply the convexity cuts judiciously and prevent the usual difficulties which arise from degeneracy and from the enormous growth of size.

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F-PM-5-V

J. L. BURROUGHS, G. MALL, Computer Sciences Corporation, Leavenworth

Mathematical Programming Techniques for Solving Weapon Allocation Problems

Weapon allocation problems in which the only constraints are stockpile constraints are among the simplest of all mathematical programming problems. Such problems do have some interesting mathematical properties which have a marked effect on the relative computational efficiencies of different mathematical programming techniques. In this paper, the effects of mathematical characteristics, such as singular Jacobian matrices and exponential sensitivity of the objective function to variations in weapon allocations, are described. These effects are demonstrated by solving example problems by means of concave simplex, geometric programming, and generalized Lagrange multiplier techniques.

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F-AM-8-V

O. L. MANGASARIAN, University of Wisconsin, Madison

* Unconstrained Lagrangians in Nonlinear Programming

The main purpose of this work is to associate a wide class of Lagrangian functions with a nonconvex, inequality and equality constrained optimization problem in such a way that unconstrained stationary points of each Lagrangian are related to Kuhn-Tucker points or local or global solutions of the optimization problem. As a consequence of this we are able to obtain duality results and two computational algorithms for solving the optimization problem. One algorithm is a Newton algorithm which has a local superlinear or quadratic rate of convergence. The other method is a locally linearly convergent method for finding stationary points of the Lagrangian and is an extension of the method of multipliers of Hestenes and Powell to inequalities.

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W-PM-2-X

R. E. MARSTEN, Northwestern University, Evanston

An Algorithm for Finding Almost All of the Medians of a Network

All of the medians of a weighted network are shown to be extreme points on the same polyhedron P. An algorithm is presented which makes a tour of P, passing through most of these special extreme points and through very few others. A problem of optimally locating facilities in a network is treated and computational results are given.

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T-PM-8-V

B. MARTOS, Hungarian Academy of Science, Budapest

* Sufficiency vs. Necessity of Smoothness and Convexity Conditions: A Challenge

In the theory of nonlinear programming we meet the following types of sufficient conditions:

1. Sufficient conditions for the feasible set to be closed-convex or polytopal in terms of the constraint functions
2. Sufficient conditions for the following kind of points:
   a) local minimum points
   b) global vertex-minimum points
   c) local vertex-minimum points
   d) virtual minimum points
   e) stationary points

   to yield global minima in terms of the objective functions.
3. Sufficient conditions that some or all the global minimum points could be identified as different kind of points a) - e) of 2 in terms of the objective function.
4. Sufficient conditions that a saddle point of the Lagrangian or a KTL (Kuhn-Tucker-Lagrange)-stationary point yield a global optimum in terms of the constraint and the objective functions.

5. Sufficient conditions that to each global minimum point belongs a saddle point of the Lagrangian or a KTL stationary point in terms of the constraint and the objective functions.

Most of these conditions refer to continuity, differentialibity and convexity properties of the functions in question. Many of the corresponding theorems have a counterpart claiming that some convexity properties are also necessary for the converse of these theorems to hold. We will dwell upon these necessary conditions from the following two points of view:

A. A comparison shows that there is often a gap between sufficient and necessary conditions, which perhaps can be reconciled by further research.

B. A schematic representation points out the missing theorems. This again may challenge researchers.

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W-AM-14-U

M. MASCHLER, Hebrew University, Jerusalem and Stanford University, Stanford

P <u>The Theory of the Bargaining Sets of Cooperative Games</u>

The bargaining set, the kernel and the nucleolus are related solution concepts for n-person cooperative games. The bargaining set reflects possible outcomes resulting from certain negotiation patterns, the kernel, which is a subset of the bargaining set is highly sensitive to symmetries which may exist in the game. It contains the nucleolus, which is a one-point solution concept, designed to minimize successive dissatsifaction of the coalitions from proposed outcomes.

These solution concepts will be discussed from a static and dynamic point of view and their properties will be surveyed. Applications to economics, political sciences and other social sciences will be outlined and methods of computation will be described. Extensions to games without side payments will be described.

F-AM-2-W

D. W. MATULA, University of Texas, Austin and Washington University, St. Louis

* A Provably Efficient Branch and Bound Search for the Maximum Subgraph Connectivity

Let κ be the connectivity of the graph $G = (V,E)$, and τ the maximum connectivity of the subgraphs of G, that is $\tau(G) = \max \{\kappa(H) | H \text{ a subgraph of } G\}$. $\kappa(G)$ can be determined by a network flow based procedure having computational complexity no worse than $O(n^5)$ in additions and comparisons, where $n = |V(G)|$, and we describe a procedure for determining $\tau(G)$.

For any graph G, we show that

$$\tau(G) = \max\{\kappa(G), \max_i \{\tau(< A_i \cup S >)\}\},$$

where $S \subset V$ is any minimum separating vertex set of G, and $A_1, A_2, \ldots, A_m$ are the vertex sets of the $m \geq 2$ components of $< V-S >$. This decomposition formula yields a branch and bound search procedure for $\tau(G)$, where a lower bound for a node of the search tree is derived from the largest $\kappa(H_j)$ encountered on the path to that node of the search tree, and an upper bound is given by $|V(H)| - 2$ if H is the non-complete subgraph at that node of the search tree. A maximal complete subgraph K_i of G has $\tau(K_i) = \kappa(K_i) = i-1$, and thus will constitute a terminal node of the search tree when reached. Branching at each node of the search tree until nodes corresponding to maximal complete subgraphs are reached can yield search trees growing exponentially in size with the number of vertices of the underlying graph G. We prove, however, that the search order determined by always branching at a node of highest upper bound will determine $\tau(G)$ in a number of branching steps less than $n = |V(G)|$, thus providing an algorithm of complexity no worse than $O(n^6)$ for determining $\tau(G)$.

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W-AM-5-Y

J. MAY, R. MIFFLIN, Yale University, New Haven

A Superlinearly Convergent Nonderivative Method for Linearly Constrained Minimization

An algorithm for minimization of a nonlinear function subject to linear inequality constraints is presented. This algorithm is an extension of Mifflin's local variations-approximate Newton nonderivative method for unconstrained optimization. It does not require explicit evaluation of partial derivatives or any exact one variable minimizations. Accumulation points of the algorithm sequence satisfy the Kuhn-Tucker optimality conditions if the objective function is continuously differentiable. Furthermore, the convergence is superlinear when the function is twice continuously differentiable and there is an optimal point satisfying second order sufficiency conditions to be a unique nondegenerate Kuhn-Tucker point.

At each iteration the coordinate system for the local variations function evaluations consists of rows of the generalized inverse of the current active constraint matrix and some rows of an orthogonal matrix. The orthogonal matrix results from QR factorization of the active constraint matrix used to obtain its generalized inverse. A method for updating the required matrices when the set of active constraints changes based on results of Gill, Golub, Murray and Saunders is employed.

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T-PM-8-V

P. MAZZOLENI, Universita degli Studi de Venezia, Italy

Nonlinear Programming with Fractional Objective Function

Some methods have been suggested to solve particular programming problems with fractional objective function.

In order to solve the general optimization problem of this kind

$$\min_{x \in X} \frac{f(x)}{g(x)}$$

the following paper applies a moving-truncation technique to reduce the original problem to a sequence without fractional objective function.

Under general assumptions of continuity on functions and compactness on sets, a sequence of truncation levels $\{s_k/t_k\}$ is constructed like in the Huard method of centers.

Starting from a point x_o, for any iteration point x_k the domain of the objective function is reduced to

$$T_k = \{x \in X : s_k g(x) - t_k f(x) \geq 0,\ t_k g(x) > 0,\ s_k = f(x_k),\ t_k = g(x_k)\},$$

the constraint $t_k g(x) > 0$ being easily satisfied. On such a set a truncation function

$$B_{k+1}(x) = s_k g(x) - t_k f(x)$$

is maximized so that a point x_{k+1} is found.

As $B_{k+1}(x)$ is a distance function the algorithm constructs a finite sequence of points $\{x_k\}$, the last one being the optimal solution.

Otherwise it is proved that any limit point of the sequence $\{x_k\}$ is the optimal solution for the fractional problem.

Some numerical tests show that this method is encouraging, as it is comparable with the known best techniques.

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W-PM-2-X

C. J. McCALLUM, JR., Bell Laboratories, Holmdel, New Jersey

A Generalized Upper Bounding Approach to a Communications Network Flow Problem

An important network optimization problem is to determine the routing of circuits and construction of additional arc capacity in a communications network so as to satisfy forecasted circuit requirements at minimum cost. This paper considers the single time period version of the problem formulated as a linear program in the arc-chain form. The special structure of this linear program is exploited to develop an efficient solution procedure. In particular, the Generalized Upper Bounding Technique (GUB) of Dantzig and Van Slyke is applied to the problem. The computer implementation of the procedure is discussed and computational experience is reported.

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Dear Garth

TH-PM-5-V

G. P. McCORMICK, George Washington University, Washington, D.C.

* Computable Methods for Obtaining Global Solutions to Nonconvex Programming Problems Which are Factorable

For nonlinear programming problems which are factorable, a computable procedure for obtaining tight underestimating convex programs is presented. This is used to exclude from consideration regions where the global minimizer cannot exist, and as a subalgorithm to a new global identification procedure which establishes that a local minimizer is a global minimizer.

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M-PM-8-V

L. McLINDEN, University of Wisconsin, Madison, Wisconsin

An Extension of Fenchel's Duality Theorem to Saddle Functions and Dual Minimax Problems

Fenchel's Duality Theorem (or more precisely, Rockafellar's extension of it) is extended here from the context of convex functions and dual convex extremum problems to that of saddle functions and dual minimax problems. The paper is written in the spirit of mathematical programming. Inequalities between optimal values are established, stable optimal solutions are characterized, strong duality theorems proved, and an existence criterion given. An associated Lagrangian saddle point problem is introduced and an extension of the Kuhn-Tucker Theorem derived. The proofs, which are necessarily different from the purely convex case, rely on recently developed pairs of dual operations on saddle functions, as well as on more widely known facts about conjugate saddle functions.

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F-PM-5-V

G. G. L. MEYER, The Johns Hopkins University, Baltimore

Inner Loops in Interior Methods

In this paper we study interior methods, i.e., methods which generate sequences of points in the constraint set, for solving the usual non-linear programming problem.

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M-PM-1-W

R. R. MEYER, University of Wisconsin, Madison

On the Existence of Optimal Solutions to Integer and Mixed-Integer Programming Problems

The purpose of this paper is to present sufficient conditions for the existence of optimal solutions to integer and mixed-integer programming problems in the absence of upper bounds on the integer variables. It is shown that (in addition to feasibility and boundedness of the objective function) (1) in the pure integer case a sufficient condition is that all of the constraints (other than non-negativity and integrality of the variables) be equalities, and (2) that in the mixed-integer case rationality of the constraint coefficients is sufficient. Some computational implications of these results are also given.

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TH-PM-8-X

R. MIFFLIN, Yale University, New Haven

A Superlinearly Convergent Algorithm for Minimization Without Evaluating Derivatives

An algorithm for unconstrained minimization of a function of n variables that does not require the evaluation of partial derivatives is presented. This method is a Newton-type second order extension of the method of local variations and it does not require any exact one variable minimizations. Accumulation points of the algorithm sequence satisfy necessary conditions for optimality if the function is continuously differentiable or if the function is convex (not necessarily differentiable). Furthermore, the rate of convergence is superlinear for a twice continuously differentiable strongly convex function.

A computational implementation of this algorithm as well as numerical results for some test problems will be described.

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combine request with paper on p. 143

TH-AM-4-V

A. L. BREARLEY, G. MITRA, H. P. WILLIAMS, Dataskil Limited, Reading, England

Analysis of Mathematical Programming Problems Prior to Applying the Simplex Algorithm

Large real life Linear and Integer Programming problems are not always presented in a form that is the most compact representation of the problem. In practice these problems are also likely to possess Generalized Upper Bound and related structures which may be exploited by algorithms designed to solve such problems.

In the first place an algorithm is presented which may be applied iteratively to reduce the rows, columns, and bounds in a problem matrix and which leads to freeing of some variables. The 'unbounded solution' and 'no feasible solution' conditions are also frequently revealed by this algorithm. An algorithm to detect structure is then discussed. This algorithm investigates sets of variables and the corresponding constraint relationships, such that the total number of GUB type constraints are maximized leading to maximum basis contraction.

Computational results of applying both these algorithms are presented and discussed.

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W-PM-5-V

P. E. GILL, W. MURRAY, Stanford University, Stanford

* Quasi-Newton Methods for Linearly Constrained Optimization

In a recent paper (Gill and Murray), a quasi-Newton method was described based on recurring the Cholesky factorization of the Hessian matrix approximation. In two other papers, methods have been described for handling linear constraints based on recurring triangular factors. In this paper all these ideas are combined to yield an algorithm for linearly constrained optimization.

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T-AM-2-X

K. G. MURTY, University of Michigan, Ann Arbor

* On the Uses of 2-Dimensional Faces in Studying Polytopes

Here we consider either simple convex polytopes or their analogues, abstract polytopes. The graph of the polytope is a regular connected graph and every two dimensional face of the polytope is a simple cycle in this graph. If we are only given the graph of the polytope, quite often it is impossible to construct the polytope from it. However, it is shown that if we are also given the simple cycles in the graph which are the 2-dimensional faces of the polytope, then all the faces of the polytope of higher dimensions can easily be constructed. This implies that two simple convex (or abstract) polytopes which have the same 2-dimensional skeleton are isomorphic.

The uses of these results in studying step conjectures on polytopes will be discussed.

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TH-PM-8-X

W. C. MYLANDER, J. D. PEARSON, U. S. Naval Academy, Annapolis

Computational Experience with the Sequential Unconstrained Minimization Technique (SUMT)

Over the past several years, the authors have used the SUMT code developed at RAC (Mylander, Holmes, and McCormick (1971)) to solve many programs of moderate size. An interesting class of problems being solved are the deterministic equivalent of a chance-constrained programming model. These problems are convex, with a quadratic objective function and nonlinear constraints of the form:

$$a_i^T x + \alpha_i (\gamma_i + x^T D_i x)^{0.5} \le \delta_i \qquad i = 1,2,\dots,m$$

where a_i is a vector of constants, D_i a positive semi-definite matrix and $\alpha_i > 0$, $\gamma_i \ge 0$, δ_i are scalars and x a n-dimensional vector of unknowns. Problems of this form with n and m in the range of 35 to 60 have been solved repeatedly using the SUMT code with varying degrees of success.

The performance of the SUMT code for solving the class of problems mentioned will be reported. Its strengths and weaknesses as a solution technique will be discussed. These problems have demonstrated the need to improve the procedures used when the Hessian matrix is not positive definite at the current point. Our experience in modifying Newton's algorithm for rapidly finding the minimum of each unconstrained function will be presented.

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F-AM-2-W

G. L. NEMHAUSER, L. E. TROTTER, JR., Cornell University, Ithaca

* Properties of Vertex Packing and Independence Systems Polyhedra

We consider two convex polyhedra related to the vertex packing problem for a finite, undirected loopless graph $G = (V,E)$ with no multiple edges. A characterization is given for the extreme points of the polyhedron $L_G = \{x \in R^n : Ax \le 1, x \ge 0\}$, where A is the $m \times n$ edge-vertex incidence matrix of G and 1 is an m-vector of ones. A general class of facets of $B_G = \text{conv. hull}\{x \in R^n : Ax \le 1,$ x binary$\}$ is described which subsumes a class examined by Padberg. Some of the results for B_G are extended to a more general class of integer polyhedra obtained from independence systems.

W-AM-13-Y

B. HALACHMI, P. NICHOLSON, University of Minnesota, Minneapolis

Inventory Models with Two-Sided Demand

We develop an efficient algorithm for computing optimal policies in the stochastic "cash balance problem." The task is to control an inventory process in which the (stationary) random "demands" may be positive or negative, a situation which precludes the standard (s,S) computational theory. Our approach is via the policy iteration algorithm of Howard and so we minimize the long-run average cost per period. The minimand involves a fixed transfer cost (no proportional term) and (distinct) per unit costs for stock levels different from zero. For such systems it is well known that the optimal policy is of the form (x,y,z) with the interpretation that levels below x or above z are to be controlled, through stock transfer, to level y at the start of a review period. A direct application of Howard's algorithm would be computationally intractable for systems of realistic size; however, by exploiting the fact that the policy improvement iterations are of (x,y,z) form at each step we are able to effectively avoid the usual limitations of a large state space. Indeed, if there are N potential states in the underlying Markov Chain, the value determination step in Howard's general algorithm involves solving an $N \times N$ linear system. In the case we analyze, the system size at iteration "t" is reduced to $(z_t - x_t + 1)\times(z_t - x_t + 1)$, which is typically much smaller than $N \times N$. Moreover, in the policy improvement routine we need only evaluate $\log_2 N$ values of the test quantity rather than N.

We note that the number of states may be taken to be very large with only negligible (i.e. logarithmic) effects on the computational process, it being recalled that the value determination step is independent of N.

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TH-PM-7-Y

H. NISHINO, M. KOJIMA, I. KANEKO, Keio University, Japan

On Applying a Complementary Algorithm to a Nonlinear Programming Problem

The following nonlinear programming problem is considered:

$$\text{Maximize } g(x)$$
$$\text{subject to } x \in X \subset R^n_+,$$

where $g(x)$ is a continuously differentiable function from R_+^n to R^1 and X is a nonconvex polyhedron which has a nonempty convex kernel. A subset $K(X)$, of X is said to be a convex kernel of X if $x \in K(X)$ and $x' \in X$ imply $\alpha x + (1-\alpha)x' \in X$ for any $\alpha \in [0,1]$. It is shown that the Extended Complementary Pivot Algorithm, which is an extension of Lemke's linear complementary algorithm to piecewise linear complementarity problems, computes a quasi Kuhn-Tucker point of the above problem.

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TH-PM-5-Z

R. P. O'NEILL, Louisiana State University, Baton Rouge

Generalized Linear Programming with Nonlinear Subproblems

This paper presents an approach to the selection of columns for the restricted master problem. The approach uses a transformation of the subproblem objective function. A class of transformations, to which the standard procedure belongs, is defined by the conditions necessary to maintain the convergence properties. Finite convergence is shown for a subset of this class. An exponential transformation, that acquires a free parameter, is used in the Dantzig-Wolfe convex programming algorithm and computational experience on standard test problems shows a vast improvement of the transformation method over the standard method of selection. If the problem is not convex, a slight modification of the method will produce convergence for a general nonlinear problem. The usefulness of this approach with separable and decomposable problems is discussed.

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TH-AM-4-V

W. ORCHARD-HAYS, National Bureau of Economic Research, Inc, Cambridge, Mass.

* Problems and Principles in the Evolution of Mathematical Programming Systems

The problems encountered in over two decades of development are reviewed, together with certain principles which have emerged. The problems are classified into Technical, System and Organizational, and Psychological, and then subclassified. Technical problems, i.e., mathematical and algorithmic, are treated only briefly, mainly with reference to implementation difficulties which they have caused. Psychological problems are treated at somewhat greater length through the device of personal experiences. The effects of attitudes and thinking habits on the evolution of a technology seems not to have been much discussed, in spite of its almost obvious importance. The concept is merely illustrated without formal treatment.

The major part of the paper is on system and organizational problems. Following a broad subclassification, the following areas are discussed at greater length:

1. A kind of duality between routines and data sets,
2. Levels of data types,
3. Management of working storage,
4. Control systems,
5. Building and using a large MPS on an interactive system: The SESAME system under development at NBER's Computer Research Center.

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T-PM-2-X

A. ORDEN, University of Chicago, Chicago

* <u>Probabilistic Estimation of the Efficiency of Some Versions of the Simplex Method</u>

Among the variants of the simplex method there are those whereby--starting with a tableau in which some or all of the rows do not yet contain a "reduced variable" (a nonartificial, non-negative basic variable)--the "nonreduced rows" are successively targeted for reduction; and the number of reduced rows is increased one by one until a solution is at hand. Algorithms for finding a solution to a system of linear inequalities, and primal-dual algorithms can be so expressed.

In such versions of the simplex method the probability at each pivot stage of occurrence of an upward step in the number of reduced rows appears to be a function of: the number of rows reduced so far, the number of pivots since the most recent upward step, and the density of the tableau. Probabilistic models of the expected value of the number of pivots required to reach a solution have been formulated on this basis for several simplex type algorithms. The models, the predictions of simplex method efficiency which they produce, and comparisons of the predictions to results of computer runs on randomly generated sets of small problems will be presented.

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T-AM-9-Z

S. S. OREN, Xerox Palo Alto Research Center and Stanford University

On the Selection of Parameters in Self Scaling Variable Metric Algorithms

This paper addresses the problem of selecting the parameter in a family of algorithms for unconstrained minimization known as Self Scaling Variable Metric (SSVM) Algorithms. This family, which was introduced recently by this author, has some very attractive properties. It is based on a two parameter formula for updating the inverse Hessian approximation, which is a member of Huang's family, and in which the parameter can be chosen freely in the interval $[0,1]$. This formula does not satisfy Dixon's conditions, and hence the performance of the resulting algorithms depends even theoretically on the choice of parameter. Earlier results obtained for SSVM algorithms are general for the entire family and give no indication of how the choice of parameter may affect the algorithm's performance. This paper, which focuses on the above problem, examines empirically the effect varying the parameters and relaxing the line search. The paper also presents some theoretical results that lend to a switching strategy for automatic selection of these parameters.

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M-PM-10-Y

G. KALAI, M. MASCHLER, G. OWEN, Rice University, Houston

* Asymptotic Stability and Other Properties of Trajectories and Transfer Sequences Leading to the Bargaining Sets

R. E. Stearns and L. J. Billera discovered transfer schemes and differential equations with solutions that always converge to various bargaining sets, thereby establishing the foundation of a dynamic theory for the bargaining sets.

In this paper we show that a point in the bargaining set is locally asymptotically stable with respect to the sequences and the solutions if and only if it is the nucleolus of the game and it is isolated relative to the bargaining set.

As by-products we obtain in a different fashion the results of Stearns and Billera and also show that along nontrivial sequences and solutions, the vector of the excess of all coalitions arranged in a decreasing order, decreases lexicographically. Thus, the bargaining sets can be viewed as resulting from certain lexicographically minimizing processes and the role of the nucleolus in the bargaining set is given a new angle.

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F-PM-2-W

M. W. PADBERG, International Institute of Management, Berlin

Perfect Zero-One Matrices

A zero-one matrix is called perfect if the polytope of the associated set packing problem has integral vertices only. By this definition, all totally unimodular zero-one matrices are perfect. In this paper we give a characterization of perfect zero-one matrices in terms of forbidden submatrices. The notion of a perfect zero-one matrix is closely related to that one of a perfect graph as well as that one of a "balanced" matrix as introduced by Berge. Furthermore, the results obtained here bear on an unsolved problem in graph theory, the strong perfect graph conjecture due to C. Berge.

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T-PM-6-Y

C. C. PAIGE, McGill University, Montreal

Stability of Matrix Updating in Mathematical Programming

In several algorithms for solving optimization problems, the general step requires the solution of linear equations involving a basis matrix, but at each new step only one row, or column, of the basis matrix is changed. For economy the solutions are not carried out by using the new basis matrix each step, instead an inverse or some decomposition of the basis matrix is updated and the solutions obtained using this. Unfortunately even with great care the inverse is likely to be in error once an ill-conditioned basis is encountered, and this error tends to remain until re-inversion. In contrast it will be shown that decompositions of the matrix into lower and upper triangular matrices, or orthogonal and triangular matrices, can be updated in a manner that ensures that the new decomposition is numerically stable in the same way that the original decomposition is. This means that numerical stability is not a factor to be worried about when deciding when to re-decompose the basis matrix using these algorithms.

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W-PM-10-Y

L. F. PAU, The Technical University of Denmark, Lyngby

Differential Games and Direct Nash Equilibrium Searching Algorithms

This report describes two numerical algorithms used for searching NASH-COURNOT equilibrium strategies in a differential game of fixed duration, and with given initial state. The first algorithm is based upon optimal weighted mean square controls. The second algorithm is based upon a hierarchical decomposition of the game into optimal control problems, with a fictive referee for the NASH-COURNOT playing rule. Each control problem, with restricted state and control functions, is solved by means of the generalized reduced gradient algorithm. A convergence result for the second NASH equilibrium searching algorithm is given tentatively.

These algorithms are applied to a non-linear dynamic sectoral model of the Danish economy. The control functions are: investments, labour, write-offs, marginal tax rates, in each sector. The state variables are: foreign debt, state budget excess. Some preliminary results for the 1947-1952 period have shown that the NASH equilibrium controls obtained were closer to the actual historical controls than those yielded by maximizing classical welfare criterions. These numerical results are reported elsewhere.

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W-AM-2-X

J. D. PEARSON, W. C. MYLANDER, III, General Research Corporation,McLean, Va.

Allocating Motive Power to Railroad Schedules

An important problem in railroad operation is the allocation of locomotives to daily schedules. Locomotives of various types each having different load characteristics must be allocated to the daily cycle of schedules defined for a railroad network. The allocation criterion is broadly speaking to minimize the number of locomotives required of each type.

This paper analyzes and presents solutions to a problem of allocating two or more types of locomotive to a system of schedules in which there is a non-mixed mode constraint which required that each schedule is run with one and only one type of locomotive.

A new multicommodity flow algorithm is proposed for the basic problem of allocating locomotives, and the resulting mixed mode schedules are resolved using a branch and bound procedure. This particular approach permits application of important network side conditions present in practical situations such as restrictions on various network links for locomotive types due to gradient or curvature.

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T-PM-2-X

C. L. J. VAN DER MEER, R. J. PETERS, University of Groningen, The Netherlands

Construction of Two-Dimensional Areas, Corresponding to Optimal Solutions, Filling Up the Parameter Space of a Parametric Programming Problem with Two Parameters in the Objective Function

Consider the problem

$$\text{Max } f = (c_1 + \lambda c_2 + \mu c_3)'x \qquad (1)$$

$$\text{under the constraints: } Ax = b \qquad (2)$$

$$x \geq 0 \qquad (3)$$

In this case c_1, c_2, c_3, x and b are vectors; A is a matrix and λ and μ are scalars.

The total parameter-set V of (λ,μ) is a subset of R^2 and consists of all elements (λ,μ) for which $(c_1 + \lambda c_2 + \mu c_3)'x$ is bounded from above for all vectors x which satisfy (2) and (3). The set $v^{(i)} \subset R^2$, for which one and the same solution $x^{(i)}$ remains optimal for all $(\lambda,\mu) \in v^{(i)}$, is a subset of V. In this article a construction-method is described and proved according to which all sets $v^{(i)}$, filling up V, may be determined in a systematic way and rendered into a construction-figure.

F-AM-8-V

E. L. PETERSON, Northwestern University, Evanston

* Decomposition in Geometric Programming

Geometric programming in its most general form provides a mechanism for transforming many important inseparable optimization problems into separable (generalized) geometric programming problems.

The key to such transformations is the exploitation of the linearities that are present in a given problem. Such linearities frequently appear as linear equations or linear inequalities, but they can also appear in much more subtle guises as matrices associated with nonlinearities.

The geometric programming problems that result from such transformations are (at least partially) separable, and many that arise from the modeling of large systems have sparse matrices. Such separability and sparsity can frequently be exploited by using appropriate decomposition principles.

These principles generalize in a very nontrivial way various decomposition principles in linear programming, and they also generalize and unify many nonlinear programming decomposition principles that appear in diverse disciplines and fields of application.

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M-PM-6-Z

J. PHILIP, Royal Institute of Technology, Stockholm, Sweden

An Algorithm for Combined Quadratic and Multiobjective Programming

The given algorithm solves $\min\{Q(x) : x \in E\}$, where Q is a positive semidefinite quadratic form in R^n and E is the set of efficient points in a vector maximization problem. The algorithm is composed of Wolfe's "Linear programming algorithm for quadratic programming" and the author's algorithm for efficient programming.

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W-PM-5-V

D. A. PIERRE, Montana State University, Bozeman

Multiplier Algorithms for Nonlinear Programming

A fundamental theorem and two solution algorithms are presented. The theorem relates constrained minima to unconstrained minima of an augmented Lagrangian on a one-to-one basis. The two algorithms differ only to the extent that one focuses on Kuhn-Tucker conditions and the other on Fritz John conditions. In both algorithms, a multiplier method of Hestenes for equality constraints is extended to account for inequality constraints: the extension is made in a unique way which results in desirable convergence properties. Namely, when applied to a general class of linear programming problems, the algorithms converge to the optimum in a finite number of iterations.

Computer programs that implement the algorithms are described, as also are test problem results. Saddle-point nonoptimal solutions to the Kuhn-Tucker conditions are shown to be unstable and are readily avoided. In one test case in which the Kuhn-Tucker first-order constraint qualification is not satisfied, the algorithm based on Fritz John conditions proved to be superior, as expected, but its rate of convergence was unsatisfactory. Suggestions are given for improving the rate of convergence in cases where the constraint qualification condition is not satisfied.

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M-PM-8-V

H. J. GREENBERG, W. P. PIERSKALLA, Northwestern University, Evanston

Quasi-Conjugacy and Nonlinear Surrogate Duality

The results in conjugate function theory, particularly the work of Rockafellar, have had a profound effect on our understanding the structure of many optimization models as well as other aspects in functional analysis. The basic results stem from the convexity structure involved in the extremum problems.

This paper develops an analogous, though less ambitious, theory of quasi-conjugate functions based upon quasi-convexity structure. Whereas conjugates relate to epigraph supports, quasi-conjugates relate to level set supports; where conjugates provide a basis for Lagrangian duality, quasi-conjugates provide a basis for linear and nonlinear surrogate duality.

F-PM-1-Y

I. POHL, University of California, Santa Cruz

A Model for Evaluating Enumerative Techniques

Artificial intelligence has treated certain constrained optimization problems from a different viewpoint than operations research. Pragmatics has preceded theory in such problem domains as automatic theorem proving and chess playing. Enumerative routines that are equivalent to branch-and-bound searches are used in a wide variety of artificial intelligence problems. Comparisons between various search techniques are empirical, without a proper distinction of the role of different pieces of the search routine.

This paper will discuss the heuristic path algorithm as a model of such searches. The algorithm suggests compromising the optimality of a solution path for a more efficient depth-first search. It does this by a non-uniform weighting of the bounding function.

Theorem: If m is the length of an optimal path in the state space, and the space is searched by HPA (Pohl, 1970), using an evaluation function

$$f(x) = g(x) + (1 + \epsilon - \epsilon\cdot w(x))\cdot h(x) \qquad 0 \leq w(x) \leq 1,$$

then m^*, the path length found by HPA, will satisfy $m^* \leq m(1 + \epsilon)$.

* * *

TH-PM-8-X

E. POLAK, University of California, Berkeley

* A Modified Secant Method for Unconstrained Minimization

In this paper we present a new gradient-secant algorithm for unconstrained optimization problems of the form $\min \{f(z) | z \in R^n\}$. Roughly speaking, in solving a problem, this algorithm uses gradient type iterations until it reaches a region where the Newton method is more efficient than the gradient method. Then it switches over to a secant form of operation. Under the assumption that f is continuously differentiable, any accumulation point z, of a sequence constructed by this algorithm, must be stationary. Under the stronger hypothesis that f is twice continuously differentiable and strictly convex, any sequence $\{z_i\}_{i=0}^{\infty}$ constructed by this algorithm converges superlinearly to the unique minimizer $\hat{z}$ of $f(\cdot)$, with rate τ^n, where τ^n is the unique positive root of $t^{n+1} - t^n - 1 = 0$, i.e., that for some $\theta \in (0,1)$ and some $R \in (0,\infty)$, $\|z_i - \hat{z}\| \leq R\theta^{\tau_n^i}$, $i = 0, 1, 2, \ldots$. Both theoretical considerations and our computational experiments indicate that this new algorithm is considerably faster than the Newton method, as well as a number of conjugate direction and quasi-Newton methods.

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M-PM-8-V

M. A. POLLATSCHEK, Technion, Israel Institute of Technology, Haifa

* Generalized Duality Theory in Nonlinear Programming

A constrained extremization problem, referred to as (D), is studied. (D) is related to the minimization of a real function (the objective) in n real variables, subject to m nonlinear inequality constraints, where the variables are confined to a set X. The latter problem is denoted by (P). Necessary conditions are derived for constructing (D) for a given (P) such that the extremal values of (P) and (D) are equal. Necessary conditions are also derived for a one-to-one correspondence between the extremizing variables of (P) and (D) under certain conditions or unconditionally. The (P) under consideration must satisfy some mild conditions, but it may involve non-(quasi-) convex functions or X may be a discrete set of points. Thus, the theory is richer than the conventional duality for convex (P). The presented framework includes the conventional duality, barrier/penalty functions and their merger as special cases of the underlying structure of generalized duality. The main tool is the perturbation function which is the optimal value of the objective as a function of the "right-hand side" of the constraints. Its extensive utilization yields global conditions on the optimum of (P) and functions having a saddle-point at the optimum of (P) and (D). A solution procedure for solving (D) is theoretically investigated for convergence and convergence-rate.

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T-PM-6-Y

S. POWELL, St. Hilda's College, Oxford

A Development of the Product Form Algorithm for the Simplex Method Using Reduced Transformation Vectors

This paper presents a product form version of the reduced basis algorithm for the simplex method.

The reduced basis is a sub-matrix of the coefficient matrix formed by the effective constraints and the basic original variables. The reduced basis algorithm for the simplex method stores, and updates at each iteration, the inverse of the reduced basis. The size of the reduced basis increases or decreases as a constraint is made effective or ineffective respectively. The product form algorithm using contracted transformation vectors given by Zoutendijk (Integer and Non-linear Programming, ed. J. Abadie) has been extended so as to be able to represent, and update at each iteration, the inverse of the reduced basis as the product of a sequence of transformation matrices.

This algorithm has been coded; time and storage comparisons with the product form algorithm, in solving some moderate sized and large linear programs, are presented.

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W-PM-11-Z

A. PRÉKOPA, Technological University of Budapest and Computer and Automation Institute of the Hungarian Academy of Sciences

* Recent Results in the Application of Logconcave Measures to Theoretical and Practical Problems of Stochastic Programming

A probability measure P defined in R^n is said to be logconcave if for every pair of convex sets $A, B \subset R^n$ and $0 < \lambda < 1$ we have $P(\lambda A + (1-\lambda)B) \geq [P(A)]^{\lambda}[P(B)]^{1-\lambda}$. The main theorem concerning these measures states that if P is generated by a probability density of the form $f(\underline{x}) = \exp[-Q(\underline{x})]$, $\underline{x} \in R^n$, where Q is a convex function, then P is a logconcave measure.

Based primarily on this theorem the author formulates stochastic programming decision problems and shows that they are convex--or equivalent to convex--programming problems. In all models probability of the following type appears $P(g_i(\underline{x}, \underline{\xi}) \geq 0,\ i = 1, \ldots, r)$, where $\underline{\xi}$ is a random vector. Three practical applications with numerical results will be presented. The first concerns the electrical energy sector of the Hungarian economy. The second is the solution of a reservoir system design problem. The third is the solution of an optimal water level regulation problem.

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T-AM-6-Y

M. RAGHAVACHARI, Indian Institute of Management, Ahmedabad, India

Efficiency of Least Square Estimates Relative to Best Linear Estimates in Regression Models

Consider the usual regression model: $Y = X\beta + U$ where Y is $n \times 1$, X is $n \times k$, β is $k \times 1$ and U is $n \times 1$. The error vector U has the properties: $E(U) = 0$, $E(UU') = \Gamma$. In this context it is well known that the efficiency of the least square estimate b relative to the best linear estimator $\hat{\beta}$ is given by $\mathrm{eff}(b) = |X'X|^2/|X'\Gamma X|\,|X'\Gamma^{-1}X|$. For $k > 1$ and $n > 2k-1$, G. S. Watson suggested the inequality, for all X of rank k,

$$\mathrm{eff}(b) \geq [4f_1f_n/(f_1+f_n)^2][4f_2f_{n-1}/(f_2+f_{n-1})^2]\cdots[4f_kf_{n-k+1}/(f_k+f_{n-k+1})]$$

where $0 < f_1 \leq f_2 \leq \cdots \leq f_n$ are the eigenvalues of Γ. For $k = 1$, this is true and can be proved by appealing to Kantorovich's inequality. For $k > 1$, it is not known whether the inequality is true. Using the tools and techniques of mathematical programming it has been shown that the inequality is true for all $k > 1$.

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T-AM-6-Y

H. RAMSIN, P. A. WEDIN, Royal Institute of Technology, Stockholm, Sweden

Numerical Treatment of the Nonlinear Least Squares Problem

In connection with nonlinear least squares problems, methods based on Gauss-Newton's method (e.g., the method due to Marquardt) sometimes show slow convergence from bad starting points. This may be due to the fact that these methods only regard information about the function at the present point in determining next direction of search. A method based on conjugate directions (e.g., a quasi-newton method) contains information from earlier steps but these methods may suffer from instability problems due to their recursive nature. Also Bard (1970) has shown that the special structure of the least squares problem should be exploited. We will discuss different existing algorithms for the nonlinear least squares problem. Some new algorithms based on a combination of the ideas mentioned above are proposed.

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F-PM-2-W

M. R. RAO, M. W. PADBERG, University of Rochester and International Institute of Management, Berlin

The Travelling Salesman Problem and a Class of Polyhedra of Diameter Two

A class of polytopes is defined which includes the polytopes related to the assignment problem, the edge-matching problem on complete graphs, the multi-dimensional assignment problem, and many other set partitioning problems. Using and modifying some results due to Balas and Padberg we give a constructive proof that the diameter of these polytopes is less than or equal to two. This result generalizes a result obtained by Balinski and Rusakoff in connection with the assignment problem. Furthermore, it is shown that the polytope associated with the travelling salesman problem has a diameter less than or equal to two. A weaker form of the Hirsch conjecture is also shown to be true for this polytope.

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TH-AM-4-V

D. C. RARICK, D. L. LINKIN, Management Science Systems, Rockville, Md.

An Algorithm for Solving Revised Models Efficiently

In the production usage of linear programming one often has an optimal or good feasible basis to model A. This model is revised, creating model B and the basis from model A is used as a starting point in solving model B. This starting point, although in some sense close to the optimal solution of model B, is often not feasible. By the time a feasible basis has been obtained, it is usually not close to the desired optimal solution.

This paper presents an algorithm for modifying model B to maintain feasibility and to apply a modest driving force where necessary to drive the columns to a feasible activity in the unmodified model. The result is that the ill effects of Phase I are avoided and savings in the number of iterations, run time, and cost are effected. This algorithm has been automated by Management Science Systems in their MPS III package and is currently being used by several different affiliates of Exxon Corp. on a production basis. Comparative performance data showing the effectiveness of the algorithm is presented. These results range from essentially no improvement (when the starting basis is feasible) to runs that require 1/5 the number of iterations and only 1/5 the time.

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W-AM-2-X

R. C. DORSEY, T. J. HODGSON, H. D. RATLIFF, University of Florida, Gainesville

A Network Approach to a Multi-Facility, Multi-Product Scheduling Problem with Backordering

A multiple-facility, multiple-product production-inventory scheduling problem with backordering is considered over a finite planning horizon. The horizon consists of discrete production periods during which at most one product can be assigned to each facility. Product demands are assumed to be constant over a period but not necessarily the same in all periods. The problem objective is to determine an assignment of products to facilities which meets all product demands on a first-come, first-served basis and minimizes the sum of production, inventory and backordering charges over the horizon. In a straightforward manner this problem is formulated as a linear, mixed integer program which can be given a network flow interpretation. However, the problem can be solved as a minimal-cost flow problem only for a very special case. It is then shown that the general problem can be reformulated as an all integer program which can be solved using any of the very efficient algorithms for finding minimal-cost flows in single-commodity networks.

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F-PM-3-X

J. K. REID, A.E.R.E., Harwell, England

Sparse Linear Programming Using the Bartels-Golub Decomposition

The Bartels-Golub algorithm has numerical stability properties which are far superior to those of the standard product-form algorithm, but usually it is associated only with small dense problems. We show here that it is also very suitable for use on sparse problems. We have developed an in-core program and report the results of its application to medium-sized problems, the largest of which has 822 constraints, 1875 variables and 11432 non-zeros in the constraint matrix. The fill-in properties are very satisfactory, being similar to those obtained with the Forrest-Tomlin algorithm and sometimes far better than those obtained by using Cholesky factorization, another stable algorithm. Direct comparisons on numerical accuracy are made with the Forrest-Tomlin algorithm. Schemes for out-of-core implementation are sketched.

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TH-PM-5-Z

G. V. REKLAITIS, D. J. WILDE, Purdue University, Lafayette

Degeneracy in Mathematical Programming Algorithms Using Implicit Variable Elimination

An important class of Mathematical Programming algorithms including the (Linear) Simplex Method, the Convex Simplex Method, the Differential Algorithm, and the Generalized Reduced Gradient Method use Implicit Function Theorem constructions both to determine which independent (non-basic) variables are to be perturbed as well as to calculate the resulting changes in the dependent (basic) variables. These constructions necessarily require the Jacobian of the constraints to have maximal row rank and the dependent variables to be non-zero (non-degenerate). In the linear case, the former condition is readily verified and if need be corrected during the initiation phase of the algorithm, while violation of the latter is conventionally accepted without corrective measures since the cycling which degeneracy can initiate rarely occurs. However, in the presence of nonlinearities, violations of these conditions do require special attention since they can either force premature termination of the algorithm, or require time consuming re-inversions, or cause the algorithm to cycle.

This paper shows that singularity and degeneracy can be treated as essentially the same phenomenon. Moreover, it presents constructions based on a differential form of Fritz John's Theorem which allow proper descent directions to be generated when these pathologies occur. The constructions are compatible with the structure of this class of algorithms and are demonstrated with numerical examples.

T-AM-9-2

M. J. RIJCKAERT, L. J. HELLINCKX, Katholieke Universiteit Leuven, Belgium

Computer Implementation of a Dual Geometric Programming Algorithm

The paper reports on the computational experience obtained with a subroutine for solving geometric programs by using the classical dual normality and orthogonality conditions together with the non-linear equilibrium conditions.

In particular the impact of different procedures for calculating starting points for such algorithms will be discussed. A relative evaluation of such procedures, which mainly use the linear dual constraints, will be presented. The influence of the problem formulation and of the numerical accuracy of the different steps of the algorithm on the numerical results and on the computing time will be closely examined too.

The results of the computational tests carried out on a IBM-370, apply as well for geometric programs in signomial as in posynomial form and can equally be used for other algorithms devised for the dual geometric program.

Finally, the use of the above algorithm for sensitivity analyses of geometric programs will be demonstrated.

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TH-AM-8-X

K. RITTER, Rutgers College, New Brunswick, New Jersey

* Accelerating Procedures for Methods of Conjugate Directions

Methods of conjugate directions have been computationally successful for the problems of minimizing an unconstrained function $F(x)$ of n variables. Under appropriate assumptions on the second derivative of $F(x)$ it has been shown that the rate of convergence is n-step superlinear. For some particular methods of conjugate directions the information accumulated in n consecutive directions of descent can be used to accelerate the convergence by performing special steps at well defined points.

Extensions of the method of conjugate directions to minimization problems with linear inequalities are given and it is shown that in this case the convergence can also be accelerated by using previously determined directions of descent in an appropriate way.

TH-PM-5-V

S. M. ROBINSON

Convex Processes and Mathematical Programming

Convex processes (multivalued functions whose graphs are convex cones containing the origin) play an important role in the analysis of several questions in mathematical programming. For example, one can use them to develop a quantitative perturbation theory for linear inequalities which generalizes the classical theory for linear equations. One can also prove generalizations of a number of well-known results, such as the inverse-function theorem. The most important advantage to be gained from the use of convex processes in many situations is that they permit one to deal with multivalued quantities (such as the solution set of a system of linear inequalities) using the same techniques and notation as are used in the single-valued case. This greatly simplifies the analysis and also furnishes a very valuable guide in formulating and proving new theorems.

In this survey we present some of the main results about convex processes and then apply these to the solution of a number of computational problems in linear and nonlinear programming.

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M-PM-8-V

R. T. ROCKAFELLAR, University of Washington, Seattle

* Augmented Lagrange Multiplier Functions and Duality in Nonconvex Programming

If a nonlinear programming problem is analyzed in terms of its ordinary Lagrangian function, there is usually a duality gap, unless the objective and constraint functions are convex. This gap can be removed by passing to an augmented Lagrangian which involves quadratic penalty-like terms. The modified dual problem then consists of maximizing a finite concave function of the Lagrange multipliers and an additional variable, which is a penalty parameter. The multipliers are not constrained a priori to be nonnegative. If the maximum in the dual problem is attained, optimal solutions to the primal can be represented in terms of global saddle points of the augmented Lagrangian. This suggests possible improvements of existing penalty methods of computation. Dual optimal solutions can be shown to exist if the primal has a "strong" optimal solution at which the standard second-order sufficiency conditions hold, or more generally if a second-order stability condition is satisfied with respect to perturbations.

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T-PM-2-X

R. R. ROSANDER, The Foxboro Company, Foxboro, Massachusetts

Multiple Pricing and Suboptimization in Dual Linear Programming Algorithms

This paper discusses the successful implementation of multiple pricing and suboptimization in a product-form dual algorithm. Dual pricing is shown to result in a larger reduction of problem infeasibilities per major dual iteration, and dual suboptimization is shown to be effective in reducing both I/O and CPU time when the LP problem resides on auxiliary storage. For thos problems solved, dual pricing and suboptimization have on the average, decreased total elapsed time by 45.7%.

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J. B. ROSEN, University of Minnesota, Minneapolis

P Interactive Computer Graphics and Mathematical Programming

The use of interactive computer graphics, combined with mathematical programming, for solving a variety of problems will be discussed. Examples of interactive computer graphics as an aid in finding global solutions to optimization problems will also be presented.

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W-PM-11-Z

I. M. ROSENBERG, Rochester Institute of Technology, Rochester

Solving the Data Base Aggregation Problem

Present information system designs are derived from a "needs" basis. Decision maker requests for information are taken as "givens" and only the storage and report generation costs are optimized.

The contribution of the work reported in this paper is that for the first time the information needs of many decision makers within an organization are considered simultaneously and jointly with the costs and benefits to the organization data system as a whole. Given a set of decision makers and the organization goals, one can determine who should receive what information, what form the information should take, and what data the central data base should contain. The analysis also points out the relative importance of specific information data elements by demonstrating the sensitivity of the organization's payoff to how accurately and who knows that information.

An algorithm is developed as an extension of the team decision theory of Marschak and Radner using mathematical programming under uncertainty. The theoretical basis, as well as procedural details, of the algorithm are discussed. Application to a specific example is used to illustrate the sub-optimality of presently considered "standard" information structures, and provides statistics on the efficiency of the method.

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W-PM-2 -X

D. KLINGMAN, A. NAPIER, G. T. ROSS, University of Texas, Austin

A Computational Study of the Effects of Problem Dimensions on Solution Times for Transportation Problems

This paper presents an indepth study of the influence of problem structure on the computational efficiency of the primal simplex transportation algorithm. The input for the study included over 1000 randomly generated problems with 185 different combinations of the number of sources, the number of destinations and the number of variables. Every problem was solved using three different starting procedures, and the following data were collected for each problem:

1) time required to obtain an optimal solution
2) time required to obtain an initial basic solution
3) number of artificial variables in the initial basic solution
4) number of basis changes
5) average time to perform a change of basis
6) average number of basic variables in the "stepping stone path" in each change of basis
7) average number of variables considered before selecting one to enter the basis

These measures of performance provide numerous insights into the computational effects of the number of constraints, the degree of "rectangularity" and the number of variables. In particular the study demonstrates that no single starting procedure dominates the others; rather the efficiency of the starting procedures vary with problem structure.

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F-PM-2-W

U. G. ROTHBLUM, Stanford University, Stanford

On the Number of Complementary Trees in a Graph

Consider the undirected network $G = (N,\alpha)$ where $|N| = n+1$, $\alpha = \{a_1, \ldots , a_n, \bar{a}_1, \ldots , \bar{a}_n\}$, $n \geq 3$, and there are no loops or repeated arcs. Let $\eta(n)$ denote the class of all networks that have the form of G. A complementary tree T of $G(G \in \eta(n)$ for some $n \geq 3)$ is a spanning tree of G with the property that $a_i \in T$ iff $\bar{a}_i \notin T$.

It was proved by Dantzig that if there exists one complementary tree in G $(G \in \eta(n))$, then there exists at least two. His proof is by means of an algorithm which finds a different complementary tree from a given one. Adler extended this result and showed by an extended form of Dantzig's algorithm that: if there exists one complementary tree in G $(G \in \eta(n))$, then there exists at least four. Moreover, Adler gives lower bounds on the number of complementary trees in a network G $(G \in \eta(n))$ which has at least one. These bounds are 8 if $n \geq 5$, 6 if $n = 4$ and 4 if $n = 3$. It is shown in this paper that using a different algorithm we get the stronger result that: if there exists one complementary tree in G $(G \in \eta(n), n \geq 4)$, then there exist at least 6 complementary trees. It is also proved that 6 is the greatest lower bound for the number of complementary trees in a network G $(G \in \eta(n), n \geq 4)$ which has at least one complementary tree.

F-PM-3-X

R. S. SACHER, Stanford University, Stanford

On the Solution of Large, Structured Linear Complementarity Problems

The linear complementarity problem (q,M) is: Given $q \in R^n$, $M \in R^{n \times n}$, find $w, z \in R^n$ such that $w = Mz + q$, $w^T z = 0$, $w \geqq 0$ and $z \geqq 0$. Previous research on this problem has yielded a large body of theory on the existence and number of solutions under conditions placed on M and q. Lemke, Cottle and others have proposed algorithms using various pivotal schemes which were based on such attributes of M as positive definiteness, positive principal minors, nonpositive off-diagonal entires, etc. Since these solution techniques must store all nonzero entries of the matrix, storage requirements usually become prohibitive for large problems, even if M is a sparse matrix. For example, if M is a tridiagonal Minkowski matrix of order n, it is possible for pivoting to cause the number of nonzero entries to become as large as n^2.

The form of large, structured, sparse matrices (e.g., tridiagonal) is exploited to mitigate this storage problem. Efficient algorithms are developed which vastly reduce the growth of nonzero entries. Problems of this type arise in engineering. Encouraging computational experience and comparisons with other solution techniques are also reported.

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TH-PM-7-Y

R. SAIGAL, D. SOLOW, L. WOLSEY, Bell Telephone Laboratories, Holmdel

* A Comparative Study of Two Algorithms that Compute Fixed Points in Unbounded Regions

We will report on an attempt to compare, empirically and theoretically, the algorithm of Merrill and Eaves-Saigal for the computation of fixed points in unbounded regions. Sufficient number of problems in various fields have been solved by the former algorithm. We will report results on the same as solved by Eaves-Saigal algorithm.

By looking at the structure of the two algorithms, we will point out the theoretical differences between them. We also find triangulations which improve the efficiency of these algorithms, and show that the Eaves-Saigal algorithm requires in the order of n^2 pivots to improve the accuracy of the solution (decrease the grid) at each step, where n is the dimension of the problem.

W-AM-1-W

H. M. SALKIN, A. TAMIR, Case Western Reserve University, Cleveland

An Exposition of Group Theory in Integer Programming

The main objective of this paper is to provide an understandable and introductory presentation of the relationships and uses of group theory in integer programming. Though introductory, the discussion is self-contained while definitions are usually illustrated by examples.

Following the derivation of the group problem from the related integer program, several solution techniques (e.g., dynamic programming, network formulations) are presented and examples are solved. Further discussion leads to the introduction of an isomorphic group. The polyhedra associated with the feasible regions of the two isomorphic groups are studied and it is shown how their faces can yield useful, valid inequalities for the integer program.

A brief and concise presentation of recent results as well as computational experience are also included.

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F-PM-3-X

C. L. SANDBLOM, The University of Birmingham, Birmingham, England

Theoretical Properties and Numerical Tests of an Efficient Nonlinear Decomposition Algorithm

A new nonlinear decomposition algorithm is presented, based on the nonlinear generalization of Benders' algorithm by T. O. M. Kronsjö. At each iteration, our algorithm produces upper and lower bounds to the true optimum. The sequence of lower bounds produced by the iterations of our method is increasing, and similarly the sequence of upper bounds is decreasing. Numerical tests show that our method performs on the average 30% better than Kronsjö's method as found by K. P. Wong, who in his turn demonstrated that the Kronsjö decomposition algorithm greatly reduced the solution time for a nonlinear decomposable problem as compared to the direct solution. Further attention is devoted to the question of the optimal degree of decomposition on the lines of the approach made by the author.

W-AM-5-V

R. W. H. SARGENT, Imperial College, London

On the Convergence of Sequential Minimization Algorithms

This note discusses the conditions for convergence of algorithms for finding the minimum of a function of several variables which are based on solving a sequence of one-variable minimization problems. Theorems are given which contrast the weakest conditions for convergence of gradient-related algorithms with those for more general algorithms, including those which minimize in turn along a sequence of uniformly linearly independent search directions.

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M-PM-8-V

H. SAYAMA, Y. KAMEYAMA, H. NAKAYAMA, Y. SAWARAGI, Univ. of Okayama, Japan

The Generalized Lagrangian Functions for Mathematical Programming Problems

For the mathematical programming problem

$$\min \{f(x) \mid g_i(x) \geqq 0,\ i = 1, \ldots , m,\ x \in E^n\} ,$$

a new type of the generalized Lagrangian function is given by

$$L(x,\sigma^k,t) = f(x) + \sum_{i=1}^{m} \begin{cases} tg_i^2(x) - \sigma_i^k g_i(x) , & \text{if } g_i(x) \leqq 0 \\ \dfrac{-(\sigma_i^k)^2 g_i(x)}{\sigma_i^k + tg_i(x)} , & \text{if } g_i(x) \geqq 0 \end{cases} ,$$

where $t > 0$ and $\sigma_i \geqq 0$, $i = 1, \ldots , m$, the Lagrange multipliers. Suppose x^k minimizes $L(x,\sigma^k,t)$, then σ_i^k is altered by the simple rule,

$$\sigma_i^{k+1} = \begin{cases} \sigma_i^k - 2tg_i(x^k) , & g_i(x^k) \leqq 0 \\ \dfrac{(\sigma_i^k)^3}{[\sigma_i^k + tg_i(x^k)]^2} , & g_i(x^k) \geqq 0 \end{cases} .$$

It is proved that a sequence of minima of $L(x,\sigma^k,t)$ converges to the local minimum value of the original problem as proceeding in this fashion. The features of this function are that it is a class of the generalized Lagrangian and is twice continuously differentiable at the boundary of the feasible region.

The method deals with the non-convex programming problems and determines the Lagrange multipliers as well as the solution of the original problem. The numerical difficulties encountered with existing penalty methods are avoided. The numerical examples are presented to illustrate the typical convergence characteristics of the method.

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F-PM-5-V

S. SCHAIBLE, University of Cologne, Cologne, West-Germany

On Nonlinear Fractional Programming: Transformation, Duality and Algorithms

The fractional programming problem

$$\text{(Q)} \qquad \operatorname*{Max}_{x \in C} \frac{\prod_{i=1}^{m} (f_i(x))^{a_i}}{(f_o(x))^{a_o}}$$

is considered where $f_o: C \to R$ is convex and positive, $f_i: C \to R$ is concave and positive, $i = 1(1)m$, $a_i > 0$ $i = 0(1)m$ and C is a nonempty convex set in R^n.

Recently problems of this kind were related to parametric concave programming problems. In a first part some simple transformations are introduced that reduce large classes of problems (Q) to parameter-free convex programming problems. The results remain valid for fractional programming problems in infinite dimensional real linear spaces.

Furthermore a dual problem of Q is defined assuming differentiability of f_i such that a weak duality theorem as well as two other duality theorems can be obtained.

It is shown that some special cases of (Q) can be solved by a dual method. For the solution of more general problems iterative procedures are proposed that use also some duality properties. Apart from convergence the rate of convergence is investigated.

F-PM-2-W

S. SCHINDLER, Technische Universitat Berlin, Berlin

Scheduling Schemata

The problem of scheduling N tasks--the operational precedence structure of which is represented as a finite, acyclic, directed, weighted graph G--on a multiprocessor system consisting of M identical processors is studied. The weight W_I of node I, $1 \leq I \leq N$, we regard as the processing time of the task represented by node I, and we want all N tasks to be processed completely within total processing time CT. We assume that preemptions of all tasks are allowed.

The paper contains the mathematical representation of the uniquely defined scheduling schemata S(G,M,CT) (intuitively used already in the author's previous papers) that belong to arbitrarily given parameters G, M and CT. Each interpretation (there are infinitely many, constituting the set $\underline{I}$(G,M,CT)) is a schedule for the M processors such that the processing of all tasks of G is completed within time CT and each schedule with these properties is an interpretation of S(G,M,CT).

To derive S(G,M,CT) an algorithm for solving a system of linear inequalities (the structure of which depends on the type of precedence relation of G) is needed which has to be very effective in order to justify the investigations by practical advantages, too. For the cases that G is a forest or an anti-forest and M and CT are arbitrary and the case that G and CT are arbitrary and M = 2 the schemata S(G,M,CT) are given and it is shown that all previously known results are elements of the corresponding $\underline{I}$(G,M,CT). The complexities of the algorithms used and the schedules derived are discussed. The correctness of all algorithms described is proven.

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TH-PM-8-X

M. D. CHOIT, G. F. SCHRACK, The University of British Columbia,
Vancouver, B.C., Canada

Optimized Relative Step Size Random Searches

This paper continues studies of Riga, Rastrigin, Mutseniyeks, Schumer, and Steiglitz concerning random search methods for optimizing functions of several variables.

We propose an algorithm called the Optimized Relative Step Size Random Search with Reversals which is an improvement of the Optimum Step Size Random Search. First we derive recursion formulas for the probability of success and the expected improvement per function evaluation for the two searches referred to above. Next we investigate a parameter called the relative step size η which plays an important role in the theory of these searches. It can be shown that optimum values of η exist which depend on the dimension N only. With a suitable strategy, the value of an initial relative step size can be held constant during the search, on the average. Furthermore, provided an initial estimate of η is available, the optimum relative step size can be held constant, thus providing a maximum decrease of the function. However, such an initial estimate is usually not available a priori to a search, and hence must be estimated. We derive a suitable estimation procedure. Numerical results are given.

Finally, we present for both searches and for a number of selected dimensions N numerical results for the probability of success and the expected improvement per function evaluation in the form of graphs, tables of the optimum relative step sizes, and a table for an updating parameter for our search. The numerical results depicting the behaviour of an implementation of the Optimized Relative Step Size Random Search with Reversals on a number of standard test functions will be given. The algorithm becomes more stable as the dimension N increases; the number of function evaluations to reduce the objective function by a fixed amount increases only linearly as the dimension increases.

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W-AM-5-V

H. SCHULTZ, University of Wisconsin, Oshkosh

Newton Projection

This work consists of an extension of the idea of projecting the gradient direction into a set to that of projecting the Newton direction (the vector obtained via the classical Newton's methods) into a set for the problem of minimizing a nonlinear function subject to linear constraints. Convergence is proved using general sufficiency conditions for algorithmic convergence and finite convergence is proven for the special case of a quadratic objective function.

M-PM-12-Y

L. E. SCHWARTZ, University of Utah, Salt Lake City

A Fixed Point Algorithm for Distributed Control Systems of Retarded Type

A fixed point algorithm is developed that solves, under certain conditions, a class of distributed parameter optimal control problems. Evolution of the model is governed by systems of functional nonlinear partial differential-difference equations with retarded arguments. It is assumed that the linear operators of the model are not self-adjoint so that no standard maximum principle is applicable to the task of deducing optimal solutions. After appropriate discretization, an approximating set of ordinary differential equations is replaced by a finite-dimensional convex program. The programming problem is then cast as a fixed point one. An algorithm similar to those developed by Hansen and Scarf, Kuhn, Eaves, and Garcia, Lemke, and Luethi is thus applicable, and one for this problem is developed. Finally, applications of the algorithm are discussed in several economic contexts.

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T-AM-13-V

J. SCHECHTMAN, Universidade Federal do Rio de Janeiro, Brazil

We will be concerned with a one-good economy. The good can be used at any period of time for production or consumption. If $\underline{x}$ units are put into production in period $\underline{t}$ then $f^t(x;\omega^t)$ units become available as outputs in period $\underline{t+1}$, where ω^t is a random variable with known distribution. If $\underline{c}$ units are consumed in period $\underline{t}$ this produces $u^t(c)$ units of satisfaction or utility to the society in that period. Our main interest is the study of qualitative properties of optimal solutions for a problem in which we maximize the total expected utility accumulated in $\underline{t}$ periods.

In deterministic cases, i.e., the output is known with certainty, the basic tools are prices and competitive policies. D. Gale has suggested the introduction of similar concepts for the stochastic cases, and it turns out that after this is done, we get a better understanding of the problem and a powerful tool in the proof of theorems. Our main purpose here is to introduce the appropriate price concept and then exploit it in several directions to obtain new information on various stochastic problems.

The concepts of price and competitive policy are introduced, and it is shown that a competitive policy is optimal. For $f(x;\omega)$ and $u(c)$ increasing, differentiable and strictly concave, optimality conditions are obtained and it is shown that every optimal policy is competitive. For the case in which $f(x;\omega) = g(x) + \omega$, $f'(x;\omega)$ and $u'(c)$ are convex functions we obtain a result that permits us to compare optimal consumption policies with the corresponding policies of a deterministic case in which $f(x;\omega) = g(x) + \bar{\omega}$, where $\bar{\omega} = E\omega$. Finally the case $f(x;\omega) = x + \omega$ is studied in more details and it is shown that the limiting policy satisfies the following inequalities $0 < c(y) < \bar{\omega}$ for $0 < y < +\infty$, and that $\lim_{y \to +\infty} c(y) = \bar{\omega}$.

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T-PM-2-X

H. D. SCOLNIK, Fundacion Bariloche, Rio Negro, Argentina

A New Approach to Linear Programming

Given a standard LP problem $\mathrm{MIN} < C,x >$ subject to $Ax = b$, $x \geq 0$, where A is a $m \times n$ matrix, a completely new algorithm is described in this paper which is able to give the solution in a fixed number of arithmetic operations which is a polynomial function of m and n.

The procedure is numerically stable and it is able to deal very efficiency with sparse matrices. Numerical experiences are presented showing that this algorithm is much faster than the simplex method. This result also follows using Wolfe's approximate formula for the number of iterations.

The method is based upon new results on convexity.

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M-PM-12-Y

S. P. SETHI, G. B. DANTZIG, R. E. DAVIS, University of Toronto, Toronto, Canada

Generalized Programming and the Bounded-State Linear Control Problem

In this paper, an attempt is made to extend the work of Dantzig, which solves the linear control problems by Wolfe's generalized program, to apply to the bounded-state linear control problems. The difficulty arises on account of the fact that in solving the subproblem, which is a parametric linear program whose cost function is supplied by the master problem, we may now encounter one or more states hitting or leaving the constraint boundary, say, at time t. We, then, introduce at time t, a cut which will have a price associated with it. It will be shown that this price is a finite jump at the point of entry to and exit from the boundary and an infinitesimal along the boundary. The main idea of the paper is to accumulate and propagate these prices along time to modify the cost function of the subproblem as we go along, and hopefully solve the problem. It is noted that this mechanism is consistent with the Pontyagin's maximum principle formulation of the problem where the adjoint variables encounter a jump at the junction of the interior and the boundary. Several questions remain open before a computational algorithm can be developed along these lines. However, short of developing a complete algorithm, the paper explores several ideas leading to it. It is certain that some sort of nesting is required. The paper concludes with an important contribution in terms of indicating a number of problems for future research.

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TH-AM-8-X

D. F. SHANNO, University of Toronto, Toronto, Canada

Quasi-Newton Methods and Brown's Method

Brown's method for solving systems of nonlinear equations uses a Gauss-Seidel type technique to provide an interesting variant on Gaussian elimination in solving the linearized system. This paper applies the method to unconstrained function minimizations, but uses Quasi-Newton methods and a new technique based on the elimination to approximate the Hessian matrix. N-step convergence to the minimum of a quadratic form is shown for both methods. Experimental results show that used properly, the method definitely improves on standard methods for some classes of standard test functions.

T-AM-1-V

M. L. FISHER, W. D. NORTHUP, J. F. SHAPIRO, Massachusetts Institute of Technology, Cambridge

Computational Experience with Duality in Discrete Optimization

Algorithmically useful dual problems have recently been derived for a number of discrete optimization problems. Computational experience is discussed using these dual problems in conjunction with enumerative algorithms. The dual problems provide strong bounds and candidate solutions for the enumerative algorithms and they also provide information about the relative cost of variables for directing the enumeration.

Two algorithms which have been developed for solving these dual problems are emphasized. The first is an ascent type algorithm which uses a synthesis of primal-dual simplex method with constraint generation procedures. The other is based on the relaxation method for solving systems of linear inequalities. Specific problems to be considered will include the general integer programming problem, the resource constrained network scheduling problem, and the traveling salesman problem. We assess the strength of the bounds obtained from the duals for these problems and the efficiency of the resulting enumerative algorithms.

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TH-PM-8-X

R. SHAPIRO, D. J. WILDE, Stanford University, Stanford

Sequential Minimax Search with Unequal Block Sizes

Consider the optimization of a unimodal, univariate function by a sequence of simultaneous function evaluations. This article removes previous restrictions on the number of simultaneous function evaluations in each sequential block. Recursive equations are derived for the optimal search strategy, determining the length of the maximal initial interval. These equations, which subsume previous results for Fibonacci, as well as even and odd block search, give the optimal plan for the possibly unequal distribution of a fixed total number of function evaluations among a given number of blocks. An example is given concerning an optimal corporate test program.

W-PM-10-Y

L. S. SHAPLEY, The Rand Corporation, Santa Monica

* A Noncooperative Game Model of Economic Equilibrium

We consider an exchange economy formulated as a noncooperative game, in such a way that each trader will take into account the effect of his trading decisions on the prices. There are N traders and $m + 1$ goods, the last good playing a special monetary role. Each trader i has a concave, nondecreasing utility function u^i and an initial bundle $a^i \geqq 0$; we assume $\bar{a}_j > 0$ for each good j. (The bar denotes summation over all traders.) All goods, except "money," must pass through the market before being consumed. A strategy for i consists of an m-vector $b^i \geqq 0$ of "bids", satisfying $\sum_1^m b_j^i \leqq a_{m+1}^i$. The final distribution of goods is given by

$$x_j^i = (\bar{a}_j/\bar{b}_j)b_j^i \quad (\text{or } 0 \text{ if } \bar{b}_j = 0) \quad \text{for } j \leq m ,$$

and

$$x_{m+1}^i = a_{m+1}^i - \sum_1^m b_j^i + \sum_1^m(\bar{b}_j/\bar{a}_j)a_j^i .$$

The payoff function of the game is then given by $P^i(b^1, \ldots , b^N) = u^i(x^i)$. Under suitable conditions, a noncooperative equilibrium in pure strategies exists. If the economy is "replicated", so that there are kN traders grouped into N types, with a^i and u^i depending only on the type of i, then if there is sufficient "money" the noncooperative solution approaches the competitive solution when k is large.

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TH-AM-4-V,

S. N. T. SHEN, Virginia Polytechnic Institute and State University, Blacksburg

Computer Solution of Linear Programming Problems Stated in English

With the technology of computer time sharing and remote terminals, more and more people are able to use computers to solve problems while sitting in their offices or homes. Recent developments in computer hardware and communication facilities have been used by researchers to predict a reasonably priced computer information utility to come into being in the near future. Considering such a utility, one can understand that it would be very convenient, if not necessary, to have a computer that can take problems stated in English and solve them.

A system has been developed to process certain types of linear programming problems stated in a prescribed grammar, with limited vocabulary and subject to certain conventions. The system is designed in a modular form aiming at a "general-purpose" natural language information system. Input sentences are parsed and interpreted by the system. Intermediate information structures are then built. It further recognizes the variables, builds the objective function, and formulates the constraints. The output of the system is the input problem in formula form, which can be easily converted into any specific format for input to a particular linear programming solution program available at a computer installation. The system is implemented in the SNOBOL 4 programming language. Many problems have been run under this system with satisfactory results.

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TH-PM-2-W

R. L. SIELKEN, JR., Texas A&M University, College Station

A Transportation Problem Involving Source-Location Optimization

A realistic generalization of the capacitated plant-location problem is formulated and a simple, concise, iterative solution algorithm proposed. The algorithm's performance characteristics in several sample problems as well as its theoretical and practical properties indicate the algorithm's attractiveness as a method for determining near optimal solutions to source-location problems. The computation of statistical confidence limits between which the value of the optimal solution is guaranteed to lie with a specified statistical confidence is also discussed.

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T-PM-5-Z

G. SILVERMAN, IBM Los Angeles Scientific Center, Los Angeles

Equipment Location in Remotely Piloted Vehicles by Integer Programming

Remotely piloted vehicle (RPV) is a term describing a small unmanned aircraft used for target, reconnaissance and other uses when a pilot is not required. As the main justification for RPV's is their small size and cost the layout of controls, propulsion and payload equipment is a particularly important part of the design process.

The problem is formulated as a three-dimensional packaging problem to minimize wasted space and complexity of electrical interconnections. The equipment geometry and interconnections are given as is the geometry of the vehicle. The constraints include fitting all the equipment in the space available as well as such environmental constraints as accessibility, vibration, heat, acceleration, center of gravity movement, and moment of inertia.

An integer programming problem is derived with special emphasis on realistic RPV design. An example is given of a real RPV design problem solved by a branch-and-bound scheme designed to take advantage of the problem structure. Preliminary computational results are given.

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TH-AM-8-X

E. SPEDICATO, CISE, Segrate (Milano)

Some Algorithms for Unconstrained Minimization Based on the Homogeneous Model

Many efficient methods for the unconstrained minimization of functions with continuous first derivatives have been recently proposed in the literature, in particular methods of quasi-Newton type (where an approximation is made to the inverse hessian) and of the Fletcher-Reeves type (where conjugate vectors are directly constructed). Both classes of methods are based on a (convex) quadratic model. Few attempts have been made to construct a minimization algorithm from a more general functional model; apart from the fine theory of geometric programming, the most promising approach seems to be the homogeneous model used by Jacobson-Oksman and Fried. Jacobson and Oksman constructed directly the unknown parameters of the model, while Fried introduced an efficient correction to the Fletcher-Reeves scheme.

Here we consider the updating of the inverse hessian starting from the following homogeneous model

$$F = \frac{1}{2r} (x^T K x)^r \tag{1}$$

where r is the degree of homogeneity and K is positive definite. Without loss of generality, the minimum value is $F = 0$ at $x = 0$. Basic to the derivation of updating formulas for the approximate inverse hessian is the consideration of the exact inverse hessian (g is the gradient of F)

$$(2) \qquad G^{-1} = (x^T Kx)^{1-r} K^{-1} - \frac{2(r-1)x^T Kx)^{2-3r} K^{-1} gg^T K^{-1}}{1+2(r-1)(x^T Kx)^{1-2r} g^T K^{-1} g}$$

showing that the homogeneous model introduces a scaling on the inverse hessian K^{-1} of the quadratic model plus a rank-one correction. The algorithms proposed are

a) the subclass of Huang's class pertaining to the Huang parameter ρ equal to $\frac{1}{1+2(r-1)}$. This subclass arises when considering the quasi-Newton equation satisfied by the homogeneous function. Here r is iteratively evaluated using Fried's method.

b) a scaled projection algorithm of the form

$$H' = \gamma H + \xi H y y^T H$$

when H, H' are the approximate inverse hessians, y is the difference in the gradients and the scalar parameters γ, ξ depend on r, H, g and the linear search parameter.

c) the subclass of Oren's class where the scaling factor γ is the same as in (b), modified by adding a rank-one term proportional to pp^T, p being the displacement vector.

Conditions of maintenance of positive definiteness for the above updates are given and numerical comparison is made with the Broyden-Fletcher-Shanno-Goldfarb algorithm and the bounded-condition-number algorithm formerly proposed by the author.

Finally a variation of the Fried algorithm is presented, which keeps conjugacy of the last two vectors even when non-exact linear searches are performed. Its superiority with respect to similar methods is experimentally shown.

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M-PM-1-W

K. SPIELBERG, IBM Philadelphia Scientific Center, Philadelphia

* Minimal Preferred Inequalities, Penalties and Structure in Zero-One Programming

At any given point in branch and bound or enumerative programming, a set of minimal preferred inequalities gives some information about the structure of the problem, such as the (integer) tightness of individual constraints. One may make predictions as to how a contemplated branch might affect the tightness of ensuing auxiliary problems.

Certain large classes of relatively tight problems are easy in the sense that they permit "double contracting" branches, i.e., choices of variables with increased tightness in each generated subproblem. When a problem does not have this property, one may at least choose variables so that the branch with lower penalty is in a contracting direction.

These considerations add further interest to an earlier proposed preferred variable branch and bound scheme, in which penalties are associated with minimal inequalities and non-basic variables play a significant role. Minimal inequalities are also useful in enumeration and in an all integer zero-one algorithm. Numerical results will be given.

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TH-PM-2-W

V. SRINIVASAN, G. L. THOMPSON, The University of Rochester, Rochester

Choosing Modes of Transportation to Minimize Total Costs and Average Shipment Times

This paper provides a framework for choosing modes of transportation (Rail, Highway, Air, etc.) by taking into account the conflicting objectives of minimizing total transportation costs and average shipment times. An efficient algorithm using the Operator Theory of Parametric Programming is presented for determining the Pareto-optimal or efficient curve denoting the minimum attainable value for the second objective for differing values of the first objective. The algorithm also provides the optimal routes, modes of transportation, and the corresponding shipping amounts for any efficient point. The methodology is also applicable in (1) assigning men to jobs where alternate selection criteria may be employed, and (2) in designing the structure and content of a job taking into account the conflicting objectives of productivity and job satisfaction.

TH-AM-8-X

J. STEIN, University of Toledo, Toledo

The Gram-Schmidt Conjugate Direction Method and the Method of Parallel Planes

These methods are iterative techniques for minimizing functions of n-variables without constraints. M. R. Hestenes and E. Stiefel developed the conjugate gradient method and more generally the conjugate direction method for minimizing the sum of a quadratic and linear function of n-variables. R. Fletcher and C. M. Reeves extended the conjugate gradient method to minimization of any function of n-variables. Recently, Hestenes has formulated a variety of conjugate direction methods based upon a Gram-Schmidt process. They minimize a quadratic function in a finite number of steps and can be extended in a variety of ways to minimization of any function of n-variables. R. Dennemeyer and E. Mookini have investigated such extensions. Some of these methods require neither gradient evaluations nor line searches. Furthermore, they appear to be much more efficient than the Fletcher-Reeves method for small values of n. This class of methods is related to the one discussed by R. Fletcher and developed by G. S. Smith in Great Britain.

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T-PM-6-Y

G. W. STEWART, IBM Watson Research Center, Yorktown Heights, New York

A Stable Implementation of the Second Method for Solving Systems of Nonlinear Equations

The secant method with successive replacements suffers from two distinct instabilities. First, the usual implementation based on rank one modification formulas is numerically unstable. Second, the points produced by the iteration may tend to collapse into an affine subspace and slow the convergence of the method. This talk describes a stable implementation of the method based on the QR factorization of certain matrices associated with the iteration. This factorization may also be used to detect and rectify degeneracies among the iterates.

TH-PM-2-W

D. KLINGMAN, A. NAPIER, J. STUTZ, The University of Texas, Austin

A Program for Generating Large Scale (Un)Capacitated Assignment, Transportation, and Minimum Cost Flow Network Problems

One purpose of this paper is to describe the development, implementation, and availability of a computer program for generating a variety of feasible network problems. In particular the code can generate capacitated and uncapacitated transportation and minimum cost flow network problems, and assignment problems. In addition to generating structurally different classes of network problems the code permits the user to vary structural characteristics within a class.

Since researchers can generate identical networks using this code, another purpose of the paper is to provide problems benchmarked on several codes currently available. In particular, the later part of the paper contains the solution time and objective function value on 40 assignment, transportation, and network problems varying in size from 200 nodes to 8,000 nodes and from 1,300 arcs to 35,000 arcs.

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T-PM-8-V

S. SUNDER, Hans Raj College, Delhi, India

A Dual for a Generalized Linear and Linear Fractional Program

In the paper "On Generalization of Linear and Piecewise Linear Programming" Teterev has proposed a simplex type algorithm for solving a class of problems whose objective function, the sum of linear and linear fractional targets, is subjected to linear restrictions only. The object of the present work is to formulate a dual for the said class of problems and to show that the well known duality principle holds. Dual has been constrained as the variable of the Primal problem. Work concludes by showing that the known duality results of linear programming are particular cases of the general results obtained here. Following is the pair of Primal and Dual Programs:

Primal Program

Maximize

$$f(X) = CX + \frac{PX}{QX}$$

subject to

$$x \in S ,$$

where

$S = \{X : AX = b, X \geq 0\}$

A is $m \cdot n$ matrix

C, P and Q are n components row vectors

b is m components column vector.

Dual Program

Minimize

$$g(U,V,W) = Wb + \frac{Ub}{Vb}$$

subject to

$$Vb \cdot QX(WA-C) + Vb(UA-P) - Ub(VA-Q)$$

$$Wb \geq 0, \quad Vb \geq 0$$

Ub and Vb cannot vanish simultaneously.

The constraint set of the Dual Program is denoted by L.

* * *

T-AM-13-V

W. R. S. SUTHERLAND, Dalhousie University, Halifax, Canada

The Target Method for the Gale-Koopmans Model of Economic Development

In this paper, Goldman's method of continual planning revision is applied to Gale's discrete-time model of economic development. The main result consists of showing that the optimal infinite-horizon plan can be determined by solving a two-period version of the model in which an appropriate target constraint has been included. Some explicit examples are presented. These are obtained by an inverse optimum argument. Next, the usual computational methods of dynamic programming are discussed and a new method is presented which solves the infinite-horizon model by finding the target constraint needed for this approach.

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W-AM-13-Y

H. TALPAZ, W. H. VINCENT, Michigan State University, East Lansing

A Population and Control Simulation of the U.S. Hog Production

The hog-pork subsector tends to follow cyclical and oscillatory patterns with harmful impacts on consumers and marginal producers. To gain new insights regarding this phenomenon, a study with the following

objectives was undertaken: (1) to identify the dominant feedback forces working in the formation of the hog cycle and to translate it into an equivalent distributive delay structure; (2) to construct a computer simulation model of the hog production industry to permit an evaluation of policy control alternatives.

A two-stage computer system was constructed. Stage I--a variable-parameter selector designed to identify and estimate elements of the state and output vectors, using stepwise recursive econometric procedures. In Stage II, using a time-variant mixed difference equations system the model includes a state hog population vector with differential age groups; a transfer matrix, composed of death loss and farm slaughter rates; biological growth rates; a lagged price vector representing market forces in relation to breeding and sales behavior. A unique slaughter allocation model to yield age and weight distributions of market hogs was developed based on normal distribution approximation adjusted for seasonal skewness.

A policy control scheme was applied demonstrating how the hog cycle could be damped yielding potential benefits for producers and consumers.

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F-PM-2-W

R. TARJAN, Cornell University, Ithaca

Analysis of Algorithms for Finding Minimum Spanning Trees and Optimum Branchings

This paper examines algorithms for finding minimum spanning trees and optimum branchings. Two implementations of Edmonds' optimum branching algorithm are presented. One has an $O(\frac{E \log v}{\min\{1, \log(E/V \log V)\}})$ running time and the other has an $O(E \log V)$ running time, if the problem graph has V vertices and E edges. The implementations use Crane's binary tree method of handling priority queues. Applying the same techniques to the Kruscal and Prim minimum spanning tree algorithms gives similar results.

TH-PM-4-V

G. S. THOMAS, J. C. JENNINGS, Scicon Ltd., London

A Blending Problem Using Integer Programming On-Line

A well known company in the photographic industry found that the manual construction of blends was proving wasteful and inefficient; it therefore decided to use Mathematical Programming to determine optimum blends. This paper describes the resulting integer programming model and its integration into an on-line production system.

The problem arises because of the variability of batch chemical processes. Output from a production "make" is stored in a pot. These pots contain essentially the same product with small quality variations. In addition the product deteriorates with age if it remains unused. Several times a day a batch of the product is required for a manufacturing process and its qualities must lie within specified ranges. A number of operational factors influence the composition of the optimum blend and these give rise to several integer constraints; these can all be incorporated into Special Ordered Sets of Type 2. The objective function is largely heuristic as it must balance the desirability of using old makes, finishing up pots and producing a blend with the specified qualities.

A shortcoming of the initial model was that it was frequently infeasible and so did not provide useful solution output. Accordingly, the formulation was amended so as to always produce a meaningful report. In order to meet peak requirements the system had to be able to handle up to 9 separate products simultaneously. The operational requirement was that results should be available on the shop floor within an hour or two of initiating a run. Consequently the entire application was designed to run on a teletype operated by factory staff situated in an office adjacent to the blending apparatus.

Statistics from a number of runs are included.

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W-AM-2-X

V. BALACHANDRAN, G. L. THOMPSON, Carnegie-Mellon University, Pittsburgh

Rim, Cost, Bound and Weight Operators for the Generalized Transportation Problem

We survey three papers that investigate the effect on the optimum solution of a capacitated generalized transportation problem when data of the problem are continuously varied as a linear function of a single parameter. First the rim conditions, then the cost coefficients, then the cell upper bounds, and finally the cell weights are varied parametrically. In each case the effect on the optimal solution, the associated change in costs and the dual solution changes are determined first for basis preserving operators and then for arbitrary operators. In the case of weight operators it is shown that the optimal cost sometimes varies in a non-linear fashion and sometimes in a linear fashion. Relevant algorithms, interpretations, and illustrations are provided.

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F-PM-5-V

J. THURBER, A. WHINSTON, Purdue University, Lafayette, Indiana

A Class of Mathematical Programming Algorithms

A new theoretical approach to mathematical programming is developed which on one hand leads to an extension of the Kuhn-Tucker conditions and on the other hand to a class of algorithms. In the first part we derive optimality conditions for optimization problems where the constraint qualification conditions are violated. The computational algorithms are based on successive linearizations of the problem to obtain improved accuracy. The linearizations are not based on a Taylor series expansion but on a new type of expansion which appears to be appropriate. Some computational results are presented to illustrate the algorithm.

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F-AM-2-W

R. L. TOBIN, General Motors Corporation, Warren, Michigan

Minimal Complete Matchings and Negative Cycles

Conditions are developed which relate the existence of negative and non-positive simple cycles in an undirected network to minimal complete matchings on a derived network. These conditions are then used to develop a test to determine whether or not an undirected network contains non-positive simple cycles.

T-PM-6-Y

J. A. TOMLIN, Stanford University, Stanford

* On Scaling Linear Programming Problems

The scaling of linear programming problems remains a rather poorly understood subject (as indeed it does for linear equations). Although many scaling techniques have been proposed, the rationale behind them is not always evident and very few numerical results are available. This paper considers a number of these techniques and gives numerical results for several real problems. Particular attention is given to two "optimal" scaling methods, giving results on their speed and effectiveness (in terms of their optimality criteria) as well as their influence on the numerical behavior of the problems.

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T-PM-8-V

D. M. TOPKIS, A. F. VEINOTT, JR., Tel Aviv University, Tel Aviv, Israel

Monotone Solutions of Extremal Problems on Lattices

A lattice is a partially ordered set in which each pair of points has a least upper bound, called their join, and a greatest lower bound, called their meet. Let $*$ be an associative, commutative, non-decreasing, binary operation on the real line. A real-valued function f on a lattice S is called sub$*$ if the $*$-composition of its function values at each pair of points in S is greater than or equal to the $*$-composition of its function values at the meet and join of the two points. For the case where S is a product of chains, join-representations will be given for sub$*$ functions on S where $*$ is $\vee$. A characterization of sub$*$ functions when $*$ is $+$ will also be given.

Next we consider the problem of minimizing a sub$*$ function $f(s,t)$, defined on a sublattice L of the product $S \times T$ of two lattices, subject to $s \in L_t \equiv \{s \in S : (s,t) \in L\}$. We show that if the $*$-composition of two real numbers is increased when both numbers are increased (this is so if $*$ is $+, \vee$, or $\wedge$) and certain compactness and lower semicontinuity conditions are imposed, then there is an s_t which minimizes $f(\cdot,t)$ over L_t, and more important, s_t is non-decreasing in t. One application of this result asserts that one optimal family of prices at every node in a weighted network flow problem is nondecreasing in the demands at all nodes. A number of other applications to inventory control and to statistical decision theory will be given.

T-AM-6-Y

L. TORNHEIM, Chevron Research Company, Richmond, California

Percentile Curves

Percentiles are considered suitable statistics to summarize certain kinds of data. If the data depends upon time, then percentile curves are appropriate. Suppose that $x = f(a_1, \dots, a_n; t)$ is the formula of the desired curve, where the a_j are the parameters to be determined. A definition of a p-th percentile curve for the data points (t_i, x_i) $(i = 1, \dots, m)$ is that choice of $a_1, \dots, a_n$ which minimizes

$$S = \sum_{t_i} \{(100 - p) \sum_{f > x_i} [f(t_i) - x_i] + p \sum_{f < x_i} [x_i - f(t_i)]\} .$$

This is a generalization, for if f is constant, i.e., $f = a$, then the answer is the p-th percentile of the x_i. If f is linear in $a_1, \dots, a_n$, then the problem has a linear programming formulation, which has a dual involving n equations and m upper-bounded variables.

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F-AM-12-W

G. L. NEMHAUSER, L. E. TROTTER, JR., R. M. NAUSS, Cornell University, Ithaca

Set Partitioning and Chain Decomposition

There is given a finite set I and a family of subsets of I. We consider the problem of determining a minimum cardinality subfamily that is a partition of I. A branch-and-bound algorithm is presented. The bounds are obtained by determining chain decompositions of directed acyclic graphs. The computation time required to determine a bound is bounded by a polynomial in the cardinality of I. Some computational experience is reported and relationships with other methods are discussed.

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W-PM-2-X

K. TRUEMPER, Case Western Reserve University, Cleveland

A Min-Cost Flow Algorithm for a Class of Networks with Gain

A min-cost flow algorithm is presented that solves a large class of networks with gain. This class is characterized by positive gain factors and a condition on arcs that form a directed cycle; hence it includes all acyclic networks with positive gains. It is shown that some networks outside of this class can also be solved by simple transformations.

The algorithm is of the primal-dual type. The restricted subproblem is a max-flow problem which is solved by an extension of an existing algorithm.

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T-AM-13-V

T. TSUKAHARA, JR., and H. BRUMM, JR., Pomona College, Claremont, California

An Analysis of the Work Incentive Effects of a Negative Income Tax: A Nonlinear Programming Approach

This paper investigates the labor supply adjustments of both primary and secondary workers, who are members of low income families, which arise in response to the enactment of a negative income tax program of income maintenance. The key assumption made is that the family is the relevant decision making unit. The analysis is conducted within the framework of a nonlinear programming model of the family. The basic conclusion derived is that on *a priori* grounds alone, one cannot predict whether a negative income tax subsidy would diminish or increase work incentives.

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M-AM-14-U: **KEYNOTE SPEAKER**

A. W. TUCKER, Princeton University, Princeton

P Simplex Algorithm and Duality

We examine these fundamentals in historical perspective and note implications they have for constructive mathematical advance.

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F-PM-1-Y Prob.

U. UEING, J. P. BARTHES, Stanford University, Stanford

Application of Branch and Bound Methods to Solve Continuous Non-Convex Optimization Problems

Non-convex optimization is a nearly unstructured domain in the field of operations research. Yet research on well-defined non-convex problems showed that the global solution cannot be determined without avoiding combinatorics. In this paper it is demonstrated how certain optimization problems with multiple stationary points can be solved globally by applying Branch-and-Bound methods. The continuous problem is transformed into an equivalent optimization problem which has only a finite number of feasible solutions, and whose structure is represented by a tree. Criteria are provided to cut down the number of nodes in the tree. Saddlepoints are excluded automatically.

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M-PM-8-V Dr. Dean Phil:

P. S. UNGER, Bell Telephone Laboratories, Holmdel, New Jersey

The Dual of the Dual as a Linear Approximation of the Primal

The (Lagrangian) dual of the dual of a twice differentiable nonlinear programming problem is formulated and investigated. It is shown that the dual of the dual of an n variable primal is a problem in $2n$ variables where the objective function and each of the constraint functions of the primal are replaced by their first-order Taylor series approximations. If the Lagrangian dual is convex, the dual of the dual is found to provide at least as good a bound on the optimal value of the objective function as the dual does. The dual of the dual of a polynomial program is shown to be equivalent to the condensed version of that program. Relationships with existing computational algorithms are presented. New conditions for the absence of a duality gap are also obtained.

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W-AM-14-U

S. VAJDA, University of Birmingham, Birmingham, England

P Sufficiency and Necessity Theorems in Mathematical Programming

Survey of such theorems in linear and in non-linear programming, and of the conditions (constraint qualifications and others) which make them valid.

TH-AM-8-Y

J. VAN REMORTEL, D. J. WILDE, Stanford University, Stanford

Asymmetric Minimization with a Convex Fourth Degree Approximation

A basic technique in unconstrained minimization is the Newton-Raphson method, which exactly minimizes a quadratic approximation to an objective function. This approximating function is symmetric, its contours being limited to n-dimensional ellipsoids having n axes of symmetry. Functions with asymmetric contours can be optimized more efficiently by exact minimization of asymmetric approximations. In this paper, a special separable convex asymmetric fourth degree function is proposed. Despite the high degree of the approximation, special properties of cubic equations lead to drastic simplifications and simple, non-iterative computations. The method may be regarded as adding a few operations to quasi-Newton methods in order to measure and correct for asymmetry.

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F-AM-2-W

R. VAN SLYKE, H. FRANK, A. KERSHENBAUM, Network Analysis Corporation
Glen Cove, New York

Network Reliability: A Case Study in Applied Computational Complexity

As part of the topological design of the ARPA computer network it was necessary to analyze the reliability of proposed configurations; that is, to relate the reliability of the network as a whole to the reliability of the communication lines and computers which constitute the network. A class of very flexible simulation techniques were developed which involve the calculation of a minimum spanning tree (MST) for each sample.

Since thousands of MST's are calculated for each analysis it was desirable to examine in detail the method of calculating MST's. Conventional wisdom is that the Prim-Dijkstra algorithm is the best method for MST calculations; this is commonly based on the "look at" argument of computational complexity. We found no less than three other algorithms which are preferable in various situations. By using one of these methods, computation time was reduced by a factor of $\frac{\text{Log } N}{N}$ for reliability problems.

First we formulate the reliability models, where we find the necessity of computing a large number of MST's on a sparse graph of moderate size. Next we discuss in detail the computational complexity of MST calculations. Then the characteristics of the required calculations are used to choose the best algorithm for the application in mind. Experimental evidence is given to support theoretical predictions. Finally, we make some comments on the application of computational complexity results in general and in particular to the situation for algorithms to find the component structure of graphs and to find MST's.

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F-PM-2-W

D. M. TOPKIS, A. F. VEINOTT, Jr., Stanford University, Stanford

* Meet-Representation of Subsemilattices and Sublattices of Product Spaces

The class $\mathcal{S}$ of all join-(resp., meet-) subsemilattices of a given join-(resp., meet-) semilattice S is, of course, closed under intersections, so $\mathcal{S}$ is a meet-subsemilattice of the lattice 2^S of all subsets of S ordered by set inclusion. We explore the possibility of obtaining meet-representations for the elements of $\mathcal{S}$ where S is a product of semilattices. The simplest result is that if S is a product of n chains and $L \in 2^S$, then $L \in \mathcal{S}$ if and only if L can be expressed as the intersection (meet) of n elements of 2^S, the i^{th} $(1 \leq i \leq n)$ of which is isotone (resp., antitone) for every fixed value of the i^{th} coordinate.

Next we apply these results where S is a lattice to obtain meet-representations for the elements of the meet-subsemilattice $\mathcal{S}$ of 2^S consisting of all sublattices of S. For example, we show that if S is a product of n chains and $L \in 2^S$, then $L \in \mathcal{S}$ if and only if L is the intersection (meet) of n^2 elements of 2^S of the following form. The ij^{th} $(1 \leq i, j \leq n)$ of these sets is a product of two sets. The first set is the product of all chains defining S except the i^{th} and j^{th}. The second set is a subset of the i^{th} chain when $i = j$, and a subset of the product of the i^{th} and j^{th} chains that is isotone and antitone respectively for each fixed value of the i^{th} and j^{th} coordinates respectively when $i \neq j$.

When specialized to the case where L is a polyhedral set in R^n, these results give characterizations of duals of pre-Leontief substitution and weighted network flow problems.

F-PM-1-Y

A. S. VINCENTELLI, M. SOMALVICO, Politecnico di Milano, Italy

Formalization and Properties of State Space Approach to Problem Solving

The purpose of this paper is to propose the utility of a deep investigation on the notion of problem solution, and on the relationship between the search algorithms and the structure of problem representation.

In the paper we restrict our investigation only to the state-space approach to problem solving, which is apted to furnish results which substantiate our claim.

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W-AM-13-X

D. A. WALKER, Federal Deposit Insurance Corporation, Washington

Effects of Imperfectly Competitive Loan and Security Markets on Bank Asset Management

In this paper a quadratic recursive programming model is presented and optimized. The purpose is to examine the effects of allowing a bank to be optimizing profits in imperfect loan and securities markets and to compare these results with the case where perfectly competitive markets are assumed.

It is virtually essentially that bank asset management models be considered in a non-static framework. Recursive programming provides such a capability since the model to be optimized in period t is conditional upon the parameters and allocations of the previous period in a recursive programming model. Changes in the environment in which a commercial bank operates can be included in a recursive bank asset management model.

In a recently published paper, a linear recursive programming model has been presented and optimized. In that paper decision rules have been presented for maximizing profits as a linear function of loans and securities. A comparison of the alternative decision rules for a quadratic versus a linear recursive model will provide some interesting results that are presented in this paper.

F-PM-2-W

D. B. WEINBERGER, Cornell University, Ithaca

On the Blocking Polyhedron of the Intersection of Two Matrices

Consider two matroids on the same set and let A be the incidence matrix of all k-sets which are independent in both. In his initial paper on the subject of blocking and anti-blocking pairs of polyhedra, Fulkerson conjectured as to the form of the blocking matrix of A. Here, this conjecture is discussed and some special cases are proved. In particular, the special case where both matroids are generalized partition matroids is discussed in detail.

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M-PM-13-X

L. E. WESTPHAL, Northwestern University, Evanston

An MIP Model for Planning Mechanical Engineering: Specification and Solution

This paper sets out the structure of a static, one region, mixed integer programming (MIP) model for appraising prospective investment projects in the mechanical engineering sector. Economies of scale and joint production, major sources of interdependence among production activities in the sector, are specified in the model, which is formulated to minimize the annual cost of meeting a fixed bill of final demands for specific products. The focal choice is between imports and domestic production; extension to choices among alternative techniques is easily accomplished. Components and sub-assemblies are costed at the lower of import price or endogenously determined production cost. The costs of capital equipment and plant are specified using a fixed charge type cost function.

A solution procedure is devised to exploit the particular structure of the model; the calculation of marginal costs places boundaries on the space in which the global optimum is contained. In its application to evaluate possible investments during the Republic of Korea's Third Five Year Plan, the model contains more than 500 zero-one variables. It was nontheless possible to solve it using less than one minute CPU time on an IBM 360/91, for the computed boundaries were sufficiently "tight" that the globally optimal solution could be obtained quickly by elementary hand calculations. The bounding process is generalizable over a number of MIP model structures and can be used in "pre-analysis" to save on formal computation using available algorithms. The paper concludes with a discussion of the relation between the efficacy of the bounding process and the degree of interdependence within the system modeled.

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W-PM-11-Z

R. WETS, University of Kentucky, Lexington

* Duality Relations for Stochastic Programs

We consider (linear two-stage) stochastic programs with resource which can be formulated as

$$\text{Minimize} = cx + E\{qy \mid Wy = p - Tx,\ y \geq 0\}$$
$$Ax = b$$
$$x \geq 0$$

where q, p and T are random matrices which known distribution function on Ξ. We also assume that the random elements are square integrable. Then the following problem:

$$\text{Maximize } \{\sigma b + E(\pi p) \mid \sigma A + e\{\pi T\} \leq c,\ \pi W \leq q\}$$
$$\sigma \in R^m,\ \pi \in L_2(\Xi)$$

is shown to be "dual" to the original problem. (This result is at variance with previous results obtained by A. Madansky in "Dual Variables in Two-Stage Linear Programming under Uncertainty.") These results can be extended in various directions, but in particular to the n-stage problem (with linear or nonlinear objectives) in which case we obtain some new results for the general class of dynamic (or sequential) optimization problems.

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W-PM-1-W

J. THURBER, A. WHINSTON, Purdue University, Lafayette, Indiana

* Primal Integer Optimization

In this paper we present a new all integer algorithm to solve the linear integer programming problem. The procedure has the following characteristics:

1. The method either starts with an initial feasible integer point or it can be used to find one.

2. Once a feasible integer solution is obtained integer feasibility is maintained throughout the course of the algorithm.

3. Finite convergence is guaranteed.

The method is comparable to ones developed by Young and Glover. Computational results are developed and comparisons with their algorithms as well as with other methods which are not primal and all integer are presented.

* * *

TH-PM-4-V Dr. Dear Bill!

W. W. WHITE, IBM Corporation, Philadelphia Scientific Center, Philadelphia

* Interactive Use of a Large Mathematical Programming System

Recent developments in computer technology have created new ways of attacking problems in mathematical programming. One of these ways is via man-machine interaction. To explore this interaction, an experimental system has been constructed, consisting of a large mathematical programming system coupled with and controlled by the highly flexible interactive APL shared variable system with supporting programs. This paper will present the structure of the experimental system, and, based upon experience obtained in its use, will discuss some advantages and disadvantages of interactive computing for some mathematical programming problems, especially with regard to the following three areas:

1. Developing and experimenting with new algorithms;
2. Model building and analysis;
3. Interactive problem solving.

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TH-PM-2-W

G. A. WICKLUND, The University of Iowa, Iowa City

Computer Experience in Generating Transportation Problems with the "More for Less" Paradox

The classical transportation model is to minimize total transportation costs $(c_{ij} \geq 0)$ subject to a set of equality constraints for the amount available (a_i) at the origins and the amount demanded (b_j) at the destinations. Several algorithms are avilable for solving this problem.

In some cases it is possible to increase the amount available at any origin $(a_i' > a_i)$ and the amount demanded at any destination such that the transportation cost found in the original solution would be less. This result is the paradox of shipping more units and reducing transportation costs.

This paper investigates the structure of transportation problems which have the "more for less" condition. Several transportation problems were randomly generated on the computer in an attempt to discover what special structure exists in a transportation problem which has the "more for less" condition. In this paper the characteristics of the generated transportation problems are discussed.

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M-PM-1-W

J. P. BARTHES, D. J. WILDE, Stanford University, Stanford

A Formalism for Branching Methods of Combinatorial Optimization

This article proposes a theoretical framework for describing, defining, and comparing the many algorithms conceived in recent years for solving combinatorial optimization problems. This formalism encompasses such known techniques as branch and bound, branch and prune, implicit enumeration, branch and exclude, bound and scan, branch search, additive algorithms, and heuristic search. It seeks to distinguish between the specific characteristics of each and to identify those fundamental properties common to them all--partitioning, search tree construction, bounding, and feasibility testing. Also it introduces objective criteria for comparing the performances of various algorithms on specific classes of problems. By making precise such concepts as node analysis, node generation, and termination, and by introducing such partial ordering relations as algebraic lattices, the formalism generates new algorithms giving improved performance on loading problems (linear integer programming with one inequality constraint) and for problems in pseudoboolean programming.

TH-PM-4-V

C. H. JOHNSON, E. L. JONES, E. P. WILLARD, Texas Instruments Inc., Dallas

TIMPS/ASC - An MPS Implementation on a Pipeline Computer

Implementation of a complete mathematical programming system - TIMPS - on Texas Instruments' Advanced Scientific Computer (ASC) is described. The ASC is a general purpose, fourth generation multiprocessor of exceptional power and capacity designed for the processing of complex technical problems. The high speed of this machine is, in part, achieved by using a pipeline architecture for instruction processing and by special vector/matrix instructions built into the hardware logic.

In 1970, Texas Instruments purchased a major mathematical programming system - FMPS - to serve as the initial skeleton for TIMPS. FMPS was particularly selected because of its code richness. Since then, extensive redesigning and coding has been performed to structure the TIMPS computational routines to be particularly suited to the ASC hardware features and the vector instruction repertoire. Various facets of the design logic for algorithms contained in TIMPS (LP, SEP, MIP, GUB) as well as computational findings are discussed.

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T-PM-1`-W

A. C. WILLIAMS, Mobil Oil Corporation, Princeton

Some Modeling Principles for MIP's

The value of a mixed integer linear program is, in general, a discontinuous function of the data, i.e., of the cost and matrix coefficients and of the right-hand side vector. Unlike the corresponding situation in linear or convex programming, small errors in the data can lead to programs which fall far short of optimal. As an example, a budget constraint (the value of which is generally highly arbitrary) may be such that a very good investment opportunity has to be passed up in favor of inferior ones, because we are a few dollars short. Or, to meet a certain small extra demand (which was only guessed at anyway) an entire new plant is forced open.

Many real world problems which are not discontinuous are thereby represented by models which do have discontinuities. The modeling techniques of linear programming, which deals with convex economics, cannot be transferred in toto to mixed integer programming, which deals with non-convex economics.

We will present several theorems which carve out classes of mixed integer programs for which the value is a continuous function of the data (and for which the optimal solution set is upper semi-continuous). These theorems have straightforward economic interpretations. Moreover, they lead to quite general rules for good modeling techniques in mixed integer programming.

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M-PM-1-W

H. P. WILLIAMS, University of Sussex, England

Experiments in the Formulation of Integer Programming Problems

Five practical problems are each formulated in two different ways as 0 - 1 integer programming models. All the models have been solved by the Branch and Bound method using a commercial package program. Full details are given of the manner of the different formulations and the computational ease of solving them. The purpose of this paper is to investigate the computational effects of different FORMULATIONS on such problems. The problems considered are a market allocation problem, a combinatorial problem, two mining problems and a problem of logical design.

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TH-PM-7-Y

R. J. WILMUTH, IBM Corporation, San Jose

A Comparison of Fixed Point Algorithms

There exist four classes of algorithms for computing fixed points on finite dimensional spaces based on pivoting along adjacent simplexes in a complex. Each class is defined by the type of complex upon which the algorithms operate.

Algorithms for computing fixed points of functions on the standard n-simplex, S^n, using complexes triangulating S^n were the first to arrive and constitute the first class. The unique element in the second class is an algorithm which operates on a complex for triangulating $S^n \times [0,\infty)$ and also computes fixed points on S^n. The third and fourth classes compute fixed points on R^n and use complexes triangulating $R^n \times [0,1]$, and $R^n \times [0,\infty)$.

The results of a study comparing the relative performance of representative algorithms from each class are presented. The study consists of executing these algorithms on several examples from economics and nonlinear programming.

A brief description of an algorithm for computing fixed points of maps on the set of continuous functions on a closed interval is given. This algorithm is an extension of Eaves' and Saigal's algorithm from the fourth class mentioned above. The important application of this algorithm to the solution of ordinary differential equations is also described.

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W-PM-5-V

M. HELD, P. WOLFE, H. CROWDER, IBM Watson Research Center,
Yorktown Heights

* Validation of Subgradient Optimization

Many problems of mathematical programming can be cast in the form of finding a real n-vector solving the problem

$$\text{Maximize } w(\pi),\ \pi \in S$$
$$\text{where } w(\pi) = \min_{k \in K} [c_k + \pi \cdot v_k]\ ,$$

c_k is a scalar and v_k is an n-vector for all $k \in K$, and S is a closed convex subset of E^n. Various authors in the U.S. and the U.S.S.R. have proposed under various names a simple iterative scheme particularly suited to problems with very large K (for example, most of those requiring column generation). The procedure is unusual in not insisting on monotonic improvement of the objective function and permitting as "direction of step" any subgradient of the objective function. We propose the term subgradient optimization for such a procedure.

We present the method as we have been using it. We give practicable formulations and computational results for the three types of problems we have tried extensively: the assignment problem, the (noninteger) traveling-salesman problem, and a multicommodity network flow problem. Our experience indicates that subgradient optimization is surprisingly effective for certain problems, and deserves to be more widely known.

TH-PM-5-Z

I. J. WEINSTEIN, O. S. YU, Stanford Research Institute, Menlo Park

Solution Procedure for a Concave Maximization Problem with a Separable Objective Function Having Interrelated Components

This paper presents an efficient solution procedure for the following concave maximization problem:

$$\text{Maximize} \quad \sum_i \sum_j f_{ij}\left(\sum_j \sum_k c_{ijk} x_{ijk},\ x_{ij1},\ x_{ij2},\ \ldots ,\ x_{ijN}\right)$$

$$\text{subject to} \qquad \sum_i \sum_j a_{ijk} x_{ijk} \leq b_k, \quad \text{for} \quad k \quad 1,2,\ \ldots ,\ N,$$

$$x_{ijk} \geq 0, \quad \text{for all} \quad i,\ j,\ k,$$

where f_{ij} is nondecreasing in the first argument and concave increasing in the remaining arguments. The difficulty of the problem arises mainly from the coupling among the f_{ij}'s. The method used in this paper represents an interesting extension of Everett's generalized Lagrange multiplier method and Dantzig-Wolfe's decomposition principle. Variations of the problem and efficiency considerations and practical experience in the computer implementation of the solution procedure will also be discussed.

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T-PM-8-V

H. M. MASSAM, S. ZLOBEC, McGill University, Montreal

The Various Definitions of the Derivative in Mathematical Programming

Optimality conditions of the Kuhn-Tucker type are stated in the literature for Fréchet differentiable functions. This report shows that there exist at least seven different classes of functions, differentiable in a weaker sense than Fréchet, for which the Kuhn-Tucker optimality conditions hold.

AUTHOR INDEX