Tenth International Symposium on Mathematical Programming

Montreal, August 27-31, 1979

Dixième Symposium international de programmation mathématique

Montréal, du 27 au 31 août, 1979

Abstracts
Résumés

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1

A Linear Programming Model for Planning Electric Energy Supply in Canada

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This paper describes the design and calibration of a linear programming model for planning the integrated expansion of electric generation and inter-regional transmission in Canada during the period 1980-2010. The model contains a four region representation of the Canadian electric power system. It features explicit supply curves for three types of coal, explicit inter-regional transportation of coal and electric energy as well as intra-regional reliability standards. We discuss the calibration process for the model and show the implications of some alternative policies for for the development of the electric power systems in Canada.

2

Employment and Efficiency in the Italian Economy: A Linear Programming Model

U. Bertele, F. Brioschi, F. Quillico, Politecnico Di Milano, Milan, Italy

The problem of unemployment, as known, is a central one in most western economies: as a matter of fact the introduction of new technologies involves a continuous increase in productivity and a continuous reduction in the number required of (direct and indirect) workers, only partially balanced from the increase of "physical" output. The governments are, hence, pushed to create"artificial" jobs or to reduce the mobility for the existing jobs (this is true, at least, for the Italian case).
A model has been built to evaluate for which sectors an excess of workers (and, therefore, a delay in the technological adjustment) may be more dangerous for the overall performance of the (Italian) economic system and for its competitivity in domestic and international markets.
From the analytical point of view this implies solving a linear programming problem, whose objective function (to maximize) is the reduction of total production costs of the economic system (the shares for domestic market and export having suitable weights), due to variations in employment levels (and connected technological adjustments) in the different industries. The main constraint concerns the maximum decrease in the total employment level, with respect to which a post-optimal analysis is carried on.
The parameters of the model are determined starting from the input-output matrix (in different years) and from the consequent partition in "vertically integrated sectors" of the economic system.

3

An Application of Goal Programming Model for Manpower Planning of Saudi Arabia

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This paper presents a formulation of a linear goal programming model to determine various skills of manpower required for the implementation of the national development plan of Saudi Arabia. In view of a set of multiple objectives specified in the Second Development Plan (1976-1980), the goal programming model is intended to be used as a decision tool by government agencies for the construction of more realistic manpower and educational policies. The model incorporates a number of important macro-economic variables, and thus allows planners to analyze linkages between several government projects. A dynamic version of the model is to be attempted once the static version with constant labor input coefficients is successfully implemented by the Ministry of Planning in Riyadh.

4

Physical Models and Methods in Mathematical Programming and Economics

B. Razumikhin, Academy of Sciences of the USSR, Moscow, USSR

Abstract not available.

5

On the Stability of a Price Adjustment Process in Planning Economy

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Processes of planning and price formation are considered. Production plan is a solution of problem I(p): $\max \{M: y \geq f(p,M), y \in Y\}$, where $f: R^n \times R^1 \to R^1$ is a demand function, $p \in R^n_+$ is a current price vector, M is a level of the consumer's income, $Y \subset R^n_+$ is the production technology set.
The prices are being formed as follows
$p^{t+1} = p^t + \alpha_t (q^t - p^t)$, $t = 0,1\ldots$
where p^0 is an initial price vector, q^t is a vector of Lagrange multipliers for the I (p^t), $\alpha_t \in (0,1] \subset R^1$, $\alpha_t \to 0$, $\sum^{\infty} \alpha_t = \infty$.
While constructing the main problem I (p) it is assumed that the consumer's demand as a function of M is known to the Planning Body (PB). When maximizing the content consumer's income at every time period (t, t+1) the PB ensures the most complete satisfaction, which is possible for the given technology and prices, of consumer's requirements.
If the producer aims to maximize the expected profit: II (p): $\max \{py: y \in Y\}$ then his interests are not the same as those of PB.
The vector p will be called the <u>equilibrium price vector</u> if the solutions of the problems I (p) and II (p) coincide.
Assume that the consumer's demand can be found by maximizing the given utility function u(x) subject to the usual budget constraints, i.e. function f(p,M) is a solution of problem $\max \{u(x): px \leq M\}$ for the fixed p and M.
Theorem. Let f(p,M) be a linear function with respect to M, u(x) be a strictly concave increasing function, $u(x) \in C_2$, Y be a convex compact set, $Y \ni y^* \geq 0$. Then the set of equilibrium prices P is nonempty and $p^t \to P$.
If $\bar{p} \in P$ then a solution of $I(\bar{p})$ is at the same time a solution of the following problem: $\max \{u(y): y \in Y\}$.
The theorem is valid if the demand function f(p,M) is given as $\Sigma^m_{i=1} f^i(p,M)$, common requirements of m consumers, and preferences of the i-th consumer can be represented by a nonnegative homogenious concave function $u_i(x)$, $i=1,\ldots,m$.

6

An Algorithm for the Vertex Packing Problem

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This paper presents several properties of the vertex packing problem which allow the original graph to be reduced. These reductions are easy to use and often very efficient. Then we present a sufficient condition for a vertex packing S to be optimal. To apply this condition we have to find an edge matching in a bipartite graph. This edge matching is built by an heuristic which gives, in most of the cases, the optimal edge matching. In the case where this condition does not prove that the considered vertex packing is optimal, we present several vertex packing properties whose cardinality is greater than the cardinality of S. We also show an important relationship between this sufficient condition and the solution of the continuous linear program associated to the vertex packing problem. All these ideas are incorporated into a branch and bound algorithm and computational experience is presented.

7

A Polynomial Algorithm for Maximum Vertex Packings on Graphs Without Long Odd Cycles

W-L. Hsu, Y. Ikura, G.L. Nemhauser, Cornell University, Ithaca, N.Y., U.S.A.

The vertex packing problem for a given graph is to find a maximum number of vertices no two of which are joined by an edge. Let G(K) be the family of graphs whose longest odd cycle is of length 2K+1, where K is any non-negative integer independent of the number of vertices in the graph. We present a polynomial algorithm for the vertex packing problem for graphs in G(K). A by-product of this algorithm is an algorithm for piecing together maximum packings on blocks to find a maximum packing on graphs that contain more than one block. We also give a polynomial algorithm for testing membership in G(K).

8

An Algorithm for the Matching Problem

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Many practical problems, related to the optimal allocation of resources, can be formalized as follows: given a finite set S, an additive real valued set function defined on the subsets of S (weight function), we are required to find a subset of S belonging to an assigned family F, which has maximum weight over all members of F. If the family F is the collection of independent sets of a matroid on S, then the so-called "greedy algorithm" of Edmonds gives a solution of the problem (weighted matching problem); this assumption on F is usually verified. We recall that the well-known Hungarian Method of Kuhn can be viewed in a matroid theory context. In this paper, we describe a new algorithm for the weighted matching problem; we call it a "dual greedy algorithm". In fact, Edmonds' algorithm obtains a sequence of feasible solutions, the last of which is an optimal one; the dual greedy algorithm, on the contrary, obtains a sequence of non-feasible, but "optimal" solutions, the last of which is a feasible one. We present some theoretical results and an automatic procedure which implements the algorithm. Finally, some examples are given.

9

Effective Procedures for Finding Transversals, Perfect Matchings and Optimal Bottleneck Assignments

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Let E be a given set of the cardinality m, and let Q be a nonempty family of n subsets of E /elements of the family Q are finite and not necessarily distinct/.
A set $T \subseteq E$ is called a <u>transversal</u> /a system of distinct representatives/ of the family Q if T consists of n different elements, not any two of them belong to the same element of Q.
Now, let us consider a bipartite graph with two sets of vertices V_1 and V_2 of the cardinality m and n respectively $(m \leq n)$. A <u>perfect matching</u> /an assignment/ on a given graph, we call any subset of m arcs such, that each vertex from V_1 is connected by an arc with at most one vertex in V_2.
There are well-known theorems of Hall and König, which state the necessary and sufficient conditions for the existance of a transversal /a perfect matching/ in a family /graph/.
In the paper we describe a procedure which finds a transversal /or a perfect matching/ if one exists or concludes that there is none. We prove that the proposed procedure has computational complexity of $O(mn)$ /set operations are considered/.
The procedure mentioned above can be used effectively in the threshold algorithms for the bottleneck matching /assignment/ problems. The bottleneck problem is defined on a weighted graph with real number - weights attached to the arcs. The task is to find a matching which maximizes the greatest arc - weight over all matchings of the maximum cardinality.
A threshold algorithm with the computational complexity $O(mn\log_2 n)$ is briefly outlined.
Another threshold algorithm of the complexity $O(n^{5/2}\log_2 n)$, which requires only the arithmetic operations is discussed, too.

10

Network Optimization: New Methods and Applications. Part I

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It is now widely known that network optimization has benefited from a remarkable series of advances during the past several years. Improved labeling algorithms and implementation strategies have produced methods of unprecedented efficiency, and have stimulated a variety of practical applications. These advances have been extended to the linear programming area, making it possible to solve LP problems with large embedded networks in a small fraction of the time required by production LP codes.
Throughout this period of innovation, a steadily clearer picture of the most effective network methods has emerged. Now, however, new results are changing this picture! In just the past year, advances in solutions methods have occurred in nearly every branch of the field, including

shortest path, maximum flow, assignment, transportation and minimum cost flow problems. The extremely important area of LP problems with large embedded network structure has also benefited from more refined methods. We survey these recent developments and describe practical network applications that are making use of them.

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Network Optimization: New Methods and Applications. Part II

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12

An Unsophisticated Implementation of the Simplex Method for Minimum Cost Network Flow Problems

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This one-phase method employs an all-artificial initial solution, gradually-increased penalties for artificial arcs, and an almost random, deliberately unsophisticated pricing strategy. No assumptions are made on the order of the input data. Extensive computation suprisingly indicates that this implementation is more efficient than other methods, including those requiring ordered input.

13

An Efficient Steepest-edge Algorithm for Maximum Flow Problems

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The steepest-edge column selection rule is incorporated in a specialization of the simplex method for maximum flow problems. The structure of the algorithm is analyzed and compared to other well known methods. Implementation aspects and computational results are presented.

14

Computational Experiments on an O-U Transshipment Model

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Glover and Mulvey have shown that any 0-1 integer programming problem can be converted into an equivalent integer generalized network or an equivalent O-U transshipment model. The advantage of the integer generalized network formulation is that it provides a tighter relaxation of the integer program. The advantage of the O-U transshipment formulation is that the pure network subproblems are easier to solve than the generalized network subproblems arising from the integer generalized network formulation. A second advantage of the O-U transshipment formulation is the wider availability of pure network codes to imbed in a branch and bound scheme. This paper reports on the implementation of an O-U transshipment code consisting of a currently available pure network code, GNET, embedded in a branch and bound scheme. Computational results are reported on randomly generated problems as well as a forestry cutting application.

15

Constructive Proof of a General Duality Theorem for Linear Inequalities and Equations

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For any given mXn matrix A and column b (over an ordered field) there is a sign-specified solution of $Ax-b=y$ or of $vA=u$ and $vb<0$ in each of the 2^{n+m} cases of n+m independent alternatives:

$x_j \geq 0$ and $u_j \geq 0$ <u>or</u> x_j free and $u_j=0$ $(j=1,\ldots,n)$;

$y_i \geq 0$ and $v_i \geq 0$ <u>or</u> $y_i=0$ and v_i free $(i=1,\ldots,m)$.

This general duality theorem is a "transposition theorem" (after Motzkin).
Three of the 2^{n+m} cases are:

(1) $Ax=b$ for some x <u>or</u> $vA=0$ and $vb<0$ for some v;

(2) $Ax \leq b$ for some $x \geq 0$ <u>or</u> $vA \geq 0$ and $vb<0$ for some $v \geq 0$;

(3) $Ax=b$ for some $x \geq 0$ <u>or</u> $vA \geq 0$ and $vb<0$ for some v.

Case (3) is a form of a classical theorem of Farkas, used three decades ago to found linear and nonlinear programming theory.
For case (1) we can construct a required x <u>or</u> v by a sequence of pivot exchanges that inverts A as much as possible. For case (2) we can construct a required x <u>or</u> v by the Lemke dual of the Dantzig simplex method (phase II), with pivots selected by the anticycling rule of Bland. For any other case, such as (3), we can construct a required solution of one of the two systems by (partial) inversion pivoting followed by dual simplex pivoting. In no case can both systems have required solutions.

16

Linear Programming Duality and Minty's Lemma

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It is well known that the Minty coloring property of directed graphs is intimately related to the max-flow min-cut theorem, which, in turn, can be regarded to be a special form of the (strong) duality theorem of linear programming. Camion, Fulkerson, and Rockafellar independently observed that the Minty property extends, in a natural way, to real vector spaces, where it again relates closely to linear programming duality. In fact, the Minty property captures precisely (and concisely) the combinatorial structure underlying linear programming duality. We will elaborate

on this equivalence, and show that this combinatorial interpretation of linear programming duality extends beyond real vector spaces, as far as oriented matroids, and no further.

17

Duality and Linear Programs with Generalized Objectives

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Let $\mathbb{Z} \subset R \subseteq \mathbb{R}$ be closed under real multiplication and real addition, $(H,*,\leq)$ an ordered commutative semigroup and $\Box: R_+ \rightarrow H$ an external composition. If certain compatibility relations hold (for example: distributivity of '$\Box$') then $(R_+;\Box;H,*,\leq)$ is called a semimodule (over real numbers) We consider the minimization of objectives $f: H^n \rightarrow H$ of the form

$$f(x) := (x_1 \Box a_1) * (x_2 \Box a_2) * \dots * (x_n \Box a_n)$$

for given coefficients $a_j \in H$, $j=1,2,\dots,n$ subject to $x \in P$,

$$P := \{x \in R_+^n \mid Cx \geq c\},$$

with a real mxn Matrix C and a real m-vector c. Such algebraic linear programming (ALP) problems are generalizations of classical LP with $f(x) = \Sigma x_j a_j$ as well as bottleneck linear programming $f(x) = \max \{a_j \mid x_j > 0\}$. Further objectives of theoretical and practical interest can be subsumed under this algebraic approach.
Duality principles and optimality criteria for ALP are given which are basic tools for theoretical understanding and practical development of various solution methods for special examples from linear and combinatorial programming. In particular feasibility and complementarity imply optimality similar to the classical case if proper generalizations of these conditions are considered. For primal and dual solution methods different generalizations are useful which coincide in the classical case.
In the general case due to the possible discrete structure of the ALP ($R=\mathbb{Z}$) a strong duality result cannot be given. Otherwise in the continuous case ($R=\mathbb{R}$) a strong duality theorem can be proved using an algebraic primal simplex method.

18

Necessary and Sufficient Conditions for Multi-Index Transportation Problems

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The multi-index transportation problem is an extension of the transportation problem (TP2) of linear programming to a large-scale problem with multiple subscripted variables. Its constraints are sums of these variables or double sums, triple sums etc. or combinations of them. Several problems can be formulated as a multi-index problem (TPn). The well known Hitchcock problem will become a three-index problem if there are different kinds of transport. It will become a four-index problem if, additionally, there are several types of the product to be transported. For a TP2 a feasible and by this an optimal solution always exists if the consistency conditions hold. These conditions can be extended to any TPn. However, for $n > 2$ they prove to be only necessary and no longer sufficient conditions.
In this paper additional necessary conditions are presented for any TPn. However, they also prove to be not sufficient. On the other hand, it is shown that there is a special class of TPn for which a feasible solution always exists if the consistency conditions hold. From this sufficient conditions for the TPn can be derived. In this case an explicite expression for a feasible solution can be given.

19

The Kairo Method and Post-Optimization

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Solving a Linear Programming Problem using the Kairo Method - where slack variables are considered neither explicitly nor implicitly - the optimal tableau would be in the following form:

$$\begin{matrix} A_{rr}^{-1} & A_{rs} & \overline{P}_r \\ A_{dr} & A_{ds} & \overline{P}_d \end{matrix}$$

Where A_{rr}^{-1} is the inverse of the "reduced" basis and A_{rs}, A_{dr} and A_{ds} are the original submatrices. The paper initiates a new treatment to deal with the different cases of post-optimization when only the inverse A_{rr}^{-1} is given.

20

Conjugate Gradients: Key to Lanczos Eigenvalue Procedures For Large, Symmetric Matrices

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Lanczos tridiagonalization procedures for computing eigenvalues of large, symmetric matrices A of order n replace A by symmetric tridiagonal matrices T_m of order m, and use eigenvalues of T_m as approximations to eigenvalues of A. Theoretically, the T_m are projections of A onto subspaces spanned by associated globally-orthogonal Lanczos vectors. Global orthogonality however, can be maintained only by using some type of reorthogonalization of the Lanczos vectors. Reorthogonalization restricts these procedures to the computation of a small number of extreme eigenvalues of A.
Empirically one observes, however, that if the Lanczos tridiagonalization procedure is utilized without any reorthogonalization, then for large enough $m>n$ all the eigenvalues of A appear as eigenvalues of T_m, even though all the typical theoretical restrictions are violated. Thus, many or even all of the eigenvalues of A could be obtained. Using an equivalence between conjugate gradients and Lanczos tridiagonalization, we offer an explanation for this interesting phenomena based upon the fact that conjugate gradient optimization procedures can converge under weak, local conditions.
Thus, we know that for large enough m, the eigenvalues of A are eigenvalues of the highly structured T_m. Due to the losses of orthogonality, however, some of the eigenvalues of T_m are extraneous and not related to A. Using the equivalence again, we devise a test for spuriousness which allows us to discard the irrelevant eigenvalues. The Lanczos tridagonalization procedure with this test for spuriousness can be very effective for computing eigenvalues of A in user-specified intervals. Numerical examples with $n \geq 1000$ are given. In many cases $m \leq 3n$ is large enough. This paper combines ideas from optimization theory and numerical algebra.

21

Trends in Sparsity Exploiting Quasi-Newton Optimization Methods

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In the past few years, there has been considerable interest in the field of Quasi-Newton updates for unconstrained minimization that could exploit any sparsity present in the second derivative matrix of the objective function. The talk will be concerned with the general projection ideas that led to useful formulae, as well as their relation with symmetrization of formulae arising in the solution of sparse non-linear systems of equations.
The convergence theory of the resulting algorithms will be briefly discussed as well as numerical behaviour. Finally, some trends for future research in this particular field will be outlined.

22

Computational Experience with Variable Metric Methods for Sparse Hessians

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Much recent study has been concerned with generating variable metric approximations to sparse Hessian matrices which maintain the sparsity pattern of the Hessian. Known methods require the solution of two sparse linear systems of equations at each step. This paper explores the overhead costs of solving these systems, both when sparse factorizations are and are not available.
Variable metric approximations to the Hessian have the further difficulty that the approximate Hessian may not be positive definite. The paper further discusses various means of generating downhill directions, together with their computational efficiency and theoretical global convergence properties.

23

On Sparse Matrix Updates for Unconstrained Optimization

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Matrix updating techniques have become an attractive method for solving unconstrained optimization problems. These methods keep an approximation of the Hessian which is updated at each iteration. The updates are made in such a way that the new approximation is symmetric and satisfies the "Quasi-Newton condition" (also called the "secant condition").
In this paper we discuss different Quasi-Newton conditions based on the interpolation properties of the local quadratic approximation of the underlying function. We also give an alternative derivation of the general rank two updates. This derivation is extended to the case where the approximate Hessian is required to satisfy a sparsity condition. However, this new sparsity condition implies that a system of linear algebraic equations has to be solved for each update and the sparsity condition might be inconsistent with the Quasi Newton condition. By introducing a k-stage update process we can easily satisfy sparsity.

24

Collinear Scaling Algorithms for Unconstrained Optimization

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A new class of algorithms for unconstrained optimization has recently been proposed by Davidon (1977). This new class of iterative methods called "Optimization by Collinear Scaling" is a natural extension of quasi-Newton methods. These methods may be derived by first constructing a local collinear scaling of the variables such that a local quadratic model to the scaled objective function can interpolate both function values and gradient values at the current and previous iterates. Then the step to the minimum of the quadratic model predicts the step to the minimum ot the scaled objective function. Thus deviation of the objective function values from quadratic behavior influences the iteration.
We also show how the scalings and the local quasi-Newton quadratic model can be maintained and updated using essentially the same amount of storage and arithmetic as existing quasi-Newton methods. A particular member of this class is shown to have a local Q-superlinear rate of convergence. Promising variants of the algorithm appropriate for large scale problems will be discussed.

25

Applications of Optimality Conditions in Convex Programming

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S. Zlobec, McGill University, Montreal, Canada

Characterizations of optimality in convex programming, requiring no constraint qualification, were given in using a feasible-direction approach. These optimality conditions are discussed here in the context of three important applications where, by definition, Slater's constraint qualification cannot hold.

1. Pareto optimality in convex programming. Let $F = \{f^k: k = 1,\ldots,m\}$ be a set of convex functions: $R^n \to R$ taken as minimization criteria. The problem is to find a Pareto minimum of F subject to convex constraints.
2. Lexicographic optimality. A lexicographic minimum of the above (ordered) set F subject to convex constraints $x \in C$, is found by solving sequentially
$\min \{f^j(x): f^k(x) \leq \alpha_k, k = 1, \ldots, j - 1; x \in C\}$,
$j = 2, \ldots, m$ where
$\alpha_1 = \min \{f^1(x): x \in C\}$
$\alpha_2 = \min \{f^2(x): f^1(x) \leq \alpha_1, x \in C\}$, etc.
3. Chebyshev solutions of convex programs. A Chebyshev solution of a (possibly inconsistent) convex program $\min \{f^o(x): f^k(x) \leq 0, k = 1, \ldots, p\}$ is a solution of $\min \{f^o(x): f^k(x) \leq \bar{\varepsilon}, k = 1, \ldots, p\}$ where $\bar{\varepsilon}$ is the minimal $\varepsilon \geq 0$ for which $\{x: f^k(x) \leq \varepsilon, k = 1, \ldots, p\} \neq \phi$.

In each case the optimality conditions can be implemented computationally to generate improving directions at each feasible solution found to be nonoptimal.

26

A Duality Theory for a Class of Problems with Essentially Unconstrained Duals

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The paper introduces a class of linearly constrained convex programs whose duals are unconstrained in the sense that their solution must be in the interior of the feasible region. A complete duality theory is developed for these problems. Several examples are discussed.

27

L-Strongly Convex Functionals and Their Applications

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Strongly convex functionals were introduced and investigated by B.T. Poljak. These functionals play an important role in optimization theory. First, strong convexity guarantees correctness of unconditional minimization (i.e., the existence of a unique minimum point of the functional, to which every minimizing sequence converges). Second, that property is important for grounds of many computational procedures. At the same time, strong convexity is a rather restrictive notion: e.g., twice differentiable strongly convex functionals exist on Hilbert space only. We consider the following generalization of this notion. Let E be a normed space which is continuously embedded in a normed space L. We say that a functional $g : E \to R^1$ is strongly convex if there is a constant $\gamma > 0$ such that

$$g(\lambda u + (1-\lambda)v) \le \lambda g(u) + (1-\lambda)g(v) - \lambda(1-\lambda)\gamma||u-v||_L^2$$

for every $u, v \in E$ and every $\lambda \in (0, 1)$. It turns out that L-strong convexity implies almost the same properties as the usual strong convexity: e.g., the problem of finding minimum for such functional is correctly posed in a generalized sense. Being more flexible tool for analysis of infinitely-dimensional extremal problems, the notion of L-strong convexity contains the usual notion of strong convexity as a special case (the spaces E and L coincide). We also show that this generalization may be naturally used for the substantiation of such computational procedures as the gradient relaxation.

28

Optimality and Regularity Conditions for the Convex Programming Problem in Banach Spaces

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In this paper we study optimality and regularity conditions for the mathematical programming problem:
(P) <u>minimize</u> $f(x)$ <u>subject to</u> $g(x) \in -S$ <u>and</u> $x \in X_0$,
where $f: X \to R$ is convex, $g: X \to Y$ is S-convex, X and Y are Banach spaces, S is a closed convex cone in Y and X_0 is a convex subset of X.
Optimality conditions without a constraint qualification have recently been given for (P) by Massam and Craven and Zlobec. These conditions use the cones of directions of constancy introduced by Ben-Israel, Ben-Tal and Zlobec. We combine this approach with the approach given by Guignard and present a class of optimality conditions for (P).
These conditions distinguish between the active and non-active part of the constraint. Furthermore, using the tangent cone of the feasible set and the cones of directions of constancy mentioned above, we isolate that part of the constraint which creates problems in the Kuhn-Tucker characterizations of optimality. This leads to weakest constraint qualifications and regularization techniques for (P).

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Duality Theorems for Modified Lagrangians in Convex Programming

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The convex programming problem

$$f(x) \to \sup, \quad g(x) \ge 0, \quad x \in X \qquad (1)$$

is considered, where X is a convex subset of E^n and the functions $f(x) : X \to E^m$ are concave.
Set $v = \sup\{f(x) | g(x) \ge 0, x \in X\}$. For the ordinary Lagrangian $L(x,y) = f(x) + \langle g(x), y\rangle$, associated with problem (1), denote $\psi(y) = \sup\{L(x,y) : x \in X\}$ $Y^* = \text{Argmin}\{\psi(y) : y \in E_+^m\}$. Further, for any function $\mu(p,y) : E^m x E_+^m \to E^1$, which is concave in x and convex in y, set $F_\mu(x,y) = f(x) + \mu(g(x),y)$ $\phi_\mu(x) = \inf\{F_\mu(x,y) : y \in E_+^m\}$ $\psi_\mu(y) = \sup\{F_\mu(x,y) : x \in X\}$ $Y_\mu^* = \text{Argmin}\{\psi_\mu^\mu(y) : y \in E_+^m\}$
Necessary and sufficient conditons on $\mu(p,y)$ are given, under which, for any problem (1), $F_\mu(x,y)$ is a weak modified Lagrangian, i.e. the problem $\phi_\mu(x) \to \sup$, $x \in X$ is equivalent to (1). These conditions imply that the equality

$$\tilde{v}_\mu = \tilde{v} \qquad (2)$$

with $\tilde{v}_\mu = \inf\{\psi_\mu(y) : y \in E_+^m\}$, $\tilde{v} = \{\inf \psi(y) : y \in E_+^m\}$ holds for "almost any" problem (1).
Somewhat more restrictive conditions on $\mu(p,y)$ are shown to be necessary and sufficient for the validity of (2) for all problems (1). In this case the duality frameworks $v = \tilde{v}$ and $v = \tilde{v}_\mu$ are equivalent, so the duality theorems based on $L(x,y)$ take place for $F_\mu(x,y)$ also. Under certain additional requirements for $\mu(p,y)$ the sets Y_μ^* and Y^* are either both empty or both non-empty as well as they are either both bounded or both unbounded. Finally, necessary and sufficient conditions on $\mu(p,y)$ are derived, under which $F_\mu(x,y)$ is a modified Lagrangian, i.e. a weak modified Lagrangian with the additional property $Y_\mu^* = Y^*$.

30

Popular Misconceptions Regarding the Application of Mathematical Programming to Planning

A.M. Geoffrion, University of California, Los Angeles, Ca., U.S.A.

Several popular misconceptions concerning mathematical programming are retarding its application to problems of planning. These include:
(1) elementary heuristics are "good enough"
(2) planning models are basically incompatible with tactical models
(3) optimizing models are idiot savants good only for the purposes for which originally designed
(4) optimizing models are less insightful than manual analysis because you can't see what they do.
The nature of and possible remedies for such misconceptions will be explored.

31

Optimization - The Curse of Mathematical Programming

M. Jeffreys, Wootton, Jeffreys & Partners, Surrey, England

Operational research has been heavily criticised over the last few years for its emphasis on optimisation. O.R.'s bedfellow, mathematical programming, is seen as the quintessence of this plague. We claim that the power of mathematical programming lies in its ability to model complex situations, the optimising process being used to select more interesting rather than less interesting feasible solutions. The ultimate aim is not optimal answers but enhanced insight into the real-world situation being modelled. We argue that this has far reaching implications both in the practical use of mathematical programming and in its teaching. The realisation that the power of mathematical programming lies in its modelling capability, suggests that it could be used in situations - such as behavioural situations - where no objective function may be determined. We conjecture that research into what constitutes "interesting" feasible solutions will lead to a sideways development of mathematical programming into areas currently considered beyond its scope. We illustrate this last point with a system containing a number of non-cooperative behavers each with a linear objective function. The feasible solutions form a closed convex polyhedral set. The behavers may only improve their objective function by varying certain of the system variables - their control variables. Interesting solutions here, are solutions where no behaver can improve his lot by change to his control variables alone. We show that these stable solutions are different to the efficient solutions of multi-criteria optimisation.

32

The Current Scope and Effectiveness of Linear Programming in a Large Firm

A. Orden, University of Chicago, Chicago, Il., U.S.A.

About 10 years ago the application of linear programming in management arrived at a plateau at which it at present still remains. The main aim of this paper is to identify factors which, in retrospect, seem to have determined this situation. The investigation centers on a firm in which there are LP applications of considerable scope and diversity. This provides a basis for consideration of model sizes and structures; the extent to which hierarchic and other structural aspects of the firm are represented within LP models vs. loosely coupled by managers; the personpower which is employed in maintenance of model structures and in data preparation; and the computer resources used in obtaining solutions. The paper aims on the basis of such observations, at an assessment of the prospects that further research--generally speaking in large-scale linear programming-will pave the way to a new round of growth of management applications, for example via wider use of multiperiod models.

33

Mathematics Through Mathematical Programming

A. Gewirtz, Brooklyn College CUNY, New York, N.Y., U.S.A.

This paper presents detailed information of the importance of Mathematical Programming as the vehicle to teach some mathematics to, and establish the importance of mathematics in society for the vast majority of Americans who have minimal knowledge of, distaste bordering on hatred for and are ignorant of the uses of mathematics in our economy. The information in this paper has been collected for a period of eight years and will include data from two classes presently being given to students under-going Drug free drug rehabilitation therapy at the Daytop Village facilities in Parksville, New York and in New York City. The students are full time students enrolled in the Daytop Miniversity Program of Brooklyn College.

34

Representation of Multi-stage Stochastic Linear Programs with Finite Distribution

R.C. Grinold, University of California, Berkeley, Ca., U.S.A.

A flexible procedure for formulating multi-stage stochastic linear programs in one of two basic formats: the contingency format or the realization format. A matrix algebra is presented to assist transforms from between formats and to construct hybrids.
The formulation stresses the stochastic nature of the problem and stochastic interpretation of the dual constraints and dual variables.
The results sharpen our understanding of stochastic linear programs, make it easier to identify a problem's special structure, and are a first step in finding ways to approximate the solutions of multi-stage stochastic linear programs.

35

Solving Multistage Stochastic Quadratic Programs

F. Louveaux, Facultes Universitaires de Namur, Namur, Belgium

We consider multistage stochastic programs with recourse, having quadratic objective function and linear constraints. In the absence of inequality constraints, the well-known certainly equivalent result applies. We recently showed that under relatively weak conditions solving a multistage stochastic quadratic program having discrete distribution and linear inequality constraints is equivalent to solving a nested sequence of piecewise quadratic programs.
We first review these conditions, then transform the existing algorithm for piecewise quadratic program to cope with this multistage situation. We then consider some aspects of the numerical implementation of the algorithm, in particular the possibility of having separable subproblems and finally report on numerical experiments illustrating the importance of these aspects in stochastic energy investment problems.

36

A Unified, Parametric Quadratic Programming Solution to Some Stochastic Linear Programming Models

M. Kojima, Tokyo Institute of Technology, Tokyo, Japan
K. Kaneko, University of Wisconsin, Madison, Wi., U.S.A.

In this paper we consider several deterministic models investigated in the literature for a stochastic linear program with a constant feasible region and stochastic cost coefficients having a multi-variate normal distribution. Relationships are examined among the solution concepts in these models. The main result is to show that solving a parametric quadratic program associated with Markowitz's mean-variance model yields solutions to all other models we consider for all relevant values of parameters, in a unified way.

37

Estimation of the Transition Matrix of a Doubly Stochastic Markov Chain

M. Raghavachari, Indian Institute of Management, Ahmedabad, India
G.L. Yang, University of Maryland, College Park, Md., U.S.A.

Consider a markov chain with a doubly stochastic transition matrix. The problem of the maximum likelihood estimation of the transition matrix from sample count data is formulated as a non linear programming problem. The form of the estimate has been derived from the theory of mathematical programming. An iterative procedure which is seen to converge to a doubly stochastic matrix of the given form has been developed to obtain the estimate. Monte Carlo simulations have also been done to study the efficiency of the maximum likelihood estimate and investigate the closeness of the estimated matrix to the population transition matrix.

38

Duality and Information in Stochastic Linear Programming Exemplified by a Dynamic Inventory Control Model

W.K.K. Haneveld, University of Groningen, Groningen, The Netherlands

We consider the single commodity N-stage nonstationary production-inventory model with stochastic demands, and assume for simplicity that the production cost, the holding cost and the shortage cost are linear. Of basic importance for the model is the assumption about the information structure, i.e. which part of the demand process has been observed by the decision maker at the time he has to decide about the production in period n. We consider the whole range from no observation (open loop control) to complete anticipative information (deterministic model). The optimal control is determined by certain optimal inventory levels which can be calculated by dynamic programming. In this paper the problem is dualized. The dual problem too can be solved by dynamic programming. Actually, this method is equivalent with solving the original problem by dynamic programming in differentiable form. The dual solution can be seen as a price system related to the information structure of the primal decisions. Since all costs are linear, we get explicit expressions for the dual solution in terms of probabilities of certain events.

39

Optimality Conditions for Programming Problems Involving Multivalued Mappings

W. Oettli, Universitat Mannheim, Mannheim, West Germany

We consider programming problems of the form $\min\{f(x) | x \varepsilon A, \Gamma(x) \cap B \neq \emptyset\}$, where Γ is a multivalued mapping from some linear topological space $X \supset A$ into another linear topological space $Y \supset B$. Using a suitable definition for the derivative of a multivalued mapping it is possible to obtain complete analogs of the usual Kuhn-Tucker-Lagrange conditions. The convex case - with and without topology - will also be discussed.

40

Infinite Dimensional Mathematical Programming Problems Arising in Communication and Control

S.K. Mitter, M.I.T., Cambridge, Ma., U.S.A.

In this talk we discuss a number of infinite dimensional mathematical programming problems arising in the areas of data communication networks, quantum communication theory and stochastic control. Some of these problems are linear programming problems where the standard regularity assumptions for the non-existence of duality gaps are not satisfied. Nevertheless a satisfactory duality theory for these problems can be given. We also present examples of mathematical programming problems in these areas which are distinctly nonconvex. It will be suggested that the methods of differential manifolds may be the "correct" way of developing a theory of optimization for nonconvex problems.

41

Post-optimality Analysis for Continuous Programming

R.N. Buie, J. Abrham, University of Toronto, Toronto, Canada

The problem under consideration is assumed to have an optimum solution and to satisfy the so-called local regularity condition. Furthermore, it is assumed that the null space of the Frechet differential of the operator in the constraint has a topological complement. The effect of a perturbation in the constraint on the optimum value of the objective functional is expressed in terms of the dual variables and the perturbation. The optimum solution is shown to be a function of the perturbation; the theory of generalized inverse operators is used.

42

Separably Infinite Programming

A. Charnes, University of Texas, Austin, Tx., U.S.A.
P.R. Gribik, K.O. Kortanek, Carnegie-Mellon University, Pittsburgh, Pa., U.S.A.

Separably-infinite programs are a class of linear infinite programs that are related to semi-infinite programs and which have applications in economics and statistics. These programs have an infinite number of variables and an infinite number of constraints. However, only a finite number of variables appear in an infinite number of constraints, and only a finite number of constraints have an infinite number of variables. Duality in this class of programs is studied and used to develop a system of nonlinear equations satisfied by optimal solutions of the primal and dual programs. This nonlinear system has uses in numerical techniques for solving separably-infinite programs.

43

A General Theory of Second Order Conditions

A.D. Ioffe, MADI, Moscow, USSR

The theory of second order conditions to be presented covers a sufficiently general class of problems in Banach spaces including "standard" smooth problems with finitely many inequality and infinitely many equality constraints. The case when the problem possesses a certain amount of nonsmoothness will be also considered. Among the results to be discussed, there are new necessary and sufficient second order conditions, exact smooth penalization and augmented Lagrangian theorems. The novelty of the theory lies in that: 1) it incorporates no additional regularity assumptions beyond those used in the first order theory; 2) it deals with the whole set of Kuhn - Tucker vectors and doesn't require this set to be a singleton; 3) in comparison with the "standard" second order conditions, necessary conditions given by the theory are stronger and sufficient conditions are weaker; interrelations among them are much like in the classical theory; 4) the theory works if the estimates for second derivatives hold only for a weaker norm than the one with respect to which the derivatives are defined. (This situation is typical for infinite dimensional problems, say, variational problems.) Most of the results are new even in the finite dimensional situation. We hope also to discuss possible applications of the theory for computational purposes.

44

Pay-off Mollifiers and the Harsenyi-Selten Valuation Function

A. Charnes, J. Rousseau, The University of Texas, Austin, Tx., U.S.A.
L. Seiford, York University, Toronto, Canada

L. Shapley raised the question of the relation of the Charnes, Rousseau, Seiford "mollifier" concepts to the Harsanyi-Selten modification of the von-Neumann-Morgenstern characteristic function construction for games in normal form to take better account of "disruption" or "threat" possibilities. We show for a large class of games that the H-S construction yields a constant mollifier. In relevant cases it is non-superadditive when the v.N-M function is superadditive.
We extend "mollification" to games in normal form. In the extended theory, the H-S construct is a constant mollifier with the preceding non-superadditive impediment. It also fails to take account of coalitional sizes. Our extended "homomollifier" concept does and always yields a superadditive constant sum characteristic function when the v.N-M construction does.

45

Games with Supermodular Characteristic Functions

D.M. Topkis, Bell Laboratories, Holmdel, N.J., U.S.A.

Shapley showed that an n-person cooperative game with a supermodular characteristic function has a nonempty core, and he gave a simple algorithm for finding elements of the core. Von Neumann and Morgenstern constructed the value of the characteristic function for each coalition from a corresponding two-person noncooperative game, but no reasonable conditions under which this construction generates a supermodular characteristic function appear to exist. This paper presents a model in which the value of the characteristic function for each coalition comes from a certain optimization problem. For this model, the characteristic function is supermodular under suitable conditions. One example involves a cooperative game version of the selection problem, where players have different sets of items available and each coalition chooses from among the items available to its members so as to maximize the value of the activities that can be performed with the chosen items minus the cost of the chosen items. Shapley's algorithm finds and element of the core of this game by solving n nested minimum cut problems. In another example, the players are producers and a coalition pays for purchases of materials for its members according to concave cost functions.

46

An Information Theory of Game Systems

S. Kano, Y. Kai, Kyushu University, Kyushu, Japan

In game theory, pay-off functions have played an important role. However, an essential point of game behaviour is that one player estimates the other players' strategies before the other players estimate the player's strategy, where strategy implies a policy and is not presented explicitly in the game. From the viewpoint, we formulate a game system where each player has a random parameter whose values denote his strategies. A state of the game system is presented by a set of all players' actions which are determined stochastically by their strategies and all past states. Each player intends to obtain the information on values of the other players' parameters by the sequential observations. We give the conditions under which one player obtains the total amount of information on the other player's strategy in two person game. This consideration is developed to 4-person game in which player 2, who competes with player 1, cooperates with player 3 or player 4, and we study a problem to decide which is better for player 1, to cooperate with player 3 or 4. It is remarkable that the above study of cooperative game gives some models of parallel information processing in brain systems and computer systems.

47

A Characterization of Vector Measure Games in pNA

Y. Tauman, CORE, Louvain La Neuve, Belgium

The space pNA plays a central role in the theory of non-atomic games. Aumann and Shapley, in their book "Values of Non-Atomic Games", have proved the existence of a unique value on pNA and presented a formula which enables us to compute the value for games in pNA of the form $f \circ \mu$ where μ is a finite vector of NA measures, and f is a real function defined on the range of μ with $f(0) = 0$. It is natural to ask which games of the form $f \circ \mu$ are in pNA ?

DEFINITIONS. Define a norm $\| \, \|_m$ on the linear space NA^m by

$$\mu = (\mu_1, \ldots, \mu_m) \Rightarrow \|\mu\| = \sum_{i=1}^{m} \|\mu_i\|_{BV}$$

Denote by $R(\mu)$ the range of μ ($R(\mu) = \{\mu(S) \mid S \in \mathcal{C}\}$).
Define $B(\mu)$ and $B(\mu,\varepsilon)$ ($\varepsilon > 0$) by

$$B(\mu) = \{\nu \in NA^m \mid R(\nu) = R(\mu)\}$$
$$B(\mu,\varepsilon) = \{\nu \in B(\mu) \mid \|\mu - \nu\|_m < \varepsilon\}.$$

Fix μ in $(NA^1)^m$ and a real function f on $R(\mu)$ with $f(0) = 0$

We will say that <u>f is continuous at μ</u> if for each $\varepsilon > 0$ there exists $\delta > 0$ such that

$$\nu \in B(\mu,\delta) \Rightarrow \| f \circ \mu - f \circ \nu\|_{BV} < \varepsilon$$

The main theorem is

THEOREM. A necessary and sufficient condition for $f \circ \mu$ to be in pNA is that f is continuous at μ.

In the course of the proof of the theorem two other propertie are established.

PROPERTY I. If $f \circ \mu$ is in pNA then there exists a sequence polynomials $(p_n)_{n=1}^{\infty}$, all of them on R^m, such that

$$\|p_n \circ \mu - f \circ \mu\|_{BV} \xrightarrow[n\to\infty]{} 0.$$

PROPERTY II. If $f \circ \mu$ pNA for each $\nu \in B(\mu)$.

48

Generalized Shapley Values by Simplicial Sampling

B. Von Hohenbalken, University of Alberta, Edmonton, Canada
T. Levesque, Wilfrid Laurier University, Waterloo, Canada

This paper describes the development of a generalization of the Shapley value, a solution concept of game theory that allows for clique formations among players. The calculation of approximate generalized Shapley values is done by a sampling technique, that is briefly summarized in Section 4 of the paper. An APL- code and an example are also given.

49

The Probabilistic Analysis of Combinatorial Optimization Algorithms

R.M. Karp, University of California, Berkeley, Ca., U.S.A.

In practice optimal or near-optimal solutions to combinatorial optimization problems are often found by heuristic algorithms. These algorithms perform well despite the fact that they are capable of failing badly on maliciously chosen problem instances. There is a growing body of work which investigates the probable performance of such algorithms on inputs drawn from well-defined probability distribution. We review some favorable results regarding heuristic algorithms for various network-flow problems, routing problems and location problems, as well as negative results about the computation of maximum cliques and optimal knapsack solutions.

50

Direct Solutions to Some Constrained Transportation Problems

P. Scobey, D.G. Kabe, St. Mary's University, Halifax, Canada

Consider the multivariate normal linear regression model $Y = \beta X + E$, in which β is subjected to the restrictions $F\beta = W_1$, $\beta H = W_2$, and $J\beta L = W_3$. Some statistical methodology available for estimating the regression coefficient matrix β under these conditions is used to derive a direct solution to a three dimensional constrained transportation problem. Also, a classical quadratic programming technique is used to derive a direct solution to a two-dimensional constrained transportation problem. Examples illustrating the methods are given.

51

A Direct Method for the Quadratic Capacitated Transportation Problem with Separable Costs

J-S. Pang, Carnegie-Mellon University, Pittsburg, Pa., U.S.A.

One of the most important reasons why the Simplex Method can be applied so successfully to a linear transportation problem is because the problem possesses a triangular basis matrix, or in other words, the subgraph associated with the basis forms a tree. In this paper, we show that in the equivalent linear complementarity formulation of the quadratic capacitated transportation problem with separable costs, there is also a certain "basis matrix" which has a similar "triangular" or "tree-like" structure. By exploiting this structure, we shall describe how the parametric principal pivoting algorithm can be applied effectively to solve this quadratic transportation problem. Finally, we shall report computational experience.

52
Minimum-Concave-Cost Network Flows
R.E. Erickson, Bell Laboratories, Holmdel, N.J., U.S.A.

The minimum-concave-cost single-source network flow problem is reduced to that of finding a stationary optimal halting policy in a finite-state branching Markov decision chain. This is accomplished by noting that an extreme flow into a node equals the sum of the requirements at some subset of the nodes. A state is a node together with the set of requirements nodes supplied by an extreme flow into that node. A decision indicates how that extreme flow is divided among the immediately accessible nodes and determines the one-period costs from the flow costs. An optimal circuitless decision then determines a minimum-cost flow. A resulting algorithm is shown to have a polynomial running time if the network is planar.

53
Solving Network Concave Optimization Problems
M. Trojanowski, Academy of Sciences, Warsaw, Poland

The problem under the consideration is how to solve network problems with concave objective function.
Most frequently, when such a problem appears/and it does in a wide variaty of optimization situations/ the usual approach to solve it is to use one of the general non-linear optimization algorithms or to develop a heuristic one. Such tools are very uneffective from many points of view.
There are reported few papers exploiting special structure of the problem embeded in it's set of constraints. The main role in these papers /authored by e.g. Katta Murty, Patrick G.Mc.Keown/ plays a procedures for ranking adjacent extreme points of the constraints polyhedron. Obtaining high performance of the procedures, enumerating neighboring extreme points is also very important for many other mathematical programming disciplines.
It seems to be the way to achieve the goal by employing data structures from the computer science to a graph theoretic characterization of the adjacency relationship. There exist a number of theorems stating correspondence between the sets of extreme points /interpreted as extreme flows in networks/ and certain network elements such as for example simple cycles. The drawbacks of methods based on these theorems are the high costs of handling data, what can be reduced /as with a great success in the large scale transshipment problems/ by adequate data structures. The attemps is to show the ways how to apply such an approach.

54
Distributions of Vertex and Path Attributes for a Few Parametrized Polytope Classes
P.C. Kettler, University of Chicago, Chicago, Il., U.S.A.

The study investigates parametrized classes of polytopes from a probabilistic point of view. Coefficients of the defining inequalities are random variables associated with several probability distributions. The research goal is to study the distributions of basic solutions, of extreme points, and of minimal step counts between arbitrarily chosen vertices, as functions of the various parameters. Three problems are discussed. In the simplest, the polytope results from cutting a simplex in R^n by n+1 hyperplanes symmetrically positioned, with each plane cutting a separate vertex. The one parameter is the dimension n, and the random variable is a number α which specifies the location,or encroachment, of the planes. An elaboration on this problem allows each plane to cut several vertices, thereby introducing another parameter. In the intermediate problem, the simplex is again cut by as many planes as vertices, but the conditions are relaxed to allow separate positioning of the planes. Now, instead of a single specifying encroachment, there is a vector of $\{\alpha_i\}$ of length n+1. In the third problem, the conditions are further relaxed to allow the planes to cut arbitrary numbers of the vertices from disjoint sets, and to allow skewness in the cuts. Additional random variables, the $\{\beta_j\}$ appear in the defining matrix to represent these new attributes. Works of Victor Klee and George J. Minty, and of David Gale, and others, impinge on these results. The paper discusses the connections.

55
Quasi-Newton Methods for Minimax
S-P. Han, University of Illnois, Urbana, Il., U.S.A.

We consider the minimization of a function which is the maximum of a finite number of smooth but nonlinear functions. It is well known that the minimax problem of this type connects naturally to a nonlinear program. Through this connection the effective Quasi-Newton methods become applicable. We show that this approach is valid and the resulting methods have global and superlinear convergence properties.

56
Updating Quasi-Newton Matrices with Limited Storage
J. Nocedal, Universidad Nacional Autonoma de Mexico, Mexico City, Mexico

We study how to use the BFGS quasi-Newton matrices to precondition the conjugate gradient method in problems where the storage is critical. We give an update formula which generates matrices using information from the last m iterations, where m is any number supplied by the user. The quasi-Newton matrix is updated at every iteration by dropping the oldest information and replacing it by the newest information. It is shown that the matrices generated have some desirable properties. Several conjugate direction algorithms are tested numerically and their relative merits are discussed.

57

An Acceptable-Point Algorithm for Function Minimization

G.E. Johnson, M.A. Townsend, Vanderbilt University, Nashville, Tn., U.S.A.

Many nonlinear programming algorithms employ a univariate subprocedure to determine the step length at each multivariate iteration. In recent years much work has been directed toward the development of algorithms which will exhibit favorable convergence properties on well-behaved functions without requiring that the univariate algorithm perform a sequence of one-dimensional minimizations. Accordingly, several univariate minimization methods based on polynomial approximation have been modified to search for "acceptable" rather than minimizing step lengths. In this paper a direct search method (the golden section search) is so modified and then used as the univariate subprocedure for a generalized conjugate gradient algorithm [Townsend & Johnson, J. Franklin Inst., V.306, No.5, 1978, pp. 325-339]. The resulting multivariate minimization method is tested on standard unconstrained test functions and constrained industrial problems. The data suggest the new method to be relatively insensitive to tuning parameters (insofar as success or failure is concerned) and also indicate favorable efficiencies (in terms of equivalent function evaluations) relative to some of the currently popular quasi-Newton methods with acceptable-point searches.

58

Computing Optimal Locally Constrained Steps

D.M. Gay, M.I.T., Cambridge, Ma., U.S.A.

In seeking to solve an unconstrained minimization problem, one often computes steps based on a quadratic approximation to the objective function. One reasonable way to choose such steps is by minimizing the quadratic approximation constrained to a neighborhood of the current iterate. We consider a new way to handle certain computational details when the approximation's Hessian is indefinite, paying special attention to a special case which may then arise. Our ideas lead to step computing modules that have proven very satisfactory in the nonlinear least-squares solver NL2SOL (which may generate indefinite Hessian approximations).

59

The Differential Geometry of the Twice Continuously Differentiable Unconstrained Optimization Surface and its Application to the Newton/Greenstadt Method

R. Frank, IBM Cambridge Scientific Center, Cambridge, Ma., U.S.A.

The twice continuously differentiable unconstrained optimization surface is analyzed using elementary classical Differential Geometry. The resulting formulae are related to the standard quadratic approximation to the surface which is the truncated Taylor series. A modified eigenproblem for the Hessian is shown to give exact curvature information anywhere on the surface. The usual eigenproblem for the Hessian yields only sign information about the curvatures except at a stationary point where it does give exact curvature information.
The modified Hessian eigenproblem is then employed to extract a new spectral expansion of the Hessian which requires the same order of magnitude operation count as the usual expansion. This new expansion is then used to geometrically interpret Newton's iteration anywhere on the surface.
The Newton iteration geometry is then analyzed from the same viewpoint as Greenstadt's modification and from the viewpoint of Generalized Inverse methods sharpening and extending the well known analytic results of those methods.
The literature is carefully related to these new developments and numerical considerations are discussed.

60

Introducing a Non-linear Optimizer into a Financial Planning Language

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L. Lasdon, University of Texas, Austin, U.S.A.

Modern financial planning languages permit interactive construction and solution of dynamic models. These are usually used to answer "What if?" questions by solving the model sequentially in time for specified values of some variables or parameters. We discuss here interfacing the language IFPS with the non-linear optimization program GRG2 to permit a selection of optimal values for the model variables.
IFPS is a language with special constructs which permit dynamic financial models with hundreds of equations to be posed in language natural to the user. GRG2 is an implementation of the generalized reduced gradient algorithm, using an explicit basis inverse, for solving non-linear programs of general form. From GRG2's viewpoint, IFPS is an evaluator of constraint and objective function values for given values of the decision variables. GRG2 also requires first partial derivatives of these functions. Means for their automatic and rapid computation from an IFPS model will be described, and some preliminary computational experience with the system will be presented. Some examples will be given of problems which prospective future users plan to pose.

61

Computational Analysis of Equivalent Formulations for Linear Product-Mix Models and Mixed-Integer Production-Investment Models

G.F. Knolmayer, Universitat Wien, Wien, Austria

This paper presents results obtained by transfering methods for comparing algorithms to the comparison of equivalent LP and MIP-formulations. Such results should support the decisions of modellers in matrix generation. An important reason for the existence of equivalent formulations is the possibility to eliminate balance equations (i.e. constraints i with b_i=0). In a model with p balance equations the 2^p equivalent formulations are often characterized by a substitutional relation between the number of rows and the number of columns (nonzeros). To test the effects of different model structures a problem generator has been written which generates "reasonable" multistage production models with many balance equations. A condensation program eliminates these constraints if the elimination will not generate more than a user specified number of nonzeros. The original and some condensed formulations were either

optimized by LP or revised with investment data. In the latter case economies of scale were modelled by SOS2 and the revised models were optimized by MIP. The characteristics of the optimization by APEX-III were automatically recorded and several regression models were tried to find "good" explanations for them. The results show that the elimination of balance equations is far less attractive than expected from the m^3-hypothesis of most textbooks. A significantly better explanation of computational effort is obtained by using both the numbers of rows and nonzeros as predicting variables in regression analysis.

62

A Goal Programming Sensitivity Analysis Approach to the Product Mix Problem

M.H. Elwany, Alfateh University, Tripoli, Libya

In production systems where a number of goals that don't have mutual compatibility and a number of substitutional goals are formulated, one often wants to know how the profit and/or cost functions change as the goals change, or the substitutional goals come into effect. The application of goal programming includes the issue of presenting suitable plans in comprehensible form to production systems. A case study on a production system has been carried out. In this case study four goals, two substitutional goals and some forty constraints are formulated. Changes in several goals at once are investigated using parametric linear programming. The changes in productivity coefficients are of practical importance in the case study because the market prices of the inputs change frequently in short run. Marginal substitution rates are used to analyse the senstivity of the solution.
The effect of some non-linearities due to practical considerations are investigated during the formulation stage. Introducing the substitutional goals provide a quite effective approach that may help in solving a wide range of product-mix problems.

63

Solving Certain New Problems of Resource Allocation by Mathematical Programming

R. Slowinski, J. Weglarz, Technical University of Poznan, Poznan, Poland

Most of the results obtained until now in resource allocation theory among activities of a network project have concerned situations in which only renewable or only non-renewable, only discrete or only continuous resources have been considered. By renewable /or non-storable/ we understand resources for which only total usage at every moment is constrained; by non-renewable /or storable/, however, those for which constraints only concern total consumptions up to given moments. On the other hand, by discrete resources we understand such resources as machines, facilities, production stages, etc., as opposed to continuous resources which are continuously divisible, for example energy, money, power, fuel flow, etc. In practice however, we often meet situations in which different types of resources must be considered in a common model. For example, the same activities simultaneously need for their performance machines and /or money, or power, etc. Moreover, in real world situations, resources often cannot be treated as either renewable or non-renewable, as in the case of power or manpower. In this paper we consider a large class of resource allocation models describing such situations. We show how the problems may be reduced to the appropriate linear or nonlinear programming problems in the most effective way taking into account the mathematical models of activities, the network structure and the optimality criterion.

64

A Generalized Application of Method "Optimization with Minimal Information"/"OMI"/to Some Type of Allocation Problems

Z.M. Farkas, Hungarian Statistical Institute, Budapest, Hungary

The purpose of this planned paper is to present one of the author's mathematical result connected with the method "OMI". The problem solved is the generalized resource allocation problem of following type:

$$\tilde{M} = \sum_{i=1}^{n} M(\underline{x}_i, \underline{y}_i) \to \min, \text{ if } \sum_{i=1}^{n} \underline{y}_i = \underline{y}, \qquad |P|$$

$$\text{where } \underline{x}_1 \subseteq \underline{x}_2 \le \ldots \le \underline{x}_n$$

$$\text{and } \underline{x}_i, \underline{y}_i \in E^n, \ \underline{x}_1 \le y_i, \quad i = 1,2,\ldots,n;$$

Here the objective function $\tilde{M}(\underline{x},\underline{y})$ is not exactly known, only well-defined properties are fulfilled by the function $M(\underline{x},\underline{y})$. These properties define a whole class of objective functions /with "minimal information" of heuristic origin/ the so called homogen separable convex functions. Upon giving a general model, a solution class is determined by the author, which class is invariant for the whole objective function class.
The author's new results are generalization of solving the above $|P|$ new type convex programming problem from scalar to n-dimensional variable, and interpret it for modelling certain modified type of transportation problem / objective with a homogen separable convex penalty function /. It is to be noted that general solution algorithm gives a chance to solve some multidimensional allocation problems connected with reliability theorem. Paper intends to present also both of above mentioned applications together with theoretical approach of this new type of optimization methods.

65

Strong Vector Minimization and Duality

B.D. Craven, University of Melbourne, Melbourne, Australia

A constrained minimum of a vector valued function is defined, in terms of an ordering cone. The definition chosen leads to a vector analog of the Kuhn-Tucker theorem, and to a duality theorem where the dual problem has a vector valued objective function, and weak duality is defined by an appropriate cone ordering. Also proved are a vector valued converse duality theorem, and a vector quasiduality theorem which does not require convexity. The results are related to perturbations of the objective function in a minimization problem.

66

Saddle-Point for Vector Valued Functions

B. Lemaire, Universite Montpellier II, Montpellier, France

The notion of saddle-point is classical for real valued functions defined over a product of two sets. Here we give extensions to functions taking values in an ordered vector space, the infimum or supremum being sought in the sense of Pareto. In some cases one of these extended notions can be reduced to the classical one by a convenient parametrization. This permits to obtain existence theorems.

67

Holistic Preference Evaluation in Multiple Criteria Optimization

J.K. Ho, Brookhaven National Laboratory, Upton, N.Y. U.S.A.

This paper presents an interactive, parametric linear programming method: HOPE (for holistic preference evaluation) to solve the multiple criteria linear programming problem. The method uses the decision maker's ordinal priority ranking of the criteria to structure the parametrizations, and by successively eliciting his holistic preference among alternative efficient solutions, refines the structure until a most preferred solution is established. Advantages over existing techniques include ease of implementation and use, absence of explicit analysis of marginal utilities (or trade-off curves), and an intuitive interpretation as a learning process for the decision maker to "weigh" the criteria. Numerical examples from forest management and university planning are given.

68

Multiple Objective Mathematical Programming with Respect to Multiple Decision-Making

R.E. Wendell, University of Pittsburgh, Pittsburgh, Pa., U.S.A.

We consider the important problem of obtaining a "majority consensus" among a number of individuals on the solution to a multiple objective optimization problem. When the number of objectives is greater than two, we observe that such a consenaus exists only under strong symmetry conditions. However, in the bi-objective case, we show that a "consensus' exists in the convex case and we show how to find it via direct and interactive approaches. In the nonconvex bi-objective case, we observe that a "local consensus" always exists under rather weak regualrity conditions.

69

An Exact L1 Penalty Function Method for Nonlinear Equations and Nonlinear Programming

R. Fletcher, University of Dundee, Dundee, Scotland, U.K.

An ideal way of solving nonlinear equations or nonlinear programming problems is to solve a subproblem in which a certain quadratic/linear L_1 function is minimized subject to a restriction on the resulting length of step. This L_1 function is obtained by approximating the L_1 penalty function for the original problem in an appropriate way. Global and second order convergence results for this algorithm can be established. However the solution of the subproblem requires quadratic programming type software, which can be expensive in computer time and storage. A related algorithm is described which avoids quadratic programming by means of an active set strategy and anti-zigzagging tests. An important feature is that only $n^2 + O(n)$ storage locations are required. Theoretical and experimental results are presented which show that the algorithm is efficient and reliable. Currently second derivatives are required but there is scope for the use of quasi-Newton updates.

70

A Technique for Forcing Convergence in Variable Metric Methods For Constrained Optimization

R.M. Chamberlain, H.C. Pedersen, M.J.D. Powell, University of Cambridge, Cambridge, England

Variable metric methods for constrained optimization are iterative. Search directions are generated in the space of the variables, and steps are taken along search directions. Successful convergence depends on an initial estimate of the solution, the choice of search directions, and the choice of step-lengths. The aim is to force convergence even when a good initial estimate of the solution is not available. One difficulty is that a trial step in the space of the variables may improve the objective function, but the constraint violations may become worse, so a suitable balance has to be found. A technique that usually gives excellent results is to use a line search function that is the objective function plus a weighted sum of the moduli of the constraint violations, but it is difficult to choose suitable weights. Han has proved a global convergence theorem for a method of this type that depends on the weights being sufficiently large and constant, but an example is given to show that the use of large weights can be highly inefficient. Therefore a "watchdog technique" is proposed. The watchdog is Han's method, for it is used to force convergence, but it is not necessary to apply it on every iteration. Therefore on several iterations one uses instead a line search objective function whose weights are of a size that is well suited to the search direction of the iteration. Numerical results show that this extension of Han's method can reduce greatly the number of function and gradient evaluations that are required in a constrained minimization calculation.

71

On the Convergence Properties of Variable Metric Recursive Quadratic Programming Methods

L.C.W. Dixon, The Hatfield Polytechnic, Herts, England

In recent years a number of authors have proposed similar algorithms based on a combination of the variable metric principle and recursive quadratic programming. Methods of this type include those proposed by Murrey (1969), Biggs (1972, 1975, 1978), Han (1975, 1977) Powell (1976, 1977, 1977) Tapia (1977) and Dixon (1979). These methods all generate the direction of search by forming a quadratic approximation to the Lagrangian and linear approximations to the constraints, but differ in regard to the function used in the linear search. In this paper a number of new results on the global convergence of this type of algorithm will be presented.

72

The Computation of Lagrange-Multiplier Estimates for Constrained Minimization

P.E. Gill, W. Murray, Stanford University, Stanford, Ca., U.S.A.

In this paper we shall discuss the computation of estimates of Lagrange multipliers for both linearly- and nonlinearly-constrained optimization. In nonlinearly-constrained optimization in particular we believe that Lagrange-multiplier estimates can be prone to gross error. When generating an estimate x of the solution of an optimization problem, it is always possible to establish simple criteria to ascertain whether or not x is better than any previous estimate. Analogous criteria for uniformly improving Lagrange-multiplier estimates are not known. In this paper however, we do give computational criteria for accepting or rejecting Lagrange-multiplier estimates and we attempt to establish the degree of confidence that can be placed in an estimate.

Most techniques for estimating Lagrange multipliers are embedded within a particular optimization method and the estimates given depend upon properties of the sequence of approximations to the solution which do not hold in other algorithms. In this paper we have attempted to derive formulae which are independent of any special properties of the point of estimation. However, the implication of applying these general formulae to particular algorithms is discussed.

73

A Superlinearly Convergent Algorithm for Constrained Optimization Problems

D.Q. Mayne, Imperial College of Science and Technology, London, England
E. Polak, University of California, Berkeley, Ca., U.S.A.

There exist many algorithms, such as those due to Levitan and Polyak, to Wilson and to Robinson, for solving constrained optimization problems which have a superlinear rate of convergence but are only locally convergent. A major difficulty in globally stabilizing these algorithms is the fact that they generate a sequence of points which are not necessarily feasible, making comparison of successive points difficult. However, a class of algorithms, proposed by Han, Powell and Mayne and Maratos, in which the search direction is determined by a suitable approximation to the original problem and step length determined by (approximately) minimizing an exact penalty function, holds considerable promise. However several difficulties, listed by Han, need to be overcome. The first is the choice of the penalty function parameter; many of the procedures proposed for choosing these parameters are heuristic and may lead to cycling or jamming. Secondly an Armijo-type procedure is needed for determining a suitable step length. Thirdly, if superlinear convergence is required, the algorithm must ensure that the step length is asymptotically unity - existing algorithms do not have this property. The final problem concerns the suitability of the search direction yielded by the quadratic approximation to the original problem. An algorithm which successfully overcomes these difficulties is presented and shown to be globally convergent and to possess a superlinear convergence rate.

74

Nonsmooth Optimization

F.H. Clarke, University of British Columbia, Vancouver, Canada

We discuss some aspects of the theory and applications of nondifferentiable, nonconvex optimization problems in mathematical programming and optimal control.

75

An Approach to Newton Method in Nondifferentiable Optimization

E.A. Nurminski, IIASA, Laxenburg, Austria

The computational efficiency of subgradient schemes in nondifferentiable (convex) optimization strongly depends on the shape of level sets and is rather slow for ill-behaved problems. The alternative approach is a Newton-like method using an analogy of second derivatives. The possible definition of second derivatives of a convex function is given and computational methods are discussed.

76

Convergence of a Modification of Lemarechal's Algorithm for Nonsmooth Optimization

R. Mifflin, Institute of Decision Science, Claremont, Ca., U.S.A.

An algorithm is given for finding stationary points of minimization problems having locally Lipschitz problem functions that are not necessarily convex or differentiable.

77
On the Surprising Effectiveness of Subgradient Optimization
J.L. Goffin, McGill University, Montreal, Canada

Subgradient optimization has proved to be an effective procedure for many large scale problems. This effectiveness has been shown through many experiments.
A theory on the rate of convergence exists, which depends upon the definition of a condition number. This theory tends to indicate that subgradient optimization should not work too well. It does, however, and more sophisticated theories are needed. The following can be shown experimentally and, in a very limited way, theoretically:
1) the method exhibits an acceleration phenomenon which appears to be similar to that of the recessive overrelaxation techniques (but it is not identical);
2) there seems to be uniform bounds on the condition number of various problems (assignment, etc...).

78
Multipliers and Duality in Nonconvex Nondifferentiable Programming
J.P. Penot, Faculte des Sciences, Pau, France

We consider a new definition of generalized subdifferential for a non convex nondifferentiable mapping. We compare it with known concepts as Clarke's, Pshenichnyi's, Rockafellar's definitions. We show by simple examples it gives more precise information than Clarke's generalized gradients especially when dealing with necessary conditions. Finally, we use it in sensitivity analysis, refining results due to J. Gauvin, J. Gauvin - J.W. Tolle, A. Auslender, V.H. N Guyen - J.J. Strodiot - R. Mifflin.

79
Stochastic Linear Programming Models of Short-Term Financial Planning Under Uncertainty
W.T. Ziemba, University of British Columbia, Vancouver, Canada

The essence of short term financial planning is to manage a firm's or bank's assets and liabilities to maximize short run profits. Seasonal, political and economic effects create uncertainty in forecasted cash requirements, interest and mortgage rates, liquidation and termination costs, etc. The problem is well formulated as a multiperiod stochastic linear programme where the objective is to maximize expected profits associated with the manipulation of the various assets employed net of constraint violation penalty costs. In each period there are typically constraints associated with sources and uses of funds, liquidity, leverage, policy and termination conditions, etc. In the banking case one must also deal with deposit flows, mortgages and various government legal regulations. The difficulty over even such a short planning horizon as four quarters is that the problem is quite large both in terms of variables and constraints and there are a significant number of random parameters. If the random variables are discrete the problem reduces to a deterministic albeit extremely large linear program. This talk is concerned with some approaches to the solution and analysis of these problems. The approaches suggested are available for other realistic sized multiperiod stochastic linear programming problems and provide insight into tradeoffs concerning their solution and analysis.

80
Stochastic Programming in Portfolio Selection
R.J. Peters, B.V.H. Van de Kieft, State University, Groningen, The Netherlands

The use of mathematical modeling techniques for finding an optimal solution of the portfolio selection problem has received a great deal of attention in the last two decades. The input data of the problem, e.g. the return from the stocks, are stochastic by nature and this would eliminate the use of deterministic approaches. A modeling technique is proposed that differs from the common stochastic schemes, proposed in literature, (e.g. Markowitz full-covariance approach), which are formulated in terms of utility functions, reflecting the attitude of the decision maker towards risk. In our approach the optimization criterion consists of minimizing the expected loss, raised to a square, at the evaluation moment afterwards, due to not having chosen the best strategy at the beginning of the time horizon. Also the cost, associated with revising the portfolio at hand (transaction cost), are taken into account. The model has been developed in close collaboration with one of the major banks in the Netherlands. To the support of the research department of this bank the program has been exercised on a real-life case.

81
Duality in Stochastic Programming Applied to the Operation of a Reservoir
H.F. Karreman, University of Wisconsin, Madison, Wi., U.S.A.

The problems encountered in the design and operation of reservoirs are inherently stochastic. The inflow of water is dependent on various environmental factors and its probability distribution can be best approximated by a multivariate gamma distribution. The water contents of the reservoir can be used for a variety of purposes, each with its own probability distribution. Still it is possible to approximate the probability distribution of the demand for water also by a multivariate gamma distribution. Consequently, the stochastic programming model does not only have a probabilistic objective function but also some of the constraints are probability functions. Discrete and continuous methods have been developed to solve the problem in this formulation. What is more, the problem, as formulated, is a convex programming problem which makes it possible to explore the duality aspects of it. This in turn may lead to more efficient ways of obtaining the optimal solution.

82

Parametric Estimation of Urban Residential Mobility Process by Linear Programming Procedure

P. Krishnan, G. Rowe, University of Alberta, Edmonton, Canada

Estimates of the urban residential mobility process (URMP) are of great interest to urban planners. Since the census data are available only after every five, or ten years, they are not of much utility for making meaningful predictions of population distribution in cities by zones. This is all the more true of rapidly growing cities, and the boom towns. Civic censuses are conducted fairly regularly by most of the cities/towns. Even though the civic censuses do not gather very comprehensive data, one can at least get the total count of population for every zone in the urban area by broad categories for a few characteristics (eg. sex, age, religion). This aggregate data set can be utilized to generate estimates of propensities of residential mobility under certain simple assumptions. We approximate the URMP by a stationary finite Markov chain. The elements of the transition probability matrix, which yield the mobility propensities, can be estimated from the time series of aggregate data by zones by the Minimum Absolute Deviations method, where the expected distributions would be supplied by the Markov process. Equivalently the estimates are provided by the solution set of a linear programming problem. An illustration is provided by estimating the parameters of the URMP in the city of Edmonton, Canada. The estimates so obtained are compared with those generated by the Edmonton Area study carried out by the Population Research Laboratory of the University of Alberta in 1977.

83

Nested Optimization in DOA Estimation for Nonlinear Dynamical Systems: Spacecraft Large Angle Manoeuvres

M.A.H. Dempster, University of Oxford, Oxford, England

This paper discusses the formulation and numerical development of an algorithm for the estimation of the domain of attraction of a general nonlinear autonomous dynamical system. The method is based on stability analysis using Lyapunov's direct method with quadratic Lyapunov functions. It requires the nesting of an unconstrained and a constrained optimization problem--both highly nonlinear. The Powell '64 conjugate direction algorithm and the BFGS quasi-Newton algorithm are used as alternatives at the outer loop, while the recent Powell-Han Lagrangean algorithm is used for the inner loop nonlinear programme. Applications of the method to stable control of large angle manoeuvres for astronomical satellites are described.

84

Computational Complexity of Parametric Linear Programming

K.G. Murty, University of Michigan, Ann Arbor, Mi., U.S.A.

We establish that in the worst case, the computational effort required for solving a parametric linear program is not bounded above by a polynomial in the size of the problem. The implications of this on the computational complexity of other related problems, will be surveyed.

85

The Steepest Edge Simplex Method: Actual Versus Worst Case Behaviour

D. Goldfarb, CUNY, New York, N.Y., U.S.A.

At each nondegenerate iteration of the steepest edge simplex method one moves from a vertex of the polytope P of feasible points to an adjacent vertex along an edge that is steepest with respect to the linear objective function Z. In this talk we briefly describe practicable implementations of this algorithm for large sparse LP problems and minimum cost network flow problems. We also show that a sequence of LP's (P_n, Z_n) in n variables can be constructed which require the method to take 2^n-1 iterations to obtain an optimal solution. Thus, in general, the steepest edge method is as bad in the worst case sense, as is the standard Simplex method.

86

A Comparison of Real World Linear Programs and Their Randomly Generated Analogs

R.H.F. Jackson, National Bureau of Standards, Washington, D.C., U.S.A.
R.P. O'Neill, Department of Energy, Washington, D.C., U.S.A.

This paper is a report of a study to determine the relationship of real-world (i.e., arising from an application) problems to their randomly-generated analogs with respect to the difficulty of computing optimal solutions. The intent of the study is to determine the ability of randomly-generated problems to simulate real-world problems for the purposes of testing, benchmarking, and comparing software implementations of solution algorithms, and further, to determine if the degree of randomness is related to the difficulty in obtaining a solution. Randomly-generated analogs are defined to be problems created with characteristics of a real-world problem, but containing data with random elements. These analogs fall into classes that can be characterized by "nearness" to the real-world problem. The first class is obtained by randomly perturbing the problem data. The second class is obtained by randomizing the ones in the Boolean image of the problem data. The third class consists of problems obtained from software that accepts the problem characteristics as input. The random elements are generated from uniform and normal distributions. The measures of difficulty include central processor time and iterations for both phase one and two. The measures of accuracy include deviations from complementary slackness and/or the deviation from a known optimal value. Several optimizers will be used in the study.

87

Implementation Experience-Active Set Selection Using a Variable Metric Generalization of the Simplex Algorithm

L.D. Pyle, University of Houston, Houston, Tx., U.S.A.

One of the objectives of the research reported in this paper is to develop and test algorithms for selecting sets of active constraints so that repeated applications of suboptimization, when restricted to the facets selected, will lead to an over-all reduction in computational effort. This paper reports on experiences in implementing an active

set selection procedure using a variable-metric generalization of the simplex algorithm, the modular structure of MINOS, and the computational facilities of the Stanford Center for Information Processing (SCIP) via the communications network, TELENET.

88

Experiments and Experience in Designing Optimal Inversion Routines

D. Ohse, Technische Hochschule Darmstadt, Darmstadt, West Germany

Efficient inversion routines for mathematical programming systems are almost as important as the simplex procedure itself. In solving real world LP-problems inversions will usually be executed every 50 - 100 iterations. The inversion time will in this case certainly be a significant part of total CPU-time.
Present routines in the mathematical programming software use more or less heuristic decision rules for choosing the next pivot. A well-known selection rule is the Markowitz criterium which locally minimizes the number of fill-ins. Another technique is that of preassigning the pivot sequence without performing any transformations. The Hellerman/Rarick approach is a good example of that strategy.
A third class of procedures is based on partitioning first the basis into block triangular form. Tarjan's algorithm which uses the depth-first search in the graph representation is very efficient when this strategy is used.
Results are represented from several experiments using different techniques and combinations of them. Proposals shall be derived for designing a near optimal inversion routine. Optimality will be measured with respect to
(i) the number of fill-in, which should be minimized in order to get a sparse inverse
(ii) the number of arithmetic operations for speeding up the inversion.

89

Spider Graph Models for Network Design and Fleet Planning in the Air Cargo Industry

R.E. Marsten, University of Arizona, Tucson, Ar., U.S.A.
M.R. Muller, The Flying Tiger Line, U.S.A.

A cargo airline serving N cities must deliver N(N-1) types of cargo every night (one for each origin, destination pair). The two main planning problems are choosing the design of the distribution network and deciding what type of aircraft to use on each part of that network. This paper describes an approach that involves building a network design out of several overlapping spider graphs. Given the network design, the selection of the fleet mix and the routing of the cargo is then a large mixed-integer programming problem. Experience with the use of this approach is presented.

90

Fleet Scheduling

O. Holst, IMSOR, Technical University of Denmark, Lyngby, Denmark
P. Allan-Andersen, Peter Matthiesen A/S, Herlev, Denmark

The paper considers a public transportation system bound to a schedule. After a general introduction to the problem of schedule construction and evaluation the paper emphasizes on the problem of rotation planning. Relations to time-tabelling, maintenance planning and to other resource planning problems are set forward. The basic problem in the paper is to find the optimal fleet size to serve a given time table and a rotation plan corresponding hereto. Two alternative models for solving this problem are presented.
The first model contains two submodels. One submodel find the optimal fleet size, but no rotation plan, by finding the minimum cost flow in a network flow model. In the other submodel an optimal rotation plan corresponding to the optimal fleet size and taking maintenance requirements into consideration is found by solving an assignment problem.
The second model is an assignment model the solution of which gives us the optimal fleet size as well as a corresponding rotation plan.
Some extensions to the basic models are given.
The two models have been applied to the scheduling of the DC-9 fleet (about 60 aircraft) of the Scandinavian Airlines System. The two network flow models are solved by a simplex based tree-indexing method which appears to be very efficient against such large scale network flow problems. Some results from the case study are presented including among other things aircraft saved, computional experience and applicability of the models.

91

Interactive Models for Aircraft Load Planning

O. Holst, O. Kessel, O. Larsen, G. Mikkelsen, IMSOR, The Technical University of Denmark, Lyngby, Denmark

The paper deals with the problem of how to generate a loadplan for a container and pallet carrying aircraft. A load plan indicates exactly where each of the containers and the pallets shall be placed.
The containers and the pallets, or units with a common name, cannot be placed at random due to the fact that some positions are allowed only for certain types of units. Furthermore, some restrictions concerning balance, combined load etc. must be met.
However, quite a lot of different load plans will normally meet the weight - and position constraints. This allows an objective function to be introduced and the obvious one is to minimize the number of units reallocated at the intermediate airports. A leg is a link between two successive airports on a route, and in the paper we consider routes with more than one leg thus introducting intermediate airports where units are to be unloaded and loaded. The problem can be presented as a linear integer programming model. However, for real life situation the model is far too big for exact solution.
Therefore, two alternative heuristics for the problem have been developed. Both heuristics apply two stages. In the first stage units are placed one by one in order to minimize the number of units to be reallocated at the intermidiate stops. If the outcome of the first stage is a feasible load plan, we are through. Otherwise, pairwise interchanges of units are done in a second stage in order to generate a feasible load plan. The two heuristics differ as regards which procedure to use for the first stage. The origin of this study was the load planning for the Boeing 747 Combi, which is a combined passenger and cargo

aircraft. The problem was initially presented to us by the Scandinavian Airlines System.
The two heuristics have been developed with reference to this problem. Therefore the case study is presented and some experiences from using the two heuristics are given in the paper, including computational effort.
The two heuristics have been implemented into an interactive programme package. The experiences from combining optimization procedures and human experience and outlook into an interactive modelling system is discussed.

92

The Optimal Sequencing of Aircraft Landings at a Two-Runway Airport

H.N. Psaraftis, M.I.T., Cambridge, Ma., U.S.A.

In this paper we extend a Dynamic Programming algorithm developed for the problem of optimally sequencing aircraft landings at a single-runway airport, for solving the two-runway configuration. As in the single-runway case, the problem is nontrivial because the permissible time between two successive landings depends on the characteristics of the leading and following aircraft and thus is not constant. Furthermore, in the two-runway case, one also has to determine what aircraft goes to each runway. The algorithm we present in this paper is a post-processing of the information created by a run of the single-runway algorithm. It exploits the fact that the airplanes, although substantially numerous, can be classified into a relatively small number of distinct "categories" (heavy jets, medium-size jets, etc.). This classification results in drastic computational savings. Two alternative objectives are examined: The first is to minimize the time at which the last landing takes place, and the second to minimize the sum of "waiting-to-land" times for all the passengers in the system. The problem is assumed "static" i.e. no intermediate arrivals are considered. The computer runs implementing the algorithm exhibit interesting partitioning and sequencing patterns.

93

Efficient Combinations of Optimization Algorithms for Flight Mechanics Problems

J. Shinar, D. Blank, Israel Institute of Technology, Haifa, Israel

The aeronautical community has an increasing requirement for computationally efficient algorithms to solve optimization problems in Flight Mechanics. These problems are characterized by multidimensional nonlinear differential equations, constrained control and state variables, terminal constraints and unspecified final time. Due to this inherent complexity there is no single numerical technique with the required low convergence sensitivity and high convergence rate. Although it is agreed that combining different numerical methods should be advantageous, no systematic study of mixed algorithms has been reported.
This paper presents efficient combinations of numerical optimization techniques summarizing their relative merits. Four basic methods handling constraints without penalty functions are considered:
1. Sequential Gradient-Restoration
2. Quasilinearization
3. Neighboring Extremals
4. Direct Shooting
The first two are modifications of algorithms introduced by Miele, avoiding numerical differentiation and eliminating the anchoring effect of the original formulation. The third one is a generalization of an algorithm due to Bryson and Ho.
The individual and combined methods are tested by solving the optimization of vertical turning maneuvers as an example. Results show that an appropriate combination of algorithms reduces the computing time requirements by more than 50%. For other optimization problems (three-dimensional turns, interception, etc.) similar saving can be achieved.

94

On the Integer Transition Property

A.J. Hoffman, IBM T.J. Watson Research Center, Yorktown Heights, N.Y., U.S.A.

A linear programming problem with a parameter in the objective function is said to possess the integer transition property if, whenever t and t+1 are successive integer values of the parameter for which optimum solutions exist, there is an integer solution optimum for both t and t+1. The first cases of this phenomenon were discovered by Greene and Kleitman, and generalized by Hoffman and Schwartz. New cases are presented arising form lattice polyhedra, and applied to matroid intersections.

95

Characterisations and Properties of Totally Unimodular and Balanced Matrices

D. de Werra, Ecole Polytechnique Federale de Lausanne, Lausanne, Switzerland

A simple characterisation of totally unimodular (t.u.) matrices is given which is derived from a result of A. Hoffman and J.B. Kruskal. This characterisation is in fact an extension of a property discovered by S. Baum and L.E. Trotter. It is expressed in terms of decomposition of an integer vector of a polyhedron into a fixed number of integer vectors lying in a related polyhedron. It is shown how this characterisation can be related to a result of Ghouila-Houri.
These properties of t.u. matrices are then applied to the case of t.u. (0, 1) matrices (unimodular hypergraphs) and interpretations in terms of colorings are given.
Similar characterisations for balanced and perfect matrices are given and interpreted in algebraic as well as in hypergraphical terms.
Finally one discusses the concept of canonical decomposition of an integer vector $\underline{x}$ lying in a polyhedron defined by a t.u. matrix.

96

Totally Unimodular Programs with Multiparametric Right-Hand Sides

C.L. Monma, J.E. Somers, Bell Laboratories, Holmdel, N.J., U.S.A.
I. Adler, University of California, Berkeley, Ca., U.S.A.

It is often desirable to determine the relationship between changes in the right-hand side of a linear program (LP) and the optimal value of the objective function. This motivates the study of a family of LP's with right-hand sides parameterized by a vector d: maximize cx, subject to $Ax \leq b+d$. The goal is to determine for which parameter vectors this LP is feasible, and for each of those vectors, to determine the optimal value of the objective function. This problem is difficult in general; however, it has been solved by Somers and Adler for the maximum-flow problem with parametric arc capacities.
Using Fulkerson's concept of generalized flows, those results are extended in this talk to LP's for which the matrix $\begin{pmatrix} c \\ \hline A \end{pmatrix}$ is totally unimodular. By duality theory our results also apply when the matrix $\left(A \mid b\right)$ is totally unimodular and the coefficient vector of the objective function is parametric, as in the shortest-path problem with parametric arc lengths and the assignment problem with parametric assignment costs.

97

Finite Checkability for Integer Rounding Properties in Combinatorial Programming Problems

S. Baum, Bell Laboratories, Holmdel, N.J., U.S.A.
L.E. Trotter Jr., Universitat Bonn, Bonn, West Germany

Let A be a nonnegative integral matrix with no zero columns. The integer round-up property holds for A if for each nonnegative integral vector w, the solution value to the integer programming problem $\min\{1\cdot y\colon\ yA \geq w,\ y \geq 0,$ $y \text{ integer}\}$ is obtained by rounding up to the nearest integer the solution value to the corresponding linear programming problem $\min\{1\cdot y\colon\ yA \geq w,\ y \geq 0\}$. The integer round-down property is similarly defined for a nonnegative integral matrix B with no zero rows by considering $\max\{1\cdot y\colon$ $yB \leq w,\ y \geq 0,\ y \text{ integer}\}$ and its linear programming correspondent. It is shown that the integer round-up and round-down properties can be checked through a finite process. The method of proof motivates a new and elementary proof of Fulkerson's Pluperfect Graph Theorem.

98

Frequency Functions in Duality

L. Papayanopoulos, Rutgers University, New Brunswick, N.J., U.S.A.

The deterministic distribution of Y = AX gives the frequency of X for each Y. This should be of interest in integer and linear programming where we usually visualize solutions in the space of X. The constraints are explicitly stated in a form such as $Y \leq b$ which defines a simple geometric figure in the space of Y. Since the frequency function is also defined in this space we gain some useful insights by studying it.
We consider the IP problem with bounded variables and its relationship to its continuous counterpart. If T is the set of solutions that satisfy the variable bounds (and integrality), then its convex hull H is the set of solutions of the corresponding LP. One of several observations made is that the polyhedron H encompasses the totality of dual optimal solutions for all possible right-hand-sides. This is the fourth paper in a series concerned with the properties and applications of the deterministic frequency function.

99

Theorems of the Alternative in Mathematical Programming

O.L. Mangasarian, University of Wisconsin, Madison, Wi., U.S.A.

The role of theorems of the alternative in linear and nonlinear programming is well known. The purpose of this talk is to briefly outline this role and to give some recent and new developments involving these theorems. Some of these developments include:
1) Enlarging the scope of theorems of the alternative to include unrestricted nonzero variables.
2) Giving a number of equivalent formulations for each of the alternatives.
3) Application of theorems of the alternative in:
a) Characterizing uniqueness of solution in linear and quadratic programs and in linear and nonlinear complementarity problems.
b) Characterizing bounded solutions of linear programs and linear complementarity problems.
c) Characterization of linear complementarity problems as linear programs.
d) Perturbation of linear inequalities, linear programs and linear complementarity problems.
e) Iterative solution of linear programs.

100

Can a General Purpose NLP Code Find Happiness in a GP World?

J.E. Fattlar, G.V. Reklaitis, K.M. Ragsdell, Purdue University, West Lafayette, In., U.S.A.

Results from a comprehensive comparative study of various algorithms for geometric programming problems is presented. Ten codes were tested including four general purpose NLP codes and six specialized GP codes. A total of fourty-two problems with up to twenty randomly generated starting points per problem was employed in the study. The convex primal formulation proved to be instrinsically easier to solve, and a general purpose GRG code, OPT proved to be the most efficient code tested.

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A Methodology for Testing Mathematical Programming Software

P.D. Domich, K.L. Hoffman, R.H.F. Jackson, P.B. Saunders, D.R. Shier, U.S. National Bureau of Standards, Washington, D.C., U.S.A.

To correct the often conflicting and confusing results of computational testing of mathematical software reported in the literature, the editorial boards of three journals (ORSA, TOMS, and Mathematical Programming) have proposed guidelines which present minimum standards to which all papers reporting such test efforts must conform. Although guidelines now exist, the development of sound methodologies for performing this testing is at the embryonic stage. This paper discusses the results to date of a multi-year effort whose ultimate goal is to develop and demonstrate methodologies for obtaining statistically valid results from thorough computational comparisons of mathematical programming software.

The paper will present the field-testing of this general methodology using four Math Programming Codes. Some of the problems involved in such testing will be addressed. Included will be a discussion of (1) a methodology for controlling timing variability; (2) the relative importance of various quantities which can be used to indicate code performance in terms of both computational effort (e.g. computer run time) and quality of results (e.g., number of accurate digits in the solution vector); (3) the uses and misuses of test problems; (4) the costs of conducting a thorough computational experiment; and (5) the conclusions one can draw from a soundly developed experimental design.

102

Testing Mathematical Programming Algorithms on Randomly Generated Polyhedra

J. Telgen, Erasmus University, Rotterdam, The Netherlands

"Random" polyhedra are used to test and compare algorithms for a wide variety of mathematical programming problems. Frequently these polyhedra are constructed by generating linear inequalities with coefficients drawn from a uniform distribution. It is noted that this class of "random" polyhedra has a special property: the angles between the faces of the polyhedron tend to be the same if the dimension of the space increases. The weird structure of these "random" polyhedra may bias the testresults.

A procedure for generating polyhedra that do not have this property, has been developed.

Finally, some remarks on the comparison of linear programming algorithms, for which results are known on different sets of testproblems will be made.

103

Ranking Mathematical Programming Techniques Using Priority Theory

F.A. Lootsma, University of Technology, Delft, The Netherlands

The paper is concerned with Saaty's priority theory in order to solve the multi-criteria decision problem of selecting a non-linear programming code to be used in a particular environment. First, so-called priorities are assigned to the relevant criteria, and thereafter some well-known comparative studies are used to compare a number of codes under each performance criterion separately. The procedure outlines what performance evaluation can do and what it cannot do. It demonstrates how priority theory can be used for an integrated assessment of codes taking into account both the performance criteria of a decision maker and the results of performance evaluation.

104

Processing Time: An Accurate Measure of Code Performance?

K.L. Hoffman, R.H.F. Jackson, National Bureau of Standards, Washington, D.C., U.S.A.

An important and commonly cited measure of performance when evaluating mathematical programming algorithms is computation time. Studies reporting timing performance often ignore the inherent variability in this measure. Characteristics affecting timing variability include: the machine environment (the number and types of jobs being run during the same time period), the location in core storage of instructions and data, and the inherent variability with the clock and the accounting procedures. This paper will discuss a variety of approaches which can be used to measure, control, and, under certain conditions, eliminate the variability in this performance measure.

105

On the Solution of 3-Dimensional Assignment Problems

R.E. Burkard, Universitat zu Koln, Koln, West Germany

The planar as well as the axial three dimensional assignment problem can be formulated as matroid intersection problem of three partition matroids. For solving this problem two approaches are considered: a transformation method based on admissible transformations, which generalizes the Hungarian method for (2-dimensional) assignment problems. Algebraic objectives can easily be taken into account by this method. The second approach uses subgradient arguments and is especially suited for sum objectives. By a Greedy algorithm a minimal basis with respect to certain transformed costs is computed. If this basis is also a basis for the other two matroids, an optimal solution has been found. Otherwise either the costs are transformed and the Greedy algorithm is applied again, or a branching is made.

Computational tests were performed for bottleneck and sum objectives. In the case of sum objectives the subgradient method yields considerably better results than the transformation method.

106

A Recursive Method for Solving Assignment Problems

G. Thompson, Carnegie-Mellon University, Pittsburgh, Pa., U.S.A.

A new non-simplex algorithm for solving assignment problems in which dual prices vary monotonically is described. The method (which was devised by the author while working on market games) maintains a pareto optimal price structure (which maximizes buyer surpluses) while considering the rows of the cost matrix (the sellers) one by one. The algorithm is shown to be polynomially bounded. It is also shown that the method achieves a theoretical computational lower bound for certain classes of problems. Computational comparisons with various primal and dual methods will be given.

107

A Dual Algorithm for the Assignment Problem

A. Weintraub, F. Barahona, Universidad de Chile, Santiago, Chile

A dual algorithm to solve the assignment problem is presented. It is based on an approach first proposed by Edmonds and Karp for the general transhipment problem.
In the programming of the algorithm, pointers were used to take advantage of the usual low density of the network, and in addition, auxiliary lists were incorporated which reduced considerably CPU time.
The algorithm was tested on an IBM 360-145 machine and compared to the alternating Basis Primal Algorithm (AP-AB) presented by Barr, Glover and Klingman, seemingly the most efficient among the primal and primal dual codes.
The comparison was made by generating problems similar to those, on which the AP-AB code was tested on a CDC 6600 machine, and by considering the approximate relative speeds of the two computers. On the test problems, the presented dual algorithm was about three to four times faster than the AP-AB code.
CPU times for the dual algorithm were quite stable with respect to the form of generating the problems. The number of iterations was well below the square root of the number of nodes and CPU times increased moderately with the size of the network, in particular for low densities.

108

Extremal Problems of Permutation Cycles

M. Hung, A. Waren, W. Rom, Cleveland State University, Cleveland, Oh., U.S.A.

Given a set of n real numbers $A = \{a_i \mid i = 1,\dots,n\}$ and a permutation I of the integers from 1 to n, define the function $f(A,I) = \frac{1}{2}\sum_{j=1}^{n} (a_{i(j)} - a_{i(j+1)})^2$. We determine the permutations Imax and Imin which maximize and minimize this function. Next, given the integer k, we find the subsets B(k)max and B(k)min which maximize and minimize f(B(k), Imax) and f(B(k), Imin) respectively, over all subsets b(k) of A with cardinality k. Then we determine the values for k, $2 \le k \le n$ which maximize and minimize the values of f(B(k)max, Imax) and f(B(k)min, Imin) respectively. These results are then applied to a special case of the assignment problem to determine the most and least improving adjacent extreme points of the worst extreme point (in terms of objective function value). This is then used to identify the "greedy path" on which every successive point is the most improving one. It is shown that the greedy path, for these special assignment problems, is of length n/2, where n is the number of rows (and columns) of the assignment cost matrix.

109

Some Results on Fixed Point Algorithmic Theory

H. Tuy, Institute of Mathematics, Hanoi, Vietnam

A new restart method for computing fixed points is presented, together with computational experiences and comparisons with other existing methods. The algorithm is applied to the solution of inclusion equations of form $0 \in F(x)$, $x \in \Omega$, under very general hypotheses about the set Ω and the multivalued mapping F. Apart from a new algorithmic proof for known fixed point theorems in finite-dimensional and infinite-dimensional spaces, some new fixed point propositions are presented.

110

On Solving Large Structured Fixed Point Problems

R. Saigal, Northwestern University, Evanston, Il., U.S.A.

Fixed point problems in several applications are found to have special structure, and also tend to be very large. This talk will report the recent progress made in solving many such problems. There are essentially three special structures, namely differentiability, separability and sparcity, that are exploitable by these algorithms. Each of these structures and its importance on the solvability of the problem will be discussed, as will computational experience of solving problems up to 150 variables will be reported.

111

A Fixed Point Algorithm for Arbitrary Complexes

W. Forster, The University, Southampton, England

A fixed point algorithm will be presented with the following properties.
Theorem: Given a continuous map f from an arbitrary complex K into itself at least one of the following is true
(i) there exists at least one compact path in K,
(ii) there exists at least one fixed point in K.
The proof is based on a novel lemma by Sperner (1978) and

utilizes among other techniques the Lemke and Howson convergence argument.
This theorem vastly extends the capabilities of pivoting algorithms. Immediate applications are e.g. fixed point computation in nonconvex regions, fixed points in graphs, etc.

112

An Enumeration Algorithm for a General LCP

K. Kaneko, W.P. Hallman, University of Wisconsin, Madison, Wi., U.S.A.

An implicit enumeration algorithm is developed for solving a general linear complementarity problem (LCP). This algorithm is based on an ordinary branch-and-exclude principle, using certain "sign configuration" fathoming/branching criteria particular to LCPs. Results of extensive computational experiments using this algorithm will be presented. Also presented are empirical results on comparing the computational performance of this algorithm with that of some alternative solution methods. The results we obtained indicated an overwhelming superiority of our algorithm over the alternatives we considered. Included among the alternatives was a standard branch-and bound algorithm for a mixed integer program.

113

Computational Efficiency of Fixed Point Algorithms

R.M. Anderson, Princeton University, Princeton, N.J., U.S.A.

We study the computation time of the algorithms introduced by Scarf to compute fixed points of continuous functions. Such algorithms proceed by subdividing the k-dimensional simplex into n^k subsimplices, where the sides of the subsimplices are of order 1/n. The algorithm labels the vertices of this subdivision, then moves from one subsimplex to another by a pivot process. It terminates with a completely labelled subsimplex, which will be an approximate fixed point.
Theoretically, the algorithm could take up to n^k pivots. Computational experience shows, however, that the number of pivots is $0(n)$ as $n \to \infty$. We provide a theoretical basis for this observed speed by showing that there is a labelling rule and an open dense family of C^2 functions on the simplex such that the number of pivots is $0(n)$. We also show that there is a one-parameter family of labelling rules such that for each C^2 function f, the set of labelling rules for which the algorithm fails to terminate in $0(n)$ pivots corresponds to a set of Lebesgue measure 0 in the parameter space.

114

Global Optimization Techniques: The State of the Art

G.P. Szego, Milano, Italy

The aim of this paper is to present a survey of the results obtained in the area of (unconstrained) global optimization. A comparison of the deterministic and the stochastic methods will be developed,a series of numerical results on a set of test problems will be presented.

115

Computing the Global Maximum of a Convex Function with Linear Constraints

J.B. Rosen, University of Minnesota, Minneapolis, Mn., U.S.A.

An algorithm will be described for finding a global maximum of a convex function subject to linear constraints. Some computational experience with this algorithm will be presented, as well as possible modification to take advantage of parallel processing.

116

Techniques for Global Optimization

L. De Biase, University of Milan, Milan, Italy
A.H.G. Rinnooy Kan, Erasmus University, Rotterdam, The Netherlands

In global optimization the objective is to find the global optimum of a function rather than a local one. For this purpose many deterministic methods have been developed (e.g., grid search and trajectory methods), but we shall concentrate on stochastic methods involving a combination of repeated sampling, grouping of sampled points into clusters and local searches. We shall discuss various refinements of this approach with special attention for a proper termination criterion, and suggest several extensions, e.g. to nonlinear 0-1 programming. Some successful applications will also be reported.

117

A Statistical Approach to Solving Concave Minimization Problems With Linear Constraints

N.R. Patel, Indian Institute of Management, Vastrapur, India
R.L. Smith, University of Pittsburgh, Pittsburgh, Pa., U.S.A.

We prove that the distribution of the minimum of a sequence of objective function values randomly generated from any concave minimization problem is asymptotically Weibull with shape parameter equal to the dimension of the feasible region. Confidence intervals are constructed for the unknown optimum to assist in the use of heuristic searches.

118

Convergent Algorithm for Minimizing a Concave Function

H. Tuy, N. Van Thoai, Institute of Mathematics, Hanoi, Vietnam

For the problem of minimizing a concave function over a polytope a class of convergent and efficient algorithms is proposed, which is based upon a combination of the branch and bound technique with the cutting method developed in an earlier paper of H. Tuy.

119

Obtaining k Best Solutions for Combinatorial Optimization Problems

T. Ibaraki, H. Mine, Kyoto University, Kyoto, Japan
N. Katoh, Osaka Center for Adult Diseases, Osaka, Japan

K best solutions of an optimization problem may be generally obtained by repeating the following procedure for $k=0,1,\dots,K-1$: Given k best solutions $x^1,x^2,\dots,x^k$ and a partition $(S_1,S_2,\dots,S_n)$ of the set of the rest of feasible solutions, find the (k+1)st best solution x^{k+1} as the best one among the best solutions of S_j's. Then partition $S_{j_0} - \{x^{k+1}\}$ into finer subsets, where $x^{k+1} \in S_{j_0}$. In order to have efficient algorithms by this approach, however, the data structure of sets S_j's and their manipulation (in addition to, of course, finding a clever way of partition) must be carefully designed. This point is discussed by actually developing three new algorithms; an $O(Kn^2)$ algorithm for K shortest paths in an undirected graph (n denotes the number of nodes), an $O(\min(n^2, m\log\log n) + Km)$ algorithm for K minimum spanning trees in an undirected graph (n denotes the number of nodes and m the number of edges), and an $O(T+K\sqrt{n \log n})$ algorithm for K best solutions of the resource allocation problem (T denotes the time required to obtain the first best solution and n denotes the number of integer variables). These algorithms are faster than the previously known ones.

120

Synthetical Analysis of Enumerative Algorithm

Y. Sekiguchi, Hokkaido University, Sapporo, Japan

Many algorithms proposed in rapidly increasing papers on various combinatorial optimization problems seem to form a jungle, mainly because of a lack of systematic views of those problems and their relations to adequate algorithms. How to overcome this jungle is the main concern here. A framework of enumerative algorithms, including most integer programming, branch-and-bound, dynamic programming, backtrack programming, heuristic programming and many other algorithms, is proposed. It is named tree programming and consists of five constituents; selection rule, branching rule, upper bounding function, elimination rule and terminating condition. Then, worth of each constituent is defined in relation to one of three algorithm-efficiency measures; amount of computation, memory space requirement and the speed of improving the best solution known so far. The conditions of finiteness and correctness of tree programming are deduced. Effects on the algorithm-efficiency of changing a constituent to a better one are discussed extensively. The results show how we can improve an algorithm in terms of each measure. On the base of this analysis, the author proposes an 'algorithm base' approach to overcome the initially mentioned difficulty. An algorithm base is an algorithm retrieval system. It tells how one can construct an effective algorithm for a certain problem in question.

121

Probabilistic Integer Programming

C. Ritz, Wharton Applied Research Center, Newton, Ma., U.S.A.

This paper presents and resolves some of the difficulties involved in the interpretation and solution of probabilistic integer optimization problems. From among the several ways of resolving the feasibility dilemna, the notion of recourse, correction of infeasibility at a penalty, is selected and defended as the most reasonable in a decision context.
The methodology of equivalent deterministic programs is expanded and applied, and several original theoretical results lead to a key formulation, a convex objective function to be minimized over a binary set, subject to a reduced set of linear constraints.
The program is solved through tangential approximation of the objective function and the solution of a set of binary linear programs. Successive solutions are shown to provide a monotonic increasing lower bound for the nonlinear minimization. After evaluating an extraordinarly small proportion of the feasible solutions, an optimum solution to the surrogate problem, and thus the probabilistic integer program, is determined.
Successful implementation through a FORTRAN computer routine demonstrates the computational efficiency of the tangential approximation technique. Testing of the algorithm over a broad range of problem sizes indicates that there should be only modest computational effort involved, even in problems of moderate to large size.

122

Admissible Transformations for Solving Combinatorial Optimization Problems with Generalized Objectives

U. Derigs, Universitat zu Koln, Koln, West Germany

For special combinatorial optimization problems different kind of objective functions as for instance the classical sum objective or the bottleneck objective are of practical interest. Mostly these objective functions are treated seperately and independently. The introduction of an "algebraic objective function" the cost coefficients of which are now elements of an ordered commutative semigroup allows a unified treatment. For standard problems like shortest path, matching and matroid intersection problems this approach has led to efficient algorithms for a broad class of objective functions.
One specific type of algorithm which came out is based on transformations of the cost coefficients and a purely combinatorial motivated optimality criterion. We will introduce this principle of "admissible transformations" for

the general combinatorial optimization problem with algebraic objective. Then we demonstrate how this general principle combined with some problem specific combinatorial observations leads to efficient and transparent algorithms for special combinatorial optimization problems.

123

On Application of Analytical Methods to Combinatorial Problems

A. Berstein, P. Buzytsky, G. Freiman, Academy of Sciences of the USSR, Moscow, USSR

The problems of combinatorial optimization are tightly connected with the question of solvability of boolean equations and inequalities. Until now analysis of the question was conducted by means of some elementary considerations.
The presented work suggests the usage of analytical methods of number theory and probability theory, which enables to get asymptotical formulae for the number of solutions of equation systems with boolean variables.
The solvability analysis of an equation system with boolean variables is being reduced to a check-up of some natural conditions, the check-up being realized effectively for broad classes of combinatorial problems.
The authors believe that the application field of the analytical methods will be large.

124

Determining Whether a System of Nonlinear Inequality Constraints Has a Feasible Point

R. Schnabel, University of Colorado, Boulder, Co., U.S.A.

A new technique is presented for determining computationally whether a general set of linear and nonlinear inequality constraints contains a feasible point ("Problem I"). The technique is based on an algorithm for determining whether a set of linear constraints and one nonlinear inequality constraint is feasible ("Problem II"). This is done essentially using a linearly constrained optimization routine, but adding features which make the routine try to find a global minimum. These seem quite effective in practice.
It is then possible to solve the main problem I by solving a parameterized set of problems of form II, in which a nonnegative parameter p is increased at each stage. One obtains theoretical results analagous to those for Lagrange multiplier methods (for constrained optimization) stating that there exists some finite q such that for any p greater than q, the solution to Problem II either is a feasible point to Problem I, or establishes that Problem I is infeasible. Thus the method is attractive computationally, and computational experience will be reported.

125

A Newton-like Method for General Nonlinear Programming Via Slack Variables

E. Spedicato, Universita di Bergamo, Bergamo, Italy

While use of slack variables to transform inequalities into equalities is well known, little attention has been paid to its effective use in the general nonlinear programming situation. We consider a Newton-like approach to solving the Kuhn-Tucker conditions for inequality constraints via slack variables. We show that a full use of the structure of the problem avoids any increase in the dimensionality; moreover the resulting linear equations are positive definite under mild assumptions. Numerical experiments are presented with comparison to the usual strategies of active set selection.

126

The Exponential Potentials Method

F.C. Incertis, A.M. Vazquez-Muniz, IBM Scientific Center, Madrid, Spain

This paper presents a new interior point method for nonlinear constrained optimization. Given, for i=1,2,...,n, a convex set of functional inequalities $g_i(\bar{x}) \le 0$, the exponential potential function is defined as $\Psi(k, \bar{x}) = \Sigma \exp k\, g_i(\bar{x})$ for all real and positive values of k. By means of the Max-Min theory a fundamental theorem is proved and a lower bound of the parameter k for which the $\mathrm{Min}_{\bar{x}}\, \Psi(k, \bar{x})$ is an interior point is obtained. Being $J(\bar{x})$ the jacobian matrix of the bounds, $\Phi(k, \bar{x})$ = Diag.Matrix $\{\exp kg_i(\bar{x})\}$ and $\bar{\Phi}(k, \bar{x})$ = Vect. $\{\exp kg_i(\bar{x})\}$ the algorithms for the unconstrained minimization of $\Psi(k, \bar{x})$ uses the iteration formula $\left(J^T(\bar{x})\, \Phi(k,\bar{x})\, J(\bar{x})\right) \Delta x = - 1/k J^T(\bar{x})\, (k,\bar{x})$, which does not require hessian matrix computations. At any point the active set of bounds is easily determined from the values of the exponential potentials $\{\exp kg_i(\bar{x})\}$ and a further simplification in the computations is possible. When implementing the method the objective function creates its own exponential potential like any other bound. At every stage the iteration formula is used to find a feasible solution and then the objective function is moved to this point. The algorithm to estimate adequate values of k is also described and computational results on test problems are presented.
The applicability of the method to solve some non-convex problems is discussed at the end of the work.

127

Elimination of Bounds in Optimization Problems by Transforming Variables

F.S. Sisser, Queens College, Flushing, N.Y., U.S.A.

The problem of minimizing a nonlinear objective function of n variables, with continuous first and second partial derivatives, subject to nonnegativity constraints or upper and lower bounds on the variables is studied. The advisability of solving such a constrained optimization problem by making a suitable transformation of its variables in

order to change the problem into one of unconstrained minimization is considered. A set of conditions which guarantees that every local minimum of the new unconstrained problem also satisfies the first-order necessary (Kuhn-Tucker) conditions for a local minimum of the original constrained problem is developed. It is shown that there are certain conditions under which the transformed objective function will maintain the convexity of the original objective function, at least in a neighborhood of the solution. A modification of the method of transformations which moves away from extraneous stationary points is introduced and conditions under which the method generates a sequence of points which converges to the solution at a superlinear rate are given.
Computational results on the effectiveness of the method are presented.

128

A Nonlinear Factorable Programming Language

G.P. McCormick, George Washington University, Washington, D.C., U.S.A.

A language has been under development for providing the interface between computer coded nonlinear programming algorithms and real world optimization models. This language provides the capability of automatic differentiation of the problem functions with respect to their variables and their parameters. The current and future status of the language and associated theoretical developments (nonconvex programming, large scale optimization, sensitivity analysis) will be discussed.

129

A Reduced LU Algorithm for the Simplex Method

S. Powell, London School of Economics, London, England

A number of methods have been proposed to represent and update at each simplex iteration the inverse basis of a linear programming problem as the product of two factors: such methods are often referred to as LU factorisation algorithms. The proposed method is also an LU factorisation but it exploits the position of the slack variables so as to reduce the number of elements needed to represent the inverse basis. A description of the theoretical basis of the method and some computational experience on some small and medium sized problems will be given.

130

Exploiting Structure and Sparsity in LP Computational Procedures: An Overview of Recent Advances and Established Methods

G. Mitra, Brunel University, Oxbridge, England

All through 1970's advances in computational procedures for solving LP problems have been regularly reported. Many of these advances exploit structure and sparsity of the constraint matrix. During reinversion the basis matrix is reordered to obtain a sparse elimination form of inverse (EFI). During a simplex iteration the LU factors of the EFI are updated in a way that inhibits the growth of the nonzeros. Three different data structures are related to these computational schemes. These comprise sparse or super sparse storage of the constraint matrix, the data structure used in the (graph theoretic) algorithms for reordering the basis matrix in the maximal block triangular form and the representation of the transformation matrices. The established algorithms concerning these problems will be reviewed and their implication when taken together with methods which exploit Generalized Upper Bound (GUB) or other strucutre will be considered. Methods which dynamically partition the basis matrix and allow addition of constraints will be discussed.

131

Structure Analysis and Partial Reinversion of Large Bases in Linear and Nonlinear Programming

J. Bisschop, A. Meeraus, World Bank, Washington, D.C., U.S.A.

The paper shows how one can take advantage of the nested spike structure in each irreducible block of a basis during the process of building an inverse representation. The structural effects of subsequent rank-1 modifications of the basis are examined and several partial reinversion schemes are proposed.

132

The Use of Sparsity by the Non-Uniqueness in Large-Scale LP Problems

I. Foltyn, Reseach Institute of Agricultural Economics and Foods, Prague, CSSR

When we solve a large-scale LP-Problem

$$\max \{c^T x : Ax = b,\ A = (\bar{A}, I),\ x \geq 0,\ b \geq 0\} \qquad (1)$$

where $c,\ x \in R^{m+n}$, $b \in R^m$, $\bar{A}$ is a (mxn)-sparse matrix, I is the (mxm)-identity matrix and $m \geq 1$, $n \geq 1$ are great integers, by the primal simplex method with the inverse basis matrix A_B^{-1} in form PFI or EFI, we frequently meet the case that the choice of incoming variable is non-unique. It is caused by the fact that the set

$$S = \{j : z_j < 0,\ j = 1, \ldots, n,\ z^T = c_B^T A_B^{-1} A_N - c_N^T\}$$

has in the majority of iterations more than one element and, especially, for large-scale problems is usually large. In this case it is necessary to give some additional criteria for the unique choice of incoming variable x_{Ns}, $s \in S$. The most-known criterion in this sense is partial pricing, where we choose x_{Ns} so that following relation $|z_s x_{Ns}| = \max \{|z_j x_{Nj}| : j \in S_1 \subset S\}$ holds. This criterion tends, in general, to the minimal number of iterations, but it does not respect the sparsity of the solved problem (1). In the contribution we propose some "sparse" criteria depending on the sparsity of matrix A of the problem (1). According to these criteria the incoming variable $x_{Ns}, s \in S$ is chosen so that the fill-in of the corresponding vector a_{Ns} by the transformation $\alpha = A_B^{-1} a_{Ns}$ would be as small as possible. The expected fill-in of sparse vectors a_{Nj} for $j \in S$ is computed as an inner-product of some "characteristic vectors" of the chain of elementary matrices (PFI or EFI) corresponding to A_B^{-1} and the Boolean representation of these vectors. It is proved that in certain cases no fill-in for some vectors will occur.
The use of these in the contribution proposed "sparse" criteria can lead to maintaining of the desired sparsity

of the used representation of the inverse matrix A_B^{-1} in the course of the solving of the problem (1) on the computer.

133

Investigation of Algorithms Used in the Restructuring of Linear Programming Basis Matrices Prior to Inversion

K. Darby-Dowman, Polytechnic of Central London, London, England
G. Mitra, Brunel University, Oxbridge, England

Restructuring of the basis matrix to create a sparse representation of the inverse is now established as an important algorithmic step in the solution of large scale LP problems. The most successful methods permute the basis matrix into block triangular form and Hellerman and Rarick were the first to propose practical algorithms. Since then the theory of block triangularisation has been studied extensively. The restructuring can be achieved by first permuting the column (or rows) of the basis matrix to produce a zero-free diagonal and then finding the strong components of the graph associated with the adjacency matrix of the permuted basis matrix. Algorithms for the first step (finding a maximum matching) have been proposed by Hall and by Hopcroft and Karp and for finding strong components by Tarjan. In practice a basis matrix can be permuted to produce the lower triangular part and the block triangularisation algorithms need only be applied to the remaining 'bump'. In this paper, the implementation of a complete restructuring algorithm and the data structures used are described. A set of real-life LP problems are used to conduct experimentation on the algorithms to investigate their behaviour with respect to problem dimension.

134

A Master Problem for Mixed Integer Programming

E.L. Johnson, IBM T.J. Watson Research Center, Yorktown Heights, N.Y., U.S.A.

A master problem is developed in order to show that every defining ineouality, other than non-negativity, of any mixed integer program is derived from a subadditive function having directional derivatives at the origin.

135

Large-Scale Mixed Integer Programming: Benders-Type Heuristics

G. Cote, Hydro-Quebec, Montreal, Canada
M.A. Laughton, University of London, London, England

In the pure integer programming field, the limited success of exact methods has prompted the development of heuristic procedures which have performed surprisingly well in their search for near-optimal solutions. The generalisation of these procedures to the mixed integer case would seem a natural extension. The Benders Partitioning method (BPM) offers an elegant approach to this problem. The efficiency of the BPM has traditionally been limited by the necessity of solving a sequence of mixed integer Master problems (with a single continuous variable). It is shown how the flexibility of the BPM can be exploited in a number of ways to transform this Master into a pure integer problem. In this manner pure integer heuristics can be used provided some additional adjustments are made; Lagrangean relaxation concepts can be called on to improve their performance. Description of these points is accompanied by an illustration of such a procedure applied to a large-scale power system planning problem. Suggestions are made to imbed these heuristics into exact solution techniques so as to strengthen the bounds on the solution.

136

Multi-Stage Benders' Decomposition Applied to Multi-Period, Multi-Commodity, Production, Distribution and Inventory System

K. Tone, Saitama University, Urawa, Japan

It becomes more and more important for some industries to have an efficient program of their activities in the long range. Such a program usually means the production, distribution and inventory plan of multi-commodity over multi-period range. The network flow model is a standard way to represent the problem. Recent advances in the computational aspect of the generalized network gives us an indication of broader areas of application. But the real world imposes upon us complicated constraints which can not be represented in the network models and even in generalized network models. In a previous paper, the author tried a decomposition of network type constraints and non-network type constraints (called pattern constraints) by using Benders' partitioning procedure. The computational experiences show that the decomposition technique works well. In this paper, the author developes a method to handle the multi-period problem, where the problems in each period and coupled with the succeeding one by the existence of the inventory activities. Our system is doubly decomposable, both by the existence of the pattern constraints and by inventory activities. The algorithm consists of two parts, one for solving the network flow problem in each period and the other for solving the pattern and coupling constraints which may be called a master problem. Finite convergence is guaranteed.

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Analysis of Composite Strategies for Relaxed MILP Problems A Case Study: Discriminant Functions Optimization

A.M. Vazquez-Muniz, IBM Scientific Center, Madrid, Spain

The computation of discriminant functions using mathematical programming when the main objective is the minimization of the number of badly classified elements, produces a mixed integer linear problem (MILP) with a number of binary (integer 0-1 variables), equal to the number of points contained in the training samples. Also this problem belongs to a very frequent class of models in which the binary variables are linked to continuous variables by implicative relations of general type: $a_j x_j \leq M_j \delta_j$ in which M_j is an estimated upper bound of x_j.
Due to the natural difficulty in the estimation of M and the weak links between the integer variables and the rest of the model, these models are very relaxed. This part produces a wide feasible region comprised between the continuous and integer optima, in which the branch and bound methods perform very badly due to the high number

of nodes to be computed to guarantee the absolute integer optimum.
The earlier work of Gomory, Johnson, Balas, Young, Ibaraki and others in cutting plane theory is revised in an attempt to reduce the feasible regions to be explored in the mixed integer phase, analyzing the behaviour and capacity of each one in this context. A composite strategy of different cutting planes and branch and bound methods is proposed to solve this class of relaxed MILP problems in a reasonable computing time.
Computational experience is reported in relaxed MILP problems of different type in comparison with strict branch and bound methods, and also some test with normal MILP problems in order to assess the value of the method, and the extension of its use before branching to B & B methods.

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A Composite Integer Linear Programming Algorithm

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Many commercial codes for general ILP and MILP are based on a branch-and-bound algorithm. Their performance can be improved by the use of cutting plane algorithm incorporated into the branch and bound scheme. Appropriate new techniques of ILP and MILP, to be applied in a computer code, are presented in the paper.
A new variant of the dual simplex method, efficient in the case of dual degeneracy, is presented. The method uses the observation that at every node of the enumeration tree (or after adding a new cut) the simplex tableau has only one infeasible basic variable.
Since the branch and bound procedure is plagued by the taxation of the computer memory, to improve the performance of the composite algorithm the cuts of integer forms for ILP and MILP with bounded variables were developed. In order to improve the efficiency of the cutting plane methods several empirical measures of the strength of the cuts, including the greatest reduction the value of the objective function, are compared.
The use of the ordinary Gomory fractional cuts has the basic disadvantage that it gives rise to severe machine round-off errors. To avoid this difficulty the simplex tableau is stored in the form of the all-integer numerator tableau and the determinant of the basic matrix separately. Since the resulting integer numbers may easily exceed the computer capacity, the numbers are stored in the floating point representation and calculations are performed in the floating point arithmetic. This technique is computationally effective for numerically ill-conditioned problems.
Finally, the numerical experiences with the algorithm and comparison with the other algorithms are presented.

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Optimal Planning of Telecommunications Networks

A. Pigott, London, England

This paper considers the problem of optimising the planned growth of a telecommunications transmission network. The problem may be summarised as minimising the total cost of equipment, subject to various constraints such as demand satisfaction, technical feasibility and network security. A mathematical programming formulation results in an integer linear (multicommodity flow) problem of great size. The scale of the problem prevents its solution by any exact method, even using auxiliary column generation procedures. However, the structure of the problem allows extensive decomposition, which, coupled with efficient heuristics to solve the sub-problems, promises to give acceptably good results.
The development of a practical system to carry out optimisation of these problems should be of great assistance in the fields of operational and strategic planning within the UKPO.

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Heuristic Algorithm for the Optimum Communication Spanning Tree Problem

P.M. Camerini, L. Fratta, F. Maffioli, Politecnico di Milano, Milan, Italy

In this paper an algorithm is presented for finding an approximate solution to the Optimum Communication Spanning Tree (OCST) problem, which appears to be a realistic enough model for a general problem often to be found in practice, i.e. that of designing a minimum cost network connecting a set of given centers, given a specified matrix A of traffic requirements to be satisfied. In general the cost of a network is given by the sum of a fixed cost and a variable (i.e. traffic dependent) cost. Special cases of this general problem which are known to be solvable in polynomial time are the minimum spanning tree, the optimum requirement spanning tree, the shortest paths tree and the flow network synthesis problems.
The OCST problem may be stated as follows. Given an undirected graph G of n nodes let two costs c_{ij}, t_{ij}, be associated to each link (i,j) of G, representing respectively the fixed cost and the cost per unit of traffic of link (i,j). Let the traffic requirement between any pair of nodes i,j be of the form $a_{ij} = b_i b_j$.
We want to build a spanning tree connecting all the nodes of G such that its cost $C = \Sigma(c_{ij} + f_{ij}\ t_{ij})$ is minimum, where the sum is taken over all links (i,j) of the spanning tree and f_{ij} represents the total traffic on link (i,j), that is the sum of all the required traffics a_{hk}'s using link (i,j).
The method proposed in this paper comprehends essentially two phases. The first phase provides a feasible solution, which is then improved by an iterative technique in the second phase. In order to find a feasible solution, two different approaches have been tested: a random approach and a greedy approach.
The second phase of the method iteratively exchanges a link of the tree with a link not in the tree, whenever this reduces the total cost. Termination occurs when all possible exchanges have been tested with negative result, thus guaranteeing the local optimality of the solution.
In both the above mentioned phases, an efficient updating and computation of the cost function are crucial for speeding up the method: a careful implementation allows the overall complexity of the algorithm to be $O(n^3)$. An extensive testing has been performed on a UNIVAC 1108. Examples have been both generated at random (in euclidean and non-euclidean metric) and taken from real world problems.

141

Mathematical Programming Models for Planning a Trans-Atlantic Communications Network

G.A. Kochman, C.J. McCallum, Bell Laboratories, Holmdel, N.J., U.S.A.

Given a projection of future circuit requirements between the United States and various European countries, the issue here is planning for the economic growth of a communications network to satisfy these requirements. Both satellite and submarine cable circuits may be used; each cable installed has a fixed, finite circuit capacity, while satellite circuits may be leased as needed. The objective is to find an optimal placement of cable (types, routes, and timing) and the routing of individual circuits between demand points (over both satellites and cables) such that the total discounted cost over a T-period horizon is minimized. This problem can be modeled as a zero-one mixed integer program. Specifically, the model takes the form of a multiperiod, capacitated facility location problem. The development, solution, and use of such a model is the subject of this paper. The evolution of the model with its various constraints is traced through its several stages of development. Solution approaches are outlined and compared by means of computational experience. Use of the model both in planning the growth of the network and in the economic evaluation of different cable technologies is discussed. Finally, extensions of the model are mentioned.

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Development of a Computerized Frequency Assignment System to Avoid Intermodulation Interferences

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The design of a computerized, frequency assignment system, that is, a procedure to assign frequencies from a given set of "available" frequencies to a known set of communication links in such a way that certain separation constraints are satisfied and also that there exist no potential threat of intermodulation interference of a given type, is discussed. The order of a given intermodulation product, which affects the power of the interfering signal, is obtained by solving a single-constraint integer program with variables unrestricted in sign, and thus the system requires the repetitive solution of a series of a specially-structured integer program.

The proposed system, designed for use by our Pacific Naval fleet, is based on efficient enumeration schemes and has two basic components. These are the detection of a largest set of intermodulation-free frequencies among available frequencies, and the identification of possible interactions of communication links considered. A unique feature of the system is the fact that it adopts a link-by-link frequency assignment strategy, as opposed to a ship-by-ship strategy. This is made possible by "inverting" the graph of communication requirements, where a node corresponds to a communication site and an arc to a link, into a graph where a node corresponds to a link, and an arc to the possibility of interactions. The graph inversion routine and its ramifications are discussed. Computational experience using the entire system is presented. Future efforts and their relative importance are described.

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The Administration of Standard Length Telephone Cable Reels

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When planning urban telephone maintenance and expansion, one has to assign reels of cable to supply for all lengths of cable needed by the network.

In the ideal world, lengths needed, and the timing of their use behave exactly as planned. The best solution for buying and using cable reels would then be: match cables exactly to the lengths demanded, and buy them accordingly; use them as needed.

In the real world, events may arise which turn desirable to buy cables in standard lengths. The administration of such a case is no longer simple: it involves the assignment of reels from which to cut the necessary lengths, in such a way as to minimize losses.

There are some model formulations to represent the problem of supplying lengths needed of some material from standard lengths. Unfortunately, most part of these models are posed from the material producer's point of view. That means: as the factories do not keep significant finished materials inventory, preferring to produce when needed for supplying, those models do not consider carrying cost.

In this paper, it is presented a model of the problem that considers both carrying and surplus costs. Some heuristics are proposed to give approximate solutions, and their performance are evaluated.

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Newton-like Methods for Nonlinear Multicommodity Network Flow Problems

D.P. Bertsekas, M.I.T., Cambridge, Ma., U.S.A.

This lecture is devoted to numerical solution of convex multicommodity flow problems often arising in optimal routing of data in communication networks and equilibrium studies of transportation networks. We will describe a class of recently developed algorithms based on ideas of Gallagher's method for distributed optimization of delay in data communication networks and gradient projection ideas from nonlinear programming. An important common feature of the algorithms which distinguishes them from other existing methods is that they utilize second derivatives and are geared towards approximation of a constrained version of Newton's method. Analysis and computational results confirm that the algorithms are convergent, and tend to employ good search directions. When used without a line search they tend to automatically generate a satisfactory stepsize regardless of the level and pattern of traffic input to the network. This latter advantage is of crucial importance for distributed routing of flow in data communication networks where the use of line search is nearly impossible.

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Large-Scale Nonlinear Network Optimization

R.S. Dembo, Yale University, New Haven, Ct., U.S.A.

Many important engineering and economic problems involving the computation of equilibrium flows on a network may be formulated as nonlinear programming models with linear (network flow) constraints. Typically these models are very large, involving anywhere from 1000 constraints and 1500 variables (in water distribution models) to 200,000 constraints and 500,000 variables (in traffic assignment models). To date, a number of NLP algorithms based-on linear subproblems have been implemented and tested on such models. They all rely heavily on the special structure inherent in linear network flow problems and display very poor theoretical and empirical rates of convergence. In this presentation we discuss how Newton-type algorithms may be specialized to this problem in a way that exploits the underlying network structure and capitalizes on the efficient data structures that make the primal simplex method so efficient in the linear case. Since these methods may be unduly expensive to implement, we outline various approaches to approximating a Newton search direction. Some encouraging preliminary computational experience is also presented.

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Coupled Multi-Commodity Flows in Generalized Networks

A. Girard, INRS - Telecommunications, Verdun, Canada

The problem discussed in this talk is of finding the minimum cost multi-commodity flow in a generalized network, where the arc multipliers are linear functions of the total flow carried on the arc.
The more simple case where multipliers affect each commodity independently of the others has a block-diagonal structure with extra capacity constraints similar to the classical M.C. flow problem and is easily amenable to standard techniques such as Dantzig-Wolf decomposition.
In order to exhibit a similar structure for the coupled problem, new flow variables are introduced to bring the system in a block-diagonal form, with extra rows and columns. A first level Dantzig-Wolf decomposition is then applied to the system, thereby eliminating the capacity constraints. The sub-problem has a block-diagonal constraint matrix, with additional columns (coupling variables). Although a second-stage Dantzig-Wolf decomposition on the dual would seem to be required, the special structure of the constraint matrix is employed to show the following results.
The dual of the sub-problem is written, with a block-diagonal matrix and coupling constraints. It is then shown that the structure of the blocks is such that they can be reduced to bounds on the (otherwise unconstrained in sign) dual variables. A change of variables then reduces the sub-problem to one where only the coupling constraints remain, with non-negative variables. Finally, it is shown that this sub-problem can be separated into independent linear programs, one for each node of the network, and of size (K x L) where K = number of arcs entering the node and L = number of commodities.
In conclusion, the structure of the master program is briefly reviewed and possible computational approaches are discussed.

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Lagrangean Relaxation and Decomposition for Optimum Multicommodity Network Synthesis with Security Constraints

M. Minoux, J.Y. Serrault, Centre National d'Etudes des Telecommunications, Issy les Moulineaux, France

This paper is concerned with the solution of the following large scale optimization problem:
given a N-node, M-arc connected graph $G=(X,U)$ determine a M vector $Y=(y_u)_{u \in U}$ of capacities associated with the arcs, meeting any one of P given (independent) multicommodity requirements, and minimizing a linear cost function $\sum_u \gamma u.Yu$. This problem, which has important applications in Telecommunication network optimization and planning, may be viewed as a generalization of a single-commodity problem treated by GOMORY and HU (1962).
As an alternative to linear programming techniques, it is shown how the use of Lagrangean relaxation for decomposition purposes, and the use of subgradient algorithms to optimize the dual problem, lead to a practical and efficient solution procedure. Though convergence is not finite, lower bounds (obtained from dual solutions) and upper bounds (via computation of primal solutions) can be determined at each step, thus providing a stop criterion. Computational results obtained show that solutions within 5 to 10% of the optimum are easily obtained at low computational cost. This is especially interesting in view of the fact that even moderate sized problems (e-g . 20 nodes 40 arcs, and P = 40) lead to very large scale linear programs (32 000 variables and 16 000 constraints in node-arc formulation) for which even the most sophisticated linear programming techniques could not provide exact solutions.

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A Bender's Decomposition Algorithm for the Combined Distribution and Assignment Problem

K. Jornsten, Linkoping Institute of Technology, Linkoping, Sweden

The system optimized combined distribution and assignment problem is solved by the adaption of generalized Bender's partitioning. The subproblems thus created are ordinary traffic assignment problems, that is multicommodity flow problems and the master problems are modified distribution problems. By modified distribution problems we mean distribution problems with a mini-max objective. The master problems are then solved by dualization resulting in a nonlinear programming problem with linear constraints.

149

Solving Dynamic Linear Programs

G.B. Dantzig, University of Stanford, Stanford, Ca., U.S.A.

Solving dynamic linear programs efficiently has remained an outstanding problem for over 30 years. Ideally one would like to solve a T-period problem in the time it would take to solve T one-period problems. The dynamic Leontief Model with substitution and certain Markov chain problems can be so solved. For the general dynamic case, a number of approaches have appeared in the literature with result than an interesting mathematical theory is now beginning to emerge. Approaches vary from compact inverse schemes for representing the inverse of the basis (using the simplex method), to imbedded decomposition and continuous time formulations in place of discrete time approximations. Suffice it to say that important applications are waiting for the development of efficient software. Some of the proposed algorithms are currently undergoing systematic trials and show promise that they will be used in place of existing software (developed for solving general linear programs).

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On Some Experiments which Delimit the Utility of Nonlinear Programming Methods for Engineering Design

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A comprehensive comparative study of nonlinear programming algorithms as applied to engineering design is presented. Linear approximation methods, interior penalty function methods and exterior penalty function methods were tested on a set of thirty problems and were rated on their ability to solve problems within a reasonable amount of computational time. The effect of the problem parameters on the solution time for the various classifications of algorithms was studied. The variable parameters included the number of design variables, the number of inequality constraints, the number of equality constraints and the degree of nonlinearity of the objective function and constraints. Also a combined penalty function and linear approximation algorithm was investigated.

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Performance Indicators for Evaluating Math Programming Software

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An important aspect in testing math programming software is the identification of properties on which the software is to be evaluated and the determination of appropriate measures with which to quantify these properties. This paper presents the results of a survey of the math programming community to identify performance indicators used in evaluating math programming algorithms and software. Examples illustrate that different performance indicators can lead to conflicting conclusions.

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A Numerical Comparison of Optimization Programs Using Randomly Generated Test Problems

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We intend to test and compare optimization software for solving constrained nonlinear programming problems of the kind

$$\min\ f(x)$$
$$g_j(x) = 0\ ,\ j=1,\dots,m_e$$
$$g_j(x) \geq 0\ ,\ j=m_e+1,\dots,m$$
$$x_l \leq x \leq x_u\ ,\ x \in R^m$$

with continuously differentiable functions f and $g_1,\dots,g_m$. Up to date (Jan. 1979), 28 qualified and widely distributed NLP programs are submitted for the comparative study.
In contrast to all earlier studies, the test problems are randomly generated allowing to predetermine an isolated local minimizer and the corresponding objective function value, furthermore the dimension, number of active constraints, Lagrange multipliers, Hessian of the Lagrangian at the given minimizer, and so on. Using this generator, 160 different test problems are computed. There are dense and sparse problems, problems with equality constraints only, with inequality constraints only, furthermore degenerate, ill-conditioned, and indefinite test problems. Since each problem has to be solved from different starting points, an optimization code under consideration has to pass 320 test runs.
The construction of artificial test problems with predetermined solutions allows to evaluate accuracy, efficiency, global convergence, and reliability. Especially it is possible to relate the efficiency of a program (execution time, number of function and gradient evaluations) to the reached accuracy.
Numerical tests obtained by (at least) 15 optimization programs will be presented. They show how the test problem generator is exploited to compare optimization software.

153

Piecewise Dynamic Programs with Applications

K. Sawaki, Nanzan University, Nagoya, Japan

This paper considers a special class of general dynamic programs which satisfies the monotonicity and contraction assumption, and in which piecewise properties hold among value functions and policies. This class of dynamic programs includes affine and quadratic dynamic programming, partially observable Markov decision processes, many sequential decision processes under uncertainty and etc. A generalized policy improvement will be presented. The algorithm has the property that it only involves distinguished subset of policies and value functions which are easily able to be represented in a computer.

154

Teaching Planning Decisions in Mine Production by Dynamic Programming

J. Elbrond, Ecole Polytechnique, Montreal, Canada

The optimal sequence of decisions on rates of production and cutoff grades is obtained by a dynamic programming model. The main parameters are: 1) the size of the deposit, average grades and grade distributions, 2) the operating costs of mining and concentration, 3) the investments in mine and mill, 4) the smelter charge, 5) the price(s) of the metal, 6) the recovery and the grade of the concentrate, 7) the rate of interest, 8) the taxation formulae.
In this application of dynamic programming the selection process starts with the first period but is limited to a specified (small) number of periods for which price and cost information is given. For each of these periods the decisions on rate and cutoff, which are optimal so far, are printed. A decision to mine at a certain rate and consequently at a certain cutoff is then made for the first period. Price and cost information is given for an additional period and the process is repeated from the state resulting from the decision on the first period, etc.
When all ore is mined a comparison is made between the result obtained by the step-wise decision making with limited knowledge about future costs and prices and the result obtained by dynamic programming using the knowledge about costs and prices during all periods.

155

Forecast-Accelerated Algorithms for Markov Decision Processes

L. Contreras, Southern Methodist University, Dallas, Tx., U.S.A.

Value convergence consumes the major portion of computation time in all mixed iterative procedures for the solution of Markov Decision Processes. Based on forecasts using two and three successive values of V(i), the expected discounted value of staring the process in state i, two new "acceleration" algorithms are presented herein. The proposed methods are compared with previously reported techniques and are shown to be both theoretically and computationally superior. Experimental results and implementation strategies are reported.

156

Lower Bounds to the Set Covering Problem from Graph Covering Relaxations

N. Christofides, A. Hey, Imperial College, London, England

The set covering problem (SCP) is the integer linear program, min $(CX|AX \geq 1, X\epsilon\{0,1\})$ where A is a 0-1 matrix. In general it cannot be solved in polynomial time, but in the case of no more than two 1's in each column of A it is a graph covering problem which can be solved by a polynomially bounded algorithm.
Two different Lagrangean relaxations of the SCP which give graph covering problems are compared. The initial lower bounds to the SCP are improved using subgradient optimization. The bounds are then improved further by "rotating" the relaxation.
The advantages of these methods are that they are particularly suitable for large sparse problems, and the bounds produced are not dominated by the bound obtained from the linear programming relaxation of the SCP.
The bounds are then used in a tree search procedure where the search is guided by information available from the relaxations.

157

Bounds for List Schedules on Uniform Processors

Y. Cho, S. Sahni, University of Minnesota, Minneapolis, Mn., U.S.A.

Bounds are derived for the worst case performance of list schedules relative to minimum finish time schedules for uniform processor systems. The tasks to be scheduled are assumed to be indepedent and only nonpreemptive schedules are considered.

158

Bounds on the Reliability Polynomial for Shellable Independence Systems

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The reliability polynomial associated with an independence system is $R(p) = \Sigma_{k=0}^{m} f_k p^k (1-p)^{m-k}$ where f_k is the number of independent sets of cardinality k and m is the cardinality of the ground set. An independence system (T,Γ), is shellable if all maximal independent sets have the same cardinality and if there exists an ordered partition of the set of independent sets, into intervals, $\{<F_i, G_i>\}_{i=1}^{n}$ (an interval $<F, G> = \{F': F \subseteq F' \subseteq G\}$) where for all n', $n' \leq n$, $G_{n'}$ is a maximal independent set and $(T, \cup_{i=1}^{n'} <F_i, G_i>)$ is an independence system. For the class of shellable independence systems, we give tight upper and lower bounds on the f_i, when the number of maximal independent sets and the number of minimum cardinality dependent sets are fixed. In addition, we apply these bounds to two network reliability analysis problems, namely, bounding the probability that a stochastic undirected network contains a spanning tree and the probability that a stochastic directed network contains a spanning arborescence.

159

On Certain Properties of the System of Linear Extremal Equations

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The author deals with some theoretical aspects of optimization in extremal vector spaces (this term was introduced by K. Zimmermann, related concepts see N.N. Vorobyev).
In his previous works the author has introduced some concepts (extremal convex set, its extreme points, assumption of anti-degeneracy precaution) using the analogy with linear programming. Extreme points of an extremal convex set are described similarly as in an obvious linear case. This characterization is used in order to introduce the independence of vectors and the dimension of the extremal vector space with respect to the system of linear extremal equations.
This paper concerns some properties of a certain system of linear extremal equations useful for getting a better notion about extreme points of an extremal convex set. A sufficient condition is given for a solution of the system to be unique.

160

Optimal Algorithms of Definitions of Pareto Optimal Set

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Assume A is an ordered set of vectors $A_r = (a_{r1}, a_{r2}, \ldots, a_{rm})$; A_i denotes i^{th} member of A; $|A|$ denotes number of elements of A; $\|A\|$ denotes a number of components of vectors A; A_{is} denotes the s^{th} component of A_i. Set $\bar{A} \subsetneq A$ is Pareto Optimal Set (POS) for A, if for all A_i

there does not exist A_j for which $a_{is} \geq a_{js}$ for all $s=1,2,\ldots,|A|$. By the words "set A is vocabulary-graphic ordered by j^{th} components" we understand that for all A_r, $A_s \in A$ and $A_r \neq A_s$, if $r < s$, then are $a_{rj} > A_{sj}$ or $a_{rj} = a_{sj}$ and $a_{ri} = a_{si}$ $(i=1,2,\ldots,t-1)$ and $a_{rt} > a_{st}$ $(t=1,2,\ldots,|A|)$; and we shall write $a_r > {}^jA_s$; ${}^jA$ denotes set A which is vocabulary-graphic ordered by j^{th} component.

Theorem 1: Assume $\|A\| = 2,3$. Then one can define ${}^1\bar{A}$ in $C|A|\lg|A|$ operations.

Theorem 2: Assume $\|A\| = 2$. Then one can define ${}^1\bar{A}$ in $C|A|\lg|A|$ operations without operational field.

161

Computation of Hyperbolic Arctangents and Hyperbolic Tangents

R.S. Mullins, London, England

This paper develops by the method of real analysis, as distinct from the technique of coordinate rotation, digit-by-digit algorithms for the direct binary computation of arctanh b/a and tanh q . The algorithms are then modified by an optimised damped oscillation method which reduces the number of computations required. These processes will be useful for hardware and software realisation in microprocessors and other digital computer systems.

162

A Survey of Models for Equitable Distributions

W.F. Lucas, Cornell University, Ithaca, N.Y., U.S.A.

There are a great number of situations in operations research and in other societal affairs in which an equitable allocation or fair division is deemed desirable. Such problems may involve a multiplicity of objectives, attributes, potential coalitions, or decisionmakers with different utility functions. Several distinct mathematical models for addressing such situations do exist. A review of several of these methods along with some recent developments will be presented. This will include the recent axiomatic approaches to the apportionment problem, fair division schemes for both divisible and indivisible objects, solution concepts relating to fairness from the multiperson game theory, and approaches to the problem of multiple objectives. These models frequently make use of concepts from linear, nonlinear and integer programming, as well as utility theory and game theory. Several of these approaches to equity have been proposed for and implemented in real-world applications. Many other problem areas are ripe for employing such analytical techniques.

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Political Elections in Europe and Shareholders' Strategies in Order to Control a Company, Studied Using the Shapley Value of a Game

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We shall describe an analytical study on the Shapley value of a game, which will allow the construction of a simplex of the n-dimentional Euclidean space, the barycentre of which corresponds to this value. We shall then present an algorithm for the automatic computing of this formula based on the aforementioned study and on a "stop-calculation" theorem. We shall evaluate the results of some recent political elections in Europe by using this algorithm. After that we shall show that the pattern of behaviour after political elections is analogous to that of shareholders' who wish to control a Company, and that the model with mathematical programming can be used to determine the different strategies.

164

An Asymmetrical Min-max Theorem for Games Against Nature

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We describe an asymmetrical min-max theorem useful for the game against nature viewpoint of optimization under uncertainty. Sufficient conditions are provided for extension of well-posed minimization problems to the min-max criterion. The introduction of mixed strategies only for nature creates a differentiable minimization problem equivalent to the min-max problem posed. Applications include extensions of deterministic optimal control results to feedback control of uncertain systems. Algorithmic implementation will also be discussed.

165

Redundancy in Systems of Linear Constraints

J. Telgen, Erasmus University, Rotterdam, The Netherlands

A system of linear (equality and/or inequality) constraints determines a convex polyhedral set of feasible solutions S. In practice it is often desirable to represent S with a system containing as few as possible constraints: such a system is defined to be a minimal representation of the set S.
We prove that a minimal representation exists for any convex polyhedral set S and that the number of equalities and inequalities in the minimal representation is uniquely determined.
Furthermore we prove a theorem stating that a minimal representation is obtained if the system contains no implicit equalities (inequalities of which the slack is zero in every feasible solution) and redundant constraints. New algorithms to identify implicit equalities and redundant constraints are given. By removing these constraints a minimal representation will be obtained according to the theorem above. Some empirical results of these algorithms are presented.
Existing theory on redundancy and known methods to reduce

the size of the system are shown to be special cases of the theory and methods introduced here.

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Improvements to a Vertex Generating Algorithm

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In a previous paper, we described a basis oriented pivoting algorithm for determining all the vertices of a convex polytope. Here we present a new version of the algorithm incorporating improvements both of a computational and a theoretical nature.
The algorithm constructs a spanning tree of the feasible basis graph using the pivoting mechanisms of the product form revised simplex method and an adjacency test for directly comparing bases which is necessary to prevent redundancy and ensure termination. Our experience with this and other pivoting methods suggests that the main computational burden lies in the adjacency test rather than in pivoting. We show how this burden may be reduced by introducing a new labelling of the search tree and a mechanism for pruning it of redundant branches. It is shown that the work involved in this adjacency test is, at worst, $0(\text{no. of vertices}^2)$.
Degeneracy presents a problem for all pivoting methods and therefore we also identify a suitable perturbation scheme for use with the above algorithm.
Finally we demonstrate how the performance of the algorithm, both in respect of execution time and storage, can be further improved by a device similar to that suggested by Mattheiss.

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Calculating the Frame of Homogeneous Equation Systems

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Consider the set of all nonnegative solutions to the set of simultaneous homogeneous equations $Ax = 0$, i.e., the cone $C = \{x \geqq 0 \mid Ax = 0\}$. C is a convex set, being the intersection of the null space of A with the nonnegative orthant. C appears above in what Rockafeller [1972] terms external representation. Although this representation allows an easy test whether a given point lies in C, it is not possible to give explicit expressions of points in C. To obtain the latter it is necessary to convert to the internal representation of C, which is dual to the external one and expresses C as the convex hull of its extreme rays, whose collection is the <u>frame</u> of C.
Let the matrix A be m by n, of rank d. It is clear that the desired frame members will be one-dimensional intersections of at least n-1 (n-1) -dimensional hyperplanes; there are, however, n+d hyperplanes available: d from Ax $Ax = 0$ and n more orthant boundaries $x_i = 0$, $i = 1,2,\ldots,n$. Furthermore the frame members must lie in the nonnegative orthant. Here, phase I of the Simplex method is employed repeatedly to solve the above problem; first a bounding constraint $ex = 1$ is introduced ($e = (1, 1, \ldots, 1)$) that converts the cone C into the polytope
$P = \{x \geqq 0 \mid Ax = 0, ex = 1\}$. Then the calculation of max ex, subject to $Ax = 0$, $ex = 1$, $x \geqq 0$ is initiated $\binom{n}{n-d-1}$ times, where the combinations represent all possible ways of selecting n-d-1 coordinate hyperplanes $x_i = 0$ out of n.

The only adaptation of the Simplex routine is a device, that <u>prevents</u> those columns of A appearing in a particular combination from entering any basis (one clearly starts with artificial variables) even if eligible according to their relative cost coefficients. A given combination is abandoned as soon as the set of allowed pivot columns becomes empty. A feasible (and here also optimal) solution will thus be reached only for that subset of combinations which correspond to vertices of P.
If more than n-d-1 coordinate hyperplanes meet at any vertex of P, several combinations will yield this same vertex (which will have more than n-d-1 zero coordinates). A simple recursive screening device weeds out duplicating combinations before any Simplex calculations are started.
The paper proves that the above modifications will yield the exact set of frame vectors for the cone C; it also gives an APL- code for the method and an interesting application in theoritical chemistry [Clarke 1976, 1979].

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What we Know About Q-Matrices

R.W. Cottle, Stanford University, Stanford, Ca., U.S.A.

A real square matrix M with the property that <u>every linear complementarity problem</u> (q,M):
$q + Mz \geq 0,\ z \geq 0,\ z^T(q + Mz) = 0$
<u>has a solution</u> is called a Q-matrix. Identifying this class of matrices has busied a number of investigators over the last decade or more. This talk will survey what is known on the subject and present some new contributions to it.

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The Linear Complementarity Problem

R.D. Doverspike, C.E. Lemke, Rensselaer Polytechnic Institute, Troy, N.Y., U.S.A.

The linear complementarity problem (LCP),
$w = Mz + q$, $w,z \geq 0$, $w^tz = 0$, is examined with respect to the classes of matrices Q_o (also called K) and subclass Q. M is in Q_o if every q for which there is a feasible solution has a complementary solution; M is in Q if there is a complementary solution for all q. Classes of matrices are introduced which establish necessary conditions for Q_o.
These conditions are labelling requirements on the faces of the positive cone Pos[I,-M] and are completely determined by the signs of the minors of M. It is shown for the case where the convex polyhedron $\{z: Mz+q \geq 0,\ z \geq 0\}$ is bounded and with certain nondegeneracy assumptions on M that these conditions are also sufficient.

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A Version of Lemke's Algorithm Using Only Elementary Principal Pivots

S.J. Byrne, R.W.H. Sargent, Imperial College, London, England

One of the authors recently presented two implementations of Lemke's algorithm for the linear complementarity problem, based on the use of orthogonal factorizations. The first algorithm was based on elementary principal pivots, but was applicable to only a very restricted class of problems. It was mentioned in passing that this could be generalized to cover Eaves' class of L-matrices, but the possibility was dismissed in favour of a "hybrid" algorithm.

In the present paper we present a new implementation of the elementary principal pivoting algorithm, covering the same class of matrices as the original Lemke algorithm. Again this algorithm deals with the generalized form of the linear complementarity problem with upper and lower bounds on the primary variables, and it will be shown that it has features making it specially convenient for solving a sequence of problems with varying "q-vectors", as required in the recursive quadratic programming approach to nonlinear programming.

Both this algorithm and the earlier hybrid algorithm have been programmed, and numerical results on some test examples are given comparing these algorithms with the implementation of the original Lemke algorithm.

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Dual Approaches in Integer Programming

J. Krarup, University of Copenhagen, Copenhagen, Denmark
S. Walukiewicz, Systems Research Institute of the Polish Academy of Sciences, Warsaw, Poland

Recent dual approaches in integer programming are briefly reviewed with emphasis on their applicability. We proceed with a general integer programming problem and its dual, where the latter is viewed as the maximization of a Lagrangean subject to an aggregation of the original primal constraints.A branch-and-bound algorithm is devised for solving both problems and it appears that the duality gap vanishes after a finite number of iterations. An estimation of the dual variables (aggregation coefficients) is given and the influence of the choice of a starting dual solution is discussed. We consider furthermore two approaches to reformulating a given problem such that the resulting formulation is more suitable for dual methods. The first approach provides a tighter equivalent formulation of the given problem by rotating of a given integer constraint without adding or eliminating any of its feasible solutions. In the second approach, the number of constraints is slightly increased by adding "strong" cuts which in certain cases bridge the duality gap. The examples of demonstration include models for capital budgeting and facility location.

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Subgradient Approach and Integer Programming

J-P. Dussault, J.A. Ferland, Universite de Montreal, Montreal, Canada

The subgradient approach is used to solve some integer programming problems. The efficiency of the approach increases when the problems structure can be used to specify an improving direction and a step size. One of these problems has a structure without this property, and the approach does not seem to be appropriate for it. When other techniques of solution are available for these problems, comparative results are reported.

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Combining Subgradients for Lagrangean Relaxation

A.M. Hey, Imperial College, London, England

Lagrangean relaxation has been widely used to provide lower bounds to a wide variety of combinatorial programming problems. Subgradient optimization can then be used to improve the bound. Although the common approach of using a single subgradient increases the bound substantially for the first few iterations, there is a difficulty in that with subsequent iterations the bound increases much less rapidly. Similar effects have been observed in non-linear programming when using a piecewise differentiable penalty function. In this case improvements were made using projection methods.

This paper shows how subgradients can be combined to enable a higher bound more quickly. This involves considering alternative solutions of the Lagrangean function. In addition an estimate of the step length can be given. This combines both theory of nonlinear programming and integer programming. Examples are given to illustrate the method.

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Solving Equations Using Orthogonal Polynomials

P.C. Cooley, Research Triangle Institute, Research Triangle Park, N.C., U.S.A.

The subject of this paper is the solution of the set of linear equations defined by $Ax = b$, using polynomial iterative methods of the form $r_i = P_i(A)b$. Here $P_i(t)$ is an orthogonal polynomial in t of degree i and r_i is the i^{th} residual vector defined by $r_i = b - Ax_i$.

The basis of the method is a property of orthogonal polynomials. This property allows successive residual vectors to be generated conveniently using a three term recurrence relation. It is shown how to estimate the parameters in this recurrence formula for a given polynomial and a number of specific polynomials are investigated, including the Conjugate Gradient Polynomial. It is shown that successive iterates of all of the selected polynomials optimize a given criterion, but that the criterion optimized by the Conjugate Gradient Method appears to be the most useful from a practical viewpoint.

A general algorithm is developed that requires two polyno-

mial specific parameters to be defined. These sets of parameters are derived for a Chebychev, Laguere as well as the Conjugate Gradient Polynomial. Limited numerical experimentation is discussed and extensions to the nonlinear case are illustrated.

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A Nonlinear-Equation-Solver for Unconstrained Optimization

H. Ohiwa, Toyohashi University of Technology, Toyohashi, Japan

In many engineering design problems, not a single but several objective functions must be optimized. These problems can be formulated as a nonlinear simultaneous equations $\underline{f}^* = \underline{f}(\underline{x})$, where $\underline{f}^*$ is the design target vector and $\underline{f}(\underline{x})$ is the objective functions. The designer can choose appropriate $\underline{f}^*$ intuitively during the optimization process. In this formulation, the number of unknowns (design parameters) is often larger than that of equations, because objective functions are better controlled by the larger number of design parameters.
From the engineering optimization point of view, we shall discuss a new nonlinear-equation-solver which adopts a kind of continuation method and finds iteratively the minimum norm correction to the design parameters. The notion of nonlinearity weights is introduced in connection with the nonlinearity of each equation and with deciding whether or not an obtained correction is promising. The imbedded parameters which are introduced in connection with the continuation method are also controlled by the nonlinearity. This method is very effective for such cases as Rosenbrock's parabolic valley in which linear and nonlinear equations are included in the system.

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Caracterisation Geometrique d'une Meilleure Approximation a l'Aide d'un Couple de Fonctions

C. Carasso, Universite de Saint-Etienne, Saint-Etienne, France

Soit f une application bornée définie sur un compact K. On considère deux espaces de dimension V_1 et V_2 engendrés par des fonctions bornées sur K. On se propose de caractériser un couple $(\overline{p}_1, \overline{p}_2) \varepsilon \overline{V}_1 \times \overline{V}_2$ tel que:
$\underline{\alpha} = \mathrm{Sup}(\mathrm{Min}\{|p_1(s) - f(s)|, |\overline{p}_2(s) - f(s)|\}: \ s \ \varepsilon \ K)$ avec
$\overline{\alpha} = \inf(\mathrm{Sup}(\mathrm{Min}\{|p_1(s) - f(s)|, |p_2(s) - f(s)|\}: \ s \ \varepsilon \ K): p_i \ \varepsilon \ V_i)$
Exemples:
1. Si V_1 et V_2 sont formés des fonctions constantes, alors p_1 et p_2 sont les deux constantes qui s'écartent le moins de la fonction f sur K. Ce type d'approximation est notament très utile en reconnaissance des formes.
2. Si V_1 et V_2 sont formés des polynômes du 1er degré sur $K = [-1,+1]$ et si $f(s) = |s|$ il est facile de vérifier que $\alpha = o$ et que $p_1(s) = -s$ et $p_2(s) = s$. L'approximation directe de f par un seul élément de V_1 nous donnerait un écart entre f et V_1 de 1/2.

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An Algorithm for Linear Programs with an Additional Reverse Convex Constraint

R.J. Hillestad, RAND Corporation, Santa Monica, Ca., U.S.A.
S.E. Jacobsen, University of California, Los Angeles, Ca., U.S.A.

A constraint $g(x) \geq 0$ is said to be a reverse convex constraint if the function g is continuous and strictly quasiconvex. The feasible regions for linear programs with an additional reverse convex constraint are generally nonconvex and disconnected. It is shown that the convex hull of the feasible region is a convex polytope and, as a result, there is an optimal solution on an edge of the polytope defined by only the linear constraints. The only possible edges which can contain such an optimal solution are characterized in relation to the best feasible vertex of the polytope defined by only the linear constraints. This characterization then provides a finite algorithm for finding a globally optimal solution.

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A Probabilistic Approach to the Solution of Global Optimization Problems

B. Betro, Universita di Milano, Milan, Italy

An increasing number of algorithms, both deterministic and stochastic, have been developed in recent years for solving optimization problems with a multiextremal objective function, the so called global optimization problems.
Deterministic algorithms are very expensive in terms of function evaluations and require a priori knowledge about the function (typically the Lipschitz constant).
Stochastic algorithms, usually based on random sampling and statistical inference, perform far better; they are, however, still lacking of a satisfactory theoretical framework.
In this paper a probabilistic approach is considered, in which a stochastic model for the objective function is assumed and algorithms are designed in order to maximize the probability that the approximation to the global optimum is kept within a prefixed bound.

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A Hybrid Algorithm to Find the Global Minimum

N. Baba, IRIA, Rocquencourt, France

There have been contrived various computational algorithms in order to find the minimum of a performance function f(x). However, most of the algorithms contrived so far can not necessarily ensure convergence to the global minimum when f(x) is a multimodal performance function.
By the way, if we use the random optimization method proposed by Matyas, we can ensure convergence to the global minimum of f(x) without regard to the location of the initial point. But, the speed of convergence of this random optimization method is not so good.
Therefore, the problem of constructing algorithms that ensure convergence to the global minimum without regard to the location of the initial point and give fast rate of convergence remains to be settled.
In this paper, we propose a new combined algorithm of the

random optimization method and the Variable Metric Algorithm. It is theoretically proved that this hybrid algorithm ensures convergence to the global minimum of f(x) without regard to the location of the initial point. Further, it is shown by various numerical examples that our hybrid algorithm converges quickly to the global minimum of multimodal performance function.

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Pivotal Exchange Methods: Theory, Computation and Applications

H.W. Kuhn, Princeton University, Princeton, N.J., U.S.A.

Pivotal exchange algorithms have been central to the development of mathematical programming. As a family they range from the Simplex Method for linear programming to extensions that apply to complementarity problems, the approximation of fixed points, and the solution of systems of nonlinear equations. The purpose of these lectures is to introduce these extensions to a wider audience. They will survey the underlying combinatorial theory, computational implementation and experience, and the more important areas of application. Each of these topics will be illustrated by representative examples. A guide to the current literature will be provided; the references below will give potential auditors an indication of the subject matter to be covered.

Pivoting and Extensions, Math. Prog. Study 1, ed. M.L. Balinski, (North-Holland, Amsterdam, 1974).
Michael J. Todd The Computation of Fixed Points and Applications, (Springer, Berlin, 1976).
Complementarity and Fixed Point Problems, Math. Prog. Study 7, eds. M.L. Balinski and R.W. Cottle, (North-Holland, Amsterdam, 1978).

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Lectures on Multiple Criteria Decision Making

S. Zionts, State University of New York, Buffalo, N.Y., U.S.A.

The purpose of these lectures is to present an introduction to the methodology and problem solving capabilities of multiple criteria decision making. I begin with a simplified framework of management decision making upon which most (but not all) of the methods are based. Then I present a typology of the methodologies and indicate the assumptions made for each. I then present a number of methodologies in some detail. There are so many different methods that I will only present a selected subset. The ones I plan to present are prototypical of the different approaches. I also plan to discuss applications of some of the methods.

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Theoretical Approaches to the Evaluation of Heuristic Combinatorial Algorithms

D.S. Johnson, Bell Laboratories, Holmdel, N.J., U.S.A.
R.M. Karp, University of California, Berkeley, Ca., U.S.A.

Every known algorithm for the exact solution of the traveling-salesman problem or the graph coloring problem experiences an exponential growth in running time as the problem size increases. This course examines the theoretical evidence that these and other classical combinatorial optimization problems are inherently complex, and then develops a theoretical framework for showing that fast heuristic algorithms can often be relied on to give good approximate solutions to these problems. The first lecture concerns reducibility techniques for showing that many problems are NP-hard, and hence not likely to be solved in polynomial time. In the second lecture we analyze certain fast heuristic algorithms, and show that they are guaranteed to give nearly optimal solutions. In the third lecture we show that certain heuristic algorithms almost surely give near-optimal solutions when their input data is drawn from a suitable probability distribution. Particular problems considered include the traveling-salesman problem, the minimum coloring and maximum clique problems in graphs, and various knapsack and bin packing problems.

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A Survey of NLP Software and Applications

L. Lasdon, University of Texas, Austin, Tx., U.S.A.

This talk will summarize two survey papers which will appear soon in Operations Research. After listing some desirable features of NLP software, we describe some available, all-Fortran NLP codes for several classes of algorithms in terms of these features. Several program libraries containing NLP software are discussed, and results of comparative computational exepriments will be summarized. Application areas discussed are: (1) Oil and Chemical Industry, (2) Nonlinear Networks and (3) Economic Models. Area (2) includes applications in the electric power industry and urban traffic assignment models. Area (3) includes optimal control of econometric models, as well as other planning applications. Brief descriptions of problem situations, NLP models, solution techniques, and results will be given.

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MPSIII Quality Assurance

J. Creegan, KETRON Inc., Arlington, Va., U.S.A.

The testing of a system of programs as complex as a mathematical programming system is ultimately a cooperative effort between the vendor and the users.
Each new release of MPSIII is subjected to a battery of tests developed by Ketron and by several clients. An advanced release is then tested at three client sites where

it encounters different operating environments. Any problems encountered are corrected before the system is distributed.

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Goals and Testing for Burroughs MP Software

D.M. Carstens, Burroughs Corporation, Radnor, Pa., U.S.A.

Results of a study to determine the best goals for Burroughs MP Software are discussed together with methods used for testing against those goals.

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Testing and Validation of IBM's Mathematical Programming Software

M. Benichou, J.M. Gauthier, G. Ribiere, IBM France, Paris, France

The following considerations address 'commerical' MP software, that is to say, MP codes used worldwide by a large number of users. Considerations can be divided into two classes:
1. Those which are applicable to the development of any Program Product:
- Very careful definition of external functional specifications, implying consulting with various types of users.
- Separation of the development into distinct phases with phase reviews and inspections.
- Use of structured programming techniques.
- Definition of a 'test plan' for extensive testing of functions in various environments.
- Highlighting documentation problems: comprehensive, easy to understand, up-to-date publications.
2. Those which address problems more specific to MP software.
a. Testing:
- Extensive tests on a large set of real-life models, covering different classes of problems (size, type, density, application areas).
- Extensive tests with various MP environments (region size, tolerances, alternate optimization strategies).
b. Validation:
- In order to be a valid 'product tool,' an MP code should have at least the three following qualities: speed, ease of use, reliability. For instance, in MPSX/370, reliability was a decisive factor in the design: automatic scaling, dynamic tolerances, and, more generally, the ability of the system to automatically adapt to difficult numeric events, according to current model characteristics.
- Validation of the code is also accomplished through the experience gained from a large number of users who continually report errors and troubles. Product maintenance should consist in both fast error correction and improvements of the product, according to users' needs and requirements.

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Validation of OMNI and PDS

C.A. Haverly, Haverly Systems, Inc., Denville, N.J., U.S.A.

Techniques and experiences in validating these large software systems will be discussed.

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Testing Procedures in Sperry Univac for its 1100 Series Mathematical Programming Product, FMPS

E.H. McCall, Sperry Univac, St. Paul, Mn., U.S.A.

Sperry Univac's 1100 Series Mathematical Programming (MP) Product, FMPS, is a large scale MP code containing LP, MIP, SEP, GUB, and some special nonlinear algorithms, has a large scale report writer/matrix generator, GAMMA 3.4, is used to solve MP problems that range from easy to hard and from small to large, is used in a wide variety of machine configurations, environments, and user applications, and was developed over a number of years by several people at three worldwide locations and in two companies. This paper will describe the testing and management procedures used in Sperry Univac for the development and release of a single FMPS product. Some unusual experiences in the testing of that product will also be described.

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On Some Optimization Problems Arising in Experimental Design

F. Archetti, Universita di Milano, Milan, Italy

The use of mathematical programming techniques in problems of statistical estimation has been gaining increasing importance in the last years. In this paper the author considers some optimization problems arising in the area of the optimal design of experiments for the estimation of parameters of non linear systems. The optimality criteria used in the practice of statistical estimation can be shown to result in multiextremal problems for which global minimum seeking methods have to be employed.
The analysis of the sensitivity of the solution to perturbations in the data and parameters of the problem has been found of great importance for evaluating the "robustness" of the estimates.
Also the particular case when few experiments can be performed, of utmost importance in environmental problems, has been considered by the author: a suitable "quality index" of the design has been derived for whose optimization a random search method is proposed.
Finally some numerical results are discussed about the application of this method to the optimal allocation of a network for seismic observation: the advantages of this approach can be stressed in terms of greater accuracy and reliability of the estimate of the earthquake location.

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Development and Testing of a Completely User-Oriented Fitting Procedure Based on a Non-Linear Least Squares Method

A. Colosimo, Universita di Roma, Rome, Italy
A. Polzonetti, Universita di Camerino, Camerino, Italy

The problem of matching experimental data and mathematical models is both a central one in scientific research and impossible to be solved exactly in the more general case. At least two major difficulties arise in finding approximate solutions for it by computer assisted methods:
a) the rather sophisticated mathematics involved, which makes heavy and cumbersome for the "average" worker in any field of experimental sciences the vast majority of the available computer programs;
b) the scarcity of handy methods useful to compare the efficiency of the different algorithms in solving each specific problem.
Having realized that, we devised a computing procedure whose main characteristics include:
i) use of a non-linear least squares method to find the best parameters for systems of equations in the absence as well in the presence of constraints;
ii) extensive use of the "structural-programming" techniques resulting in a completely user-oriented conversational procedure.
Examples of application to some non linear models extremely popular in physical-chemistry and biosciences, like the sum of exponentials and of bell-shaped curves, are fully discussed and the efficiency of the suggested algorithm analysed as a function of the signal/noise ratio and of the number of experimental points: such quantitative tests are easily applicable to any experimental situation or mathematical model or alternative algorithm.

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Development of SALS System - Statistical Analysis with Least Squares Fitting

T. Nakagawa, University of Tokyo, Tokyo, Japan
Y. Oyanagi, University of Tsukuba, Tsukuba, Japan
H. Ohiwa, Toyohashi University of Technology, Toyohashi, Japan

SALS is a program system for statistical data analysis in the fields of experimental sciences in general. It has been developed by a group of people consisting of chemists, physicists, numerical analysists and statisticians. The group was organized in 1975 and the first version of the system coded in standard Fortran was made open to public in 1978 at the Computer Centre, University of Tokyo. The objective of the development is to include best techniques currently available and the system has the following properties:
1) general-purpose and user-oriented;
2) high precision for linear analysis, and stable and fast convergence for nonlinear fitting;
3) robust estimation techniques including Tukey's biweight method;
4) various statistical computations including Akaike's information criterion;
5) easy-to-use commands for controlling various facilities.
A user of the system is requested to supply a subroutine for computing his model function whose parameters are to be fitted to the observed data. The working area is dynamically allocated according to the numbers of parameters and observables. The program is written on more than fourteen thousands cards including comments and is fully documented in flowchart form.

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Fitting Multifactor Data: A Case Study

T.M. Simundich, University of California, Los Angeles, Ca., U.S.A.

Power Spectral Densities [PSD's] as measured by a satellite sensor depend upon several satellite-earth-sun variables. The utility of extrapolating PSD's as a function of these variables is in saving satellite operation time and procuring more data in the same operation time. This gives rise to the problem of fitting equations to scattered multifactor data. Considerations included in the paper are: (1) the equations to be fitted; (2) the fitting methodology; (3) partitioning of the data for the fit; (4) the effect of the allocation of data in the factor space; (5) effects of errors in the dependent and independent variables on the fit; (6) handling outliers; (7) non-parametric testing of the validity of the fitted equation.

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Recursive Estimation of Time Varying Parameters in Regression Models

J.P. Indjehagopian, Ecole Superieure des Sciences Economiques et Commerciales, Cergy Pontoise, France

In this paper we discuss the recursive estimation of parameters of linear regression models in which the parameters are time varying according to a nonstationary multivariate Autoregressive Integrated Moving Average (ARIMA) models. The procedure uses Kalman filtering techniques. In the first part of this paper, we show how the parameter estimation problem can be brought into the state variable representation of dynamic and stochastic systems, and then we apply the Kalman filtering methodology in the cases where parameters are constant or changing over time. Since the stochastic model for the parameters changing over time is rarely known, we consider in the second part of the paper particular parameters models such as random walk, multivariate ARIMA (1,1,0) and ARIMA (1,1,1). Besides, due to the fact that Kalman filter requires an exact knowledge of the process noise covariance matrix and the measurement noise covariance matrix (Jazwinski 1970, Mehra 1971 and 1972, Young 1972) a macroeconomic example is developed using Mehra's approach to identify covariance matrix of noise for model where parameters are varying over time according to multivariate ARIMA (1,1,0) process.

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Problems in Regional and Global Modelling: Would Math Programmers be Interested?

A.N. Elshafei, The Institute of National Planning, Cairo, Egypt

This paper aims at bringing some problems in the field of Regional and Global Modelling to the attention of Mathematical Programmers. This field, being rather new, is still plagued with difficulties and problems which deserve some attention from them.
With a brief introduction to the field: its history, philosophy, and a quick survey of the literature, we then move to some detailed explanation of the system of models

which was developed for countries in the Middle East. We explain in some detail the Math Prog parts of these models. We also describe the stages of data assessment, model building, data collection, building of various scenarios, computer implementation and data management.
This leads to an exposé of several new concepts which emerged during the study. We classify them according to:
a) methodology (hybrid models vs simulation models, problems of convergence, problems of consistency, invalidity of classical economic tools), b) building of scenarios (multi-dimensional tree-time path analysis), and c) database management.
From this exposé, we conclude by suggesting several new research areas which fit the interest of mathematical programmers and are rather challenging. We demonstrate (by examples from the literature of Long Range Planning and Global Modelling) that these problems are also faced by other workers in the field and remain unsolved. We invite math programmers to work in these areas and hence enhance (and accelerate) the development of this new field.

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Multiobjective Multilevel Programming Models for Regional Planning

P. Nijkamp, P. Rietveld, Vrije Universiteit, Amsterdam, The Netherlands

An integrated planning framework for a system of regions has to take into account inter alia the following elements:
(1) interrelationships (spatial spillovers, e.g.) between regions.
(2) intraregional conflicts among the set of relevant regional objectives.
(3) interregional conflicts due to diverging interests of the regions within the system at hand.
(4) the availability of relevant and appropriate data.
Thus far, multiobjective decision methods concentrated mainly on the element (2) and to a lesser extent on (3). Multilevel planning models mainly dealt with elements (3) and (4).
This paper tries to integrate these two approaches by providing an analytical framework for integrated regional planning for a spatial system. An interesting aspect of this combination of methods is that it may include the possibility of double interactive procedures. In multilevel planning the interaction pertains to the exchange of information between a central planning unit and regional planning authorities. In multiobjective decision methods the interaction takes place between the decision-making committee and the analyst(s) providing scientific assistance in identifying efficient points and in suggesting compromise solutions.
The results will be illustrated by means of a numerical application to a multiregional model with economic and environmental objectives.

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Applications of Mathematical Programming to Economic Policy Making: Some Empirical Results

C-L. Sandblom, Concordia University, Montreal, Canada

Although the disciplines of automatic control and of macroeconometric modelling have been well established, separately, for a number of years, it is only fairly recently (in the past ten years or so) that much interest has been devoted to the optimal use of economic policy, based on econometric models. In this paper we argue, that when real-life economic policy problems are studied, knowing the "true" optimum is of less value than finding near-optimal solutions and the sensitivity of these to changes in the models used, to the weights and targets of the objective functions and even to the choice of the particular economy studied. As illustrations, optimal control policy calculations are presented for the Canadian and US economies (we use a condensed version of the Canadian RDX2 model and the small 1973 US model by Pindyck). We also consider simple linear policy rules, with various lag structures, to find how near-optimal they are. Stochastic as well as deterministic simulation techniques are employed.

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OMER: A Technoeconomic Energy Model for Israel

M. Avriel, Israel Institute of Technology, Haifa, Israel

In recent years there is a growing interest in analyzing the economy in terms of energy. Mathematical modelling of the economy with special emphasis on the energy sector can provide valuable analysis for policy makers at national and international levels. In this paper an integrated approach to energy modelling, pioneered by G.B. Dantzig in the PILOT model for the U.S., is described. In this approach a wide spectrum of economic activities of Israel from energy conversion to industrial production, consumer demand and foreign trade, is modelled. The data for the OMER model are extracted from many different sources such as input-output tables, consumer surveys, export-import figures, and engineering and production data. These are used as coefficients in a multistage linear program with a suitably selected objective. The methodology developed is general enough to make the model easily adaptable to other countries.

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Rim Analysis - Statistical Techniques Applied to the Evaluation of the National Energy System by Multiple Criteria

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A.S. Kydes, Brookhaven National Laboratory, Upton, N.Y.

Evolving national energy supply/demand distribution systems rely, at least in part, on quantifiable factors such as local and national environmental restrictions, resource availability (type, price and quantity) and the associated transportation infrastructure, the amount and price of capital available to consumers and suppliers of energy, total annualized system cost, including the annualized cost of end-use devices and the demands for energy and their price/supply responsiveness. The evolution also depends on non-quantifiable factors such as personal, regionally aggregated, or even national "utility functions" and institutional or social barriers.
This paper describes a systematic statistical methodology for capturing both visually and quantitatively, the trade-offs between competing quantifiable, differentiable objective functions. Several previously documented approaches to multi-objective analysis, as they relate to energy systems analysis, are examined. The aim is to provide decision makers with a more easily understood tool and a more easily defensible methodology on which trade-offs between certain sensitive and competing energy issues can be based. The Brookhaven Energy System Optimization Model was used to provide data for the analysis through the application of pseudodata generation techniques. The statistical methodology incorporated two major steps: (1) exploratory data analysis and (2) regression analysis.

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Extensions of the Gauvin-Tolle Optimal Value Differential Stability Bounds to General Parameter-Dependent Mathematical Programs

A.V. Fiacco, W.P. Hutzler, The George Washington University, Washington, D.C.

Gauvin and Tolle have obtained bounds on the directional derivative limit quotient of the optimal value function for mathematical programs containing a right-hand side perturbation. In this paper, we extend the results of Gauvin and Tolle to the general mathematical program in which a parameter appears arbitrarily in the constraints and in the objective function. An implicit function theorem is applied to transform the general mathematical program to a locally equivalent inequality constrained program, and, under conditions used by Gauvin and Tolle, their upper and lower bounds on the optimal value function directional derivative limit quotient are shown to pertain to this reduced program. These bounds are then shown to apply in programs having both inequality and equality constraints where a parameter may appear anywhere in the program.

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Differential Properties of the Marginal Function in Mathematical Programming

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F. Dubeau, Universite de Montreal, Montreal, Canada

This paper consists of a study of the differential properties of the marginal or perturbation function of a mathematical program where a parameters or perturbations vector is present. The program, with equality and inequality constraints, is not assumed convex neither in its variables or in its parameters. This study is a generalisation of some published works on this subject for the special case of right-hand side parameters or perturbations. Some possible application of the results to a certain method of decomposition is also indicated.

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On the Parametric Variation of Nonlinear Programmes and the Convergence of Nonlinear Programming Algorithms

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The paper is concerned with nonlinear programmes in which the objective and constraint functions depend on a set of parameters, and falls naturally into three parts:
In the first part we study conditions for the upper and lower semicontinuity of the feasible set and the solution set, regarded as mappings from the parameter space, and it is shown that the Mangasarian-Fromovitz constraint qualification plays a key role.
In the second part various sufficient conditions for a point to be an isolated local minimizer are considered. For certain conditions bounds are derived for the distance of a given point from an isolated local minimizer in terms of the violations of the Kuhn-Tucker conditions at the point in question. Similar bounds are obtained for the distance of a given point from the feasible set, and for the distance between local minimizers for two different perturbations of a given nonlinear programme. These results parallel those obtained earlier by Robinson, but under significantly weaker conditions.
In the third part the results are applied to prove global convergence and local superlinear convergence of a broad class of nonlinear programming algorithms. The class includes reduced-gradient and projection-type feasible point algorithms, as well as those based on augmented Lagrangians and exact penalty functions.

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Perturbation Theory for Functions Over Convex Sets

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Convex multivalued functions exhibit a certain type of regular behaviour when their ranges have internal points. Their properties are applied to study the behaviour of the solution set of a differentiable nonlinear system over convex sets in Banach spaces when the system is subjected to arbitrary but small perturbations.
A very simple regularity condition is sufficient and, under a reasonable additional assumption, also necessary for the perturbed system to be solvable. The regularity condition is related to the linearization of the system about a point; it reduces to several of the constraint qualifications of the mathematical programming literature in the finite-dimensional case.
Also, we are able to obtain a bound for the distance from a point solving the original system to the solution set of the perturbed system in terms of the magnitude of the perturbation.

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Quantitative Analysis of the Stability Sets in Convex Programming with Parameters in the Objective Function

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A great deal of work has been done in the field of parametric linear programming from the theoritical as well as from the computational point of view. In Nosicka & al. the notions of the set of feasible parameters, the solvability set and the local stability set have been defined and analyzed qualitatively. The same notions have been defined and analyzed for convex quadratic programs in Guddat.
In Osman and in subsequent work, the author introduced a qualitative analysis for the problem
(P) $\min|f(x) + \Sigma p_i h_i(x)|$, subject to
$M = \{x \in R^n / g_r(x) \leq 0, r = 1,2,\ldots,k\}$, where $f(x)$; $h_i(x)$, $i=1,2,\ldots,m$; $g_r(x)$, $r = 1,2,\ldots,k$ are assumed to be convex functions possessing continuous first order partial derivatives on R^n, p_i, $i=1,2,\ldots,m$ are any nonnegative real numbers.
The stability set of the first kind for problem (P) is defined as the set of all parameters for which an optimal point for one parameter rests optimal for all parameters. The stability set of the second kind for problem (P) is defined as the set of all parameters for which an optimal solution rests always on one side of the set M.
In this paper, the author presents a quantitative analysis for the stability sets of the first and second kinds for problem (P), which is based on obtaining all feasible points of a given polytope. An illustrative example is given in which a modification of the algorithm introduced in Balinsky is used.

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An Algorithm to Find All the Shortest Path Trees of a Network

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The determination of Shortest Paths Trees (SPT) is required in a large number of quantitative transportation models.
In these models a network is defined, with a set of origins, a set of intermediate nodes and a set of destinations, and the SPT from each of the origins to all the destinations needs to be determined.
Although simple and fast methods to compute shortest paths are available, due to the size of real life transportation networks and to the fact that the computation of shortest paths is performed repeatedly, often such a task constitutes the most expensive part of a transportation study. Usually the computation is performed independently for each origin. In the paper an algorithm is presented which makes use of the SPT relative to origin k+1. Actually a proper use of information from past computations might result in an appreciable decrease in the effort required by each single SPT computation.
The computational effort required by the proposed procedure depends strongly on the order in which the single SPTs are determined. An heuristic procedure to find a minimum length Hamiltonian Path is used to find a good ordering for the origins.
The results from a set of computational experiments on some real life transportation networks are presented.

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Adaptation of the Algorithm of d'Esopo-Pape for the Determination of All Shortest Paths in a Network: Improvements and Simplifications

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The d'Esopo-Pape adaptation of the Ford, Bellman and Moore algorithm is actually considered the most efficient for obtaining the shortest paths in a transportation network. The network consists of origin and destination nodes, called centroids, which are connected to the real network by access and egress links. Using the node labelling correcting method, the algorithm determines the shortest path tree connecting an origin with all destinations. By repeating the algorithm for each origin, all shortest paths are obtained.
In this paper simplifications made possible by repetition of the algorithm for each origin node are investigated. In particular a new technique for inserting corrected nodes in the list of "labelled nodes" is presented together with a new method for memorizing the nodes already reached. Another improvement is obtained by treating the destination nodes separately from the intermediate nodes. Tests on two large scale transportation networks showed an improvement of the order of 10% over the d'Esopo-Pape method.
Program listings and test results are included at the end of the paper.

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A Dual-Simplex Based Algorithm for Finding All the Shortest Path Trees on a Network

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We present the adaptation of the dual-simplex algorithm to compute all shortest path trees by using a reoptimization approach. Given the shortest path tree rooted at node r, the change of the root node to node s, makes the initial tree a dual feasible and primal unfeasible solution. The adaptation of the dual-simplex algorithm results in an algorithm that resembles a label setting algorithm but does not necessarily require the exploration of all the nodes of the network. The computational complexity of the algorithm is similar to that of label setting methods. We report some computational results which demonstrate the efficiency of the method.

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A Computational Study of Floyd's Algorithm

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Floyd's algorithm provides a simple and elegant technique for finding all shortest paths in a network. We study the empirical computational complexity of five different computer codes for realizing this algorithm, using a class of randomly generated test problems. The focus of the present work is to investigate objective and reproducible measures of computational effort, as well as to indicate how rather substantial reductions in this computational effort (as much as 30%, asymptotically) can be achieved. Both CPU time (obtained using totally dedicated computer runs) and predicted computation time (based on cycle times for elemental computer instructions) are discussed and compared. These results indicate a strong interaction between code, computer, compiler, and some empirical measures of computational effort. Moreover, it is found that matrix-subscripting operations (often ignored in performing "operation counts" for assessing computational complexity) are profoundly important in explaining observed computational behavior.

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Oracle Algorithms for Combinatorial Optimization Problems and Related Topics

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An oracle algorithm is a conceptual framework for deriving lower bounds on the complexity of approximative or exact solution procedures for some general classes of combinatorial optimization problems and related ones. In general, a "combinatorial problem over an independence system (or matroid)" can be defined as a mapping from the set of all classes of isomorphic independence systems (or matroids) into a suitable image set. In case of the decision problem whether an independence system has a certain property or not this image set is {Yes, No} while an optimization problem has the real numbers as image. Using different kinds

of general theorems for the complexity of oracle algorithms we can demonstrate for many combinatorial problems over independence systems (or matroids) that there is no "good" algorithm within a certain framework, i.e. any oracle algorithm which solves this problem has exponential complexity. Beside this we investigate polynomial reducibility among different kinds of oracles which allows us distinct different levels of difficulties among oracles and problems.

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Scheduling Two-Machine Flow Shops and Open Shops

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We investigate the computational complexity of two-machine scheduling problems. Each of n jobs has to be processed on each of two machines for a given amount of time. In a flow shop, each job has to be completed on the first machine before it can start on the second one; in an open shop, the processing order of a job is immaterial.
It is well known that, when the maximum completion time is to be minimized, optimal nonpreemptive schedules for flow shops and open shops can be found in O(n log n) and O(n) time, respectively. In both cases, there is no advantage to preemption.
When the maximum lateness with respect to given due dates of the jobs is to be minimized, finding optimal nonpreemptive schedules is unary NP-hard both for flow shops and open shops. In the preemptive case, the flow shop problem is still unary NP-hard, but the open shop problem can be solved in O(n) time.

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Approximation Algorithms for Clustering Problems

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A partitioning of the set of vertices of an undirected network, i.e. an undirected graph in which there is a distance associated with each edge, into k nonempty disjoint sets is called a k-clustering. The sets in the partitioning are called clusters. A k-clustering problem may be formulated as follows: Given k find a k-clustering which minimizes some objective function defined on the set of all k-clusterings. The objective functions considered are the sum or the maximum of values associated with each cluster. These values are given by the sum, the maximum or the mean of all distances within the clusters.
Combining maximum - or sumoperation with maximum -, sum- or mean-operation we get the six objective functions taken under consideration. With two exceptions for $k \geq 2$ all k-clustering problems are shown to be NP-complete. The exceptions are: the max-max 2-clustering problem which is polynomial solvable and the sum-max 2-clustering problem where the complexity is unknown. For the NP-complete problems approximation algorithms are given and their worst-case behaviour is analysed.

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On Computational Complexity of Unitary Transforms

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A new unitary transform utilizing an (nxn)-matrix $\mathcal{L}$ such that $\mathcal{L} = \mathcal{L}^{-1} = \mathcal{L}^*$ is developed for an arbitrary integer n, using the characters of Abelian groups. $\mathcal{L}$ is generated by the shifts of its first column, it is extremely easy to implement and store in the computer because of the small number of non-zero values and its computation requires only 3n real additions and n real multiplications which is substansially fewer than all known unitary transforms (Fina and Algazi, 1977). Moreover it is asymptotically optimal in the sense that if $|\mathcal{L}|^{(n)}$ is the number of non-zero entries of $\mathcal{L}$ then $\lim n^{-2}|\mathcal{L}|^{(n)} = 0$.
Then a straightforward generalization is done on the continuous case and a new complete orthonormal system $\{\mathcal{L}_i\}_{i=0}^{\infty}$ in L^2 is obtained. Its superiority over the known bases like trigonometric functions, Walsh's functions (Walsh, 1923; Paley 1932), Haar's functions (Haar, 1910) is based on the fact that the functions $\mathcal{L}_i$, $i=0,1,2,\ldots$ depend of arbitrary numbers l_w, $w=0,1,\ldots,n-1$; $n=1,2\ldots$; which can be chosen in such a way that $\{\mathcal{L}_i\}$ has the desired properties. This basis is then compared with other orthogonal bases in L^2 in some problems of digital signal processing and approximation theory.

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Complexity of Approximation Algorithms for Combinatorial Optimization Problems

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For the allocation problem it is shown that to find ε - approximation solutions is the NP - hard problem for any $\varepsilon > 0$. For the covering and traveling salesman problems (even if the distances in the latter obey the "triangle inequality"), it is shown that to find a fast (i.e. polynomial in the problem size n and $1/\varepsilon$) ε- approximation algorithm is NP - hard. For the knapsack to find the "k -th in succession" suboptimal solution is NP - hard for any k. Fast ε - approximation algorithms are presented for modifications of the knapsack problem, the time complexity T and space complexity S of the algorithms being bounded as follows:
- for the sum of subset problem $T \sim S \leq 0(n/\varepsilon)$ or $T \sim S \leq 0(n + 1/\varepsilon^2)$;
- for the maximization multiple - choice knapsack problem $T \leq 0\,(nm/\varepsilon)$, $S \leq 0\,(n + m^2/\varepsilon)$, m being the number of equivalence classes, n being the number of items; the same bounds are valid for the "obligatory multiple-choice knapsack", where exactly one item is to be chosen from each of the equivalence classes:
- for the minimization multiple-choice knapsack problems $T \leq 0\,(mnlogm + nlogn + mn/\varepsilon)$, $S \leq 0\,(n + m^2/\varepsilon)$;
- for the "arborescent" knapsack (i.e. one with the constraints that $\Sigma_{i \in J_k} a_i x_i \leq b_k$, where $\forall\, J_k, J_{k'}$ if $J_k \cap J_{k'} \neq \phi$, then $J_k \subset J_{k'}$ or $J_{k'} \subset J_k$), we have $T \leq 0(n^3/\varepsilon^2)$, $S \leq 0(n^2/\varepsilon)$. The same bounds are obtained for the fixed-charge and non-linear knapsack problems.

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Linkage of Optimization Models

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Separate optimization models for separate areas of activities are now widely used in operations research and systems analysis. Examples are energy, water and other resource supply systems, industrial and agricultural production models, manpower and educational planning models, etc. These models are mostly analysed independently without linking them into a system. Hence, the investigation of interactions between separate models is becoming more and more urgent. Examples are energy supply — economic development, manpower planning — production, water supply — agriculture production, and forestry — wood processing interaction models. To investigate such interaction the separate, and already existing models, should be linked into some integrated model. In many cases these integrated models can be formalized as structured linear programming problems with blocks describing separate models. Hence decomposition methods can be applied in principle for the solution of such integrated problems. However, there is a difference between decomposition and linkage approaches. In decomposition we begin with an integrated model and a specific decomposition algorithm implies the corresponding partitioning scheme of the whole model. In linkage we begin with separate sub-models and a linkage algorithm determines the linking of sub-models into an integrated model. Therefore, linkage algorithm should be heavily based on the information which can be obtained from the solution of the sub-models. In this paper three approaches are analysed for linking models. First, a special partition scheme for the inverse of the integrated problem has been developed leading to an algorithm in which subsequent solutions of sub-problems are used for optimal linking. In this approach information which can be obtained from the solution of the sub-problems by the revised simplex method is used. The second approach is based on the generalized gradient technique which leads to another iterative algorithm. In this algorithm only information on dual variables (associated with linking constraints) is used at each iteration. The third approach is based on feasible direction methods and may be placed between the first two approaches. These approaches are analysed from a theoretical, numerical and practical point of view. An illustrative example for linkage of a forest management — wood processing industry system is given.

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Advanced Implementation of the Dantzig-Wolfe Decomposition Algorithm

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Since the original work of Dantzig and Wolfe in 1959, the idea of decomposition has persisted as an attractive approach to large-scale linear programming. However, empirical experience reported in the literature over the years has not been encouraging enough to stimulate practical application. Recent experiments indicate that much improvement is possible through advanced implementations and careful selection of computational strategies. This paper describes such an effort based on state-of-the-art, modular linear programming software (IBM's MPSX/370). Extensive computational results, especially with multinational energy models for the European Economic Community, are reported.

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A Decomposition Procedure for a Production Planning Model

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Given the timing and sizing of final product demand requirements (sometimes referred to as a master schedule), the problem is to determine the timing and sizing of manufactured and/or purchased component quantities. When the set-up costs are ignored the resulting problem is a large linear programming problem with a special structure. Exploitation of this special structure, using a number of problem manipulations and solution strategies, results in subproblems of transportation type, for which very efficient algorithms exist. In addition to the computational feasibility, the decomposition approach simplifies the sensitivity analysis of the parameters of the problem. The model presented can be visualized as a normative version of the descriptive methodology of Materials Requirements Planning. A natural extension of the model that incorporates the set-up costs is the general disaggregation model in production planning. Since a binary variable is associated with each product in each period, the computational feasibility is limited by the resulting integer program (with one continuous variable).

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Two-Level Linear Optimization Via Aggregation

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Large-scale linear optimization models are suffering from the difficulties arising in the context of centralized planning systems. Therefore attempts have been made to achieve partially decentralized planning and optimization procedures. Here, the concept of aggregation - well-known in econometrics - is used to face these problems. The partial models for the divisions are aggregated to one activity each. The aggregated information is the input for a rough central model which is able to perform a preliminary optimization. The information gained in the interaction of the aggregated activities in the macromodel are now used in the rearrangement of the partial micromodels.
Thus an iterative optimization procedure was formulated, which is able to solve large structured as well as unstructured linear programming problems. The properties of convergence are investigated. Computing experiences on medium-size problems will be reported.

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Solving Large-Scale Structured Linear Programs by Aggregation and Disaggregation

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Complementary to the concepts and techniques of decomposition in solving large-scale mathematical programming problems is that of the aggregation-disaggregation approach. The approach aggregates a large-scale mathematical program into a "macro-problem" and solve it. Then it determines the potentially advantageous ways of disaggregating the

macro-problem into partially more detailed problems and solve them sequentially. When certain consistency assumptions are satisfied, both the sequence of partially disaggregated problems and their solutions can be obtained efficiently. The dynamic and hierachical procedure may be terminated at any level of the disaggregation process when solution bounds are satisfied. For large-scale highly structured problems, this approach appears very promising both in terms of problem generation efforts and solution bounds. This approach will be explored using a class of multi-item, multi-plant, multi-period production inventory problems.

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A Projected Lagrangian Method for Nonlinear Programming Using the Algorithmic Tools of Linear Programming Codes

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Fast algorithms for non-linear constrained optimization calculations can be obtained by applying Newton's method to the non-linear equations that come from the first order Lagrangian conditions for a solution. However, this technique requires the calculation of some second derivatives and it requires a good initial estimate of the solution, and the final solution holds not only at constrained minima, but also at constrained maxima and constrained saddle points of the objective function.
An algorithm for solving large-scale non-linear programs with non-linear/linear constraints is presented. The objective function is directly the Lagrangian function. The method optimizes iteratively a quadratic approximation of this function, combining efficient sparse-matrix techniques in the revised simplex method with stable Quasi-Newton methods to revise automatically a suitable positive definite approximation of the hessian matrix of the Lagrangian function. First order multiplier estimates are obtained by minimizing the error on the Lagrangian gradient optimality condition using the modified Gram-Schmidt algorithm. Also the method uses alternatively a procedure for updating the hessian so as to retain any sparsity present in the original matrix; although this method does not guarantee that the approximate hessian is positive definite, the next step direction is descendent.
We show the differences between our algorithm and those based on the reduced gradient of the objective function and the algorithms based on sequential optimizations of the augmented lagrangian function. An experimental code using the MPSX/370 algorithmic tools, is described.

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A Global and Superlinear Method to Solve the Nonlinear Programming Problem

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We present a new globally and superlinearly convergent algorithm to solve the nonlinear programming problem. The method solves the NLP by minimizing an exact penalty function. Whereas Conn and Pietrzyknowski have suggested a first order method based on an orthogonal projection, here a second order method is based on a non-orthogonal projection.
In a neighbourhood of the solution this algorithm behaves in a fashion similar to the recently popular recursive quadratic programming methods (and thus a local superlinear convergence rate is achieved). This algorithm has the additional feature that it has a natural interpretation far from the solution and thus global convergence is normally obtained.
The method employs stable factorizations and updating techniques. The algorithm has been implemented and we present some numerical results.

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Constrained Quasi-Newton Method with Differentiable Objective Function

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The recently developed quasi-Newton method for constrained optimization has local and superlinear convergence properties To ensure global convergence of the method a nondifferentiabl objective function with exact line search has been proposed. In this paper, we present a method in which a differentiable objective function is used. By adopting a differentiable objective function use of finite line search such as Armijo-type procedure becomes possible. The proposed objective function has the form similar to the augmented Lagrangean function. The algorithm for updating two parameters in the objective function is given and the global convergence property of the method is proved.

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An Efficient Method of Feasible Directions

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The past ten years have seen the development of new methods for solving the general nonlinear programming problem, namely augmented Lagrangian methods, and more recently extended Newton's methods. Although these methods are efficient they nevertheless have some drawbacks: (1) the iterates generated by the new methods are usually not feasible; (2) the constraints must satisfy stringent continuity requirements for the new methods to be applicable; and (3) the new methods usually require the evaluation of the gradients of all the constraints at each iteration. Classical methods of feasible directions which have been overshadowed by these new methods may therefore be still useful and competitive provided that: (4) the iterates are required to be feasible; (5) the constraints do not satisfy strong continuity hypotheses; (6) the nature of the constraints is such that the computation of their gradients is very time consuming; and (7) the efficiency of the methods is improved. In this paper, we present a new feasible directions algorithm which satisfies (4), (5), (6) and (7). The main difference between the new method and the more classical ones lies in the use of an efficient antizig-zagging procedure. We select the "almost active" constraints according to a procedure which possesses several attractive properties: (8) at each iteration, only the gradient of the cost function and the gradients of the constraints active at the current iterate need to be evaluated; (9) the redundant constraints are automatically ignored; (10) the sequence of iterates generated depends on the shape of the constraints set and not its description; and (11) the method can handle non-differentiable constraints.

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Linearly Constrained Pseudo-Newton Method

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This paper presents a new method of dealing with second derivative information when solving the linearly constrained nonlinear programming problem. Previous Newton-like methods have, generally, either been of the modified Newton variety, updating an entire Hessian approximation at each iteration, or of the quasi-Newton variety, approximating all of it from information. Our pseudo-Newton methodology allows the user to select the number of columns of the projected Hessian to be explicitly updated at each iteration. Thus, given the difficulty of the problem to be solved, the user can trade off the benefits of accurate derivative approximations versus low work per iteration. The Hessian approximation is recurred using the symmetric indefinite factorization of Bunch and Parlett, so that negative curvature information can be kept and exploited in the determination of descent directions. Our method does not require analytic derivatives, and is shown to have desirable convergence and rate of convergence properties. Computational results support the hypothesis that a pseudo-Newton method may be superior to either a modified Newton or quasi-Newton approach.

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On the Nature of Optimization Problems in Engineering Design

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The purpose of this paper is to survey the distinctive features of optimization problems arising in engineering design and some of the special techniques which we developed for coping with them. The optimization problems in engineering design to be discussed can be grouped, roughly, into three categories, characterized by the type of constraints present.

a) In this first category of problems we find constraints on the average or pointwise system dynamic behavior of the form $f^j(x) \leq 0$, with $f^j: R^n \to R^1$ continuously differentiable, but requiring integration of a differential equation for function and gradient evaluations. In such problems, computation is totally dominated by function and derivative evaluations. Furthermore, the optimization algorithm must contain a feature which sets the precision of these evaluations at each iteration. This requirement is also present in the next two categories of problems.

b) In the second category of problems we find constraints of the form $\max\{\phi^j(x,\omega): w \epsilon \Omega\} \leq 0$, with $\phi^j: R^n \times R^m$ continuously differentiable and Ω compact. Such constraints arise from specifications on time or frequency responses or from the need to take production or identification tolerances into account. The form of these constraints makes them amenable to solution by outer approximation techniques as well as by straightforward extensions of methods of feasible directions.

c) The third, and most difficult category of problems contain constraints of the form $\max(\min|\max\{\zeta^j(x,\omega,\tau): j \epsilon J\}: \tau \epsilon T|: \omega \epsilon \Omega) \leq 0$, where $\zeta^j: R^n \times R^m \times R^\ell \to R^1$ are continuously differentiable, Ω and T are compact and J is discrete. Constraints of this kind commonly arise in electronic circuit design in which both production tolerances and post manufacture tuning must be taken into consideration. These constraints are nonconvex, nondifferentiable, nonsemismooth and there are no formulas for the computation of their generalized gradients. As a result, algorithms were proposed which depend on weaker optimality conditions than those proposed by F. Clarke and R. Mifflin.

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On the Feedback Between Mathematical Programming and the Engineering Design Process

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The benefit of the application of mathematical programming (mp) within an engineering design process (edp) can be only fully utilized by a thorough use of their different feedbacks which are encountered on at least two levels:
in the first level fundamental ideas, definitions and terms of mp are projected onto the edp while on the second level of constructing algorithms and software for the edp the feedback is more in the direction from edp to the software. Examples of the first level range from questions of proper model-building to the definition of local and global optima, Pareto-optima or the application of the Kuhn-Tucker-conditions to obtain some basic informations about properties of optimal designs, thus helping to formalize the "art" of optimal design. Examples of the second level are the modification of the existing algorithms to the needs of this class of problems, the use of heuristics and the construction of special algorithms such as special but highly efficient Kuhn-Tucker-algorithms, the use of analysis acceleration techniques for the repeated system analysis or some questions on the confidence in the computed results. Thus these feedbacks are used to reduce the computational costs. Some of these topics will be discussed in more detail and their impact on real world applications will be shown.

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A Non-Linear Programming Algorithm for Structural Optimization Problems

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In structural optimization it is necessary to solve a large scale non-linear constrained programming problem. This work describes an algorithm developed on the basis of our experience with a method used in structural optimization and oriented to that kind of problems.
The problem we are dealing with is to minimize $f(\underset{\sim}{x})$ subject to $g_i(\underset{\sim}{x}) < 0$, $i=1,.,p$ and $h_j(\underset{\sim}{x}) = 0$, $j=1,.,m$.
The Kuhn-Tucker points $(\underset{\sim}{x}^*, \underset{\sim}{\lambda}^*, \underset{\sim}{\mu}^*)$ are found by using the following iterative procedure:
Let $\underset{\sim}{x}^\nu$ be a feasible point for inequality constraints. We set $\underset{\sim}{x}^{\nu+1} = \underset{\sim}{x}^\nu + s^\nu \underset{\sim}{\delta}^\nu$ where $0 < s < 1$ and δ is the vector

$$\underset{\sim}{\delta} = \gamma(\nabla f + \Sigma\lambda_i \nabla g_i + \Sigma\mu_j \nabla h_j) \quad (1)$$

Here γ is a fixed positive constant and $(\underset{\sim}{\lambda}(\underset{\sim}{x}), \underset{\sim}{\mu}(\underset{\sim}{x}))$ is the solution of the linear system obtained by substituting (1) in:

$$\underset{\sim}{\delta} . \nabla h_j = -h_j \quad (2)$$

$$\underset{\sim}{\delta} . \nabla g_i \quad -\gamma \; \lambda_i \; g_i \quad (3)$$

Stepsize factor s is evaluated in each iteration in such a way to maintain feasibility for inequalities. We show that this algorithm has $\underset{\sim}{x}^*$ as a fixed point. Equations (2) force the equality constraints to be satisfied (s becomes 1 as iterations proceed). Equations (3) perform an automatic selection of the set of active constraints at the optimum; these constraints correspond to positive λ_i^*'s. This is so because Eqs (3) force δ to point towards constraints with positive λ_i's, and we expect λ_i to have the same sign as λ_i^* in a region around the optimum as a consequence of $\underset{\sim}{\lambda}(\underset{\sim}{x}^*) = \underset{\sim}{\lambda}^*$.
This algorithm behaves as the steepest descent method for an interior optimum and as the projected gradient method when all the inequality constraints are active. It was applied to several structural problems and to test functions, showing good results as compared to other methods

found in the literature. We will also discuss further generalizations and improvements on the method.

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An Algorithm for Optimum Structural Design Using Duality

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Optimum Structural Design is concerned with the problem of finding the "best" structure for some given purposes. To attack this formidable problem with mathematical methods one must restrict the search to some subset of the possible structures.
In this paper we restrict ourselves to consider the following problem: Minimize the weight of a structure subject to given limitations on stresses and displacements under different load conditions. The design variables are generalized thicknesses of the structural members.
The problem is a nonlinearly constrained optimization problem. However, most of the functions involved are not explicitly given. Instead, each function evaluation includes an expensive finite element method (FEM) calculation. Since, in addition, the size of the problem in most cases is very large, it is hardly realistic to use some general method for nonlinear optimization. Instead one must develop methods that make use of the specific structure of the problem under consideration.
In this paper such a method is suggested. Each step of the iterative process consists of one FEM-calculation and the solution of a subproblem. The subproblems are in themselves nonlinear optimization problems, but due to their special structure their corresponding dual problems are easily solved, and the overall algorithm is computationally efficient.

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Design of Optimal Decoupled Low Sensitivity Controllers for Industrial Multivariable Systems

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In this paper the method of Implicit Optimal Model-Following is applied to multivariable control systems in order to achieve decoupled closed-loop systems with prespecified dynamic responses. Moreover, the proposed controllers preserve the desired closed-loop responses even in the presence of small system parameters variations.
In the method of Implicit Optimal Model-Following a model of the desirable system dynamics is constructed. A cost functional of the squared error of the closed-loop system dynamics from the desired model dynamics is, then, minimised to get best matching between the model and the obtained closed-loop system. In order to reduce the effect of small changes in system parameters on the designed controller, the state equations are augmented by introducing trajectory sensitivity vector in the system dynamics and cost functional.
After a brief exposure of the mathematical solution, the technique is adopted to design controllers for the longitudinal motion as well as the lateral-directional motion of the F-8 aircraft. Figures illustrating the specifications of the resulting closed-loop systems are also included.

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The Significance of Numerical Analysis and Computer Science in Mathematical Programming

W. Murray, Stanford University, Stanford, Ca., U.S.A.

It is the intention of most, if not all researchers in mathematical programming that their work will eventually aid the solution of real problems. Since the solution of real problems invariably involves the use of a digital computer this fact needs to influence both the design of algorithms and the type of theorems of interest. Unfortunately this is not always the case and algorithms have been suggested that are intrinsically unstable. This talk will discuss the care that needs to be exercised in implementing algorithms on a computer in order to minimize the effects of finite precision arithmetic. The talk will also include a discussion on how the environment in which software is used affects the software design and the desirable properties required of the original algorithm.

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Inscribing and Circumscribing Convex Polyhedra

B.C. Eaves, R.M. Freund, Stanford University, Stanford, Ca., U.S.A.

Two problems related to inscribing and circumscribing convex polyhedra are discussed. The first involves linear programming and the second involves complementary pivot theory.

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Decomposition of Complexes and Diameters of Polyhedra

L.J. Billera, Cornell University, Ithaca, N.Y., U.S.A.
J.S. Provan, State University of New York, Stony Brook, N.Y., U.S.A.

We discuss the property of k-decomposability for simplicial complexes which, if satisfied by the dual simplicial complex of a simple convex polyhedron, implies that the diameter of the polyhedron is bounded by a polynomial of degree k+1 in the number of facets. These properties form a hierarchy, each one implying the next. The strongest, vertex decomposability, implies the Hirsch conjecture; the weakest is equivalent to shellability. We show that several cases in which the Hirsch conjecture has been verified can be handled by these methods. We conclude with a strengthened form of the property of shellability which would imply the Hirsch conjecture for polytopes.

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Faces of Polar Polyhedra and Extension of Facets

J. Araoz, Universidad de Simon Bolivar, Venezuela
J. Edmonds, University of Waterloo, Waterloo, Canada
V.J. Griffin, Universidad de Carabobo, Valencia, Venezuela

A generalization of classical theorems relating the faces of polar polytopes and polar cones is given and applied to the problem of determining facet inequalities of convex hulls of finite sets of (integer) points. The results generalize recent results by several authors on extensions and lifts of facets of polytopes with (0-1)-vertices.

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Faces of Polar Polyhedra

V.J. Griffin, Universidad de Carabobo, Valencia, Venezuela

Let X and Y be two linear spaces and r any binary relation between X and Y which is linear with respect to both x in X and y in Y. The r-polar of a subset P of X (Y respectively) is the set of elements of Y (X respectively) which are related by r to every element of P. When P is a polyhedron, the r-polar of P is also a polyhedron and we describe the manner in which the faces of P are related to the faces of the r-polar of P. The results generalize classical theorems by Minkowski and others relating the faces of polar polytopes and polar cones. Applications of the theory to combinatorial optimization are suggested.

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Simplicial Partitionings of Internally Represented Polytopes

B. Von Hohenbalken, University of Alberta, Edmonton, Canada

Consider a polytope P given in internal representation [Rockafellar 1972] , i.e., as the convex hull of a finite point set. Such polytopes play important roles in many areas of applied mathematics [Clarke 1978] and other fields, e.g., chemical physics [Clarke 1979] . Algorithms to find the complete facial structure of internally represented polytopes have recently been devised by the author [Von Hohenbalken 1978] . To obtain nonnegative barycentric coordinates of given points in P w.r.t. fixed sets of affinely independent vertices of P, partitionings of the d-polytope P into d-simplices is necessary.

This paper describes a simple algorithm that yields such partitionings, which also can be induced to have certain desirable characteristics (e.g., a given vertex of P will lie in as few simplices as possible); the calculation of the volume and the centroid of P are also discussed.

The partitioning algorithm requires as input the complete scheme of the polytope P (a matrix containing the vertex sets of the faces of all dimensions, as found by the APL-codes in Von Hohenbalken 1978). It selects an arbitrary vertex $x^o \epsilon P$ and treats it as the common apex of a set of d-pyramids, whose bases are the facets of P not incident on x^o. If these facets are all simplices, the partitioning is achieved. If not, each such facet that is not a simplex is treated in the same way as P, i.e., an apex is selected, partitioning in pyramids follows, etc. Following this course recursively down the dimensions, the bases of "pyramids" in 2-faces are edges, thus automatically simplices, and one stops. Now, the set of all apices in the relevant d-, (d-1)-,... pyramids, together with the vertices of the terminal simplex (which, at the worst, will be an edge) make up the vertex set of one d- simplex. Since the only computational labor involved is the identification of vertices in the given scheme of P, all simplices can be foun simultaneously and negligible CPU time is used.

It is obvious that the vertex selected as the apex of the initial set of d-pyramids will lie in all simplices of the partitioning. It is shown in the paper that a given vertex x^* will lie in a minimal number of simplices, if the apices of the d-, (d-1)-, ... pyramids are chosen from among the neighbors of x^*.

APL-codes that make operational the above ideas are included as well as ancillary methods to calculate the volume and the centroid of P. Applications are discussed and examples given.

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A Constrained Optimization Algorithm for Solving Certain Convex Systems of Equations

D. Solow, Case Western Reserve University, Cleveland, Oh., U.S.A.

A constrained optimization algorithm is proposed for solving convex systems of equations. Convergence is established by assuming differentiability and nonsingularity of the derivative matrix. A global algorithm for finding the zero of a real valued convex function of one variable (or determining no such point exists) will also be presented.

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Sphero-Convex Sets

J-P. Vial, CORE, Universite Catholique de Louvain, Louvain, Belgium

A new notion is presented which strengthens the notion of strict convexity. We give two alternative definitions of sphero-convexity. Then, we study the problem of maximizing a linear from $< x,p>$ over a strictly convex and compact set C and prove that the solution point x(p) is a Lipschitz continuous function of p, where p is a normed vector in the Euclidean space R^n, if and only it C is sphero-convex.

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Methods for the Decomposition of Variational Inequalities Via the Proximal Point Algorithm

D. Gabay, Laboria-Iria, Le Chesnay, France

A Convex Program, consisting of minimizing a convex functional ϕ over a closed convex subset K of a Hilbert space V, are a special case of the variational inequality.

(1) Find $u \in K$ such that $(A(u), v-u) \geq 0$ for all $v \in K$,

involving a (possible multivalues) maximal monotone A from

V to $V^* \simeq V$, with $A = \partial\phi$, the subgradient of ϕ, for convex programming problems; this problem is viewed as a particular case of a general multivalues equation

(2) Find $u \in V$ such that $0 \in T(u)$,

where T is a maximal monotone operator on V.
Similarly to the conjugate duality for convex programs, it is possible to associate a dual variational inequality to (1) and a dual multivalued equation to (2), involving T^{-1} the inverse of T in the sense of multivalues operators.
We then review the properties of the proximal point algorithm of Martinet-Rockafellar

(3) $x^{k=1} = J_T^{\varepsilon}(z^k) \qquad k = 1,2,\dots,$

where the resolvant of T $J_T^{\varepsilon} = (I+\varepsilon T)^{-1}$ is a contraction defined on the entire space V; in particular the sequence generated by (3) converges weakly to a particular solution of (1). We illustrate the proximal point algorithm by applying it to the dual of a convex program with linear equality constraints and showing that it generates the same sequence that Hestenes-Powell multiplier method for the Augmented Lagrangian. We also give an interpretation of (3) as an implicit discretization scheme for the multi-valued evolution partial differential equation

$$0 \in \frac{\partial u}{\partial t} + T(u).$$

We turn to the case where the operator T can be split into the sum $T = A + B$, where A and B are "simpler" maximal monotone operators on V. This situation includes the convex programs à-la-Feuchel and their duals. We propose for this problem algorithms which approximates the proximal point algorithm (3) and involving the simpler resolvants J_A^{ε} and J_B^{ε}, thus allowing to decouple the original problem into problems relative to A and to B.
When A is univoque, a possible scheme is

(4) $v^{k+1} = J_B^{\varepsilon}(I-\varepsilon A)v^k$

which, applied to (1), yields the well-known algorithm of gradient with projection onto K. Applied to its dual, we obtain a new algorithm.
In the general case, another scheme is

(5) $v^{k+1} = [\,J_A(2J_B^{\varepsilon}-I) + I-J_B^{\varepsilon}]\; u^n$

which reduces to a method of alternated directions when (2) is an equation. Applied to a separable convex program, algorithm (5) corresponds to an implementation of the method of multipliers for the Augmented Lagrangian, where the decomposition property of ordinary dual methods is preserved, while the speed of convergence is improved. New insights are obtained by applying (5) to the dual problem. Again the sequence $\{v^k\}$ converges weakly to V such that $u = J_B v$ is a particular solution of (2); strong convergence can be established under additional assumptions on A or B. The rate of convergence is linear and rate estimates are discussed.
Finally the algorithm (5) is applied to several problems, including variational formulations of partial differential equations arising in Mechanics, systems of mildly non-linear equations and variational inequalities over the intersection of convex sets, and numerical experiments reported.

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Applications of Sublinear Operators to Subdifferential Calculus and to Convex Mathematical Programming

M. Thera, Universite de Limoges, Limoges, France

For the last few years, many articles have been published in which the theory of scalar optimization has been carried over to the vector case. (see for instance M. VALADIER, J. ZOWE, KUTALELADZE and the others).
When we are interested in vector optimization problems, we realize that sublinear operators acting in an ordered topological vector space $F^{\bullet} = F \cup \{+\infty\}$, with a largest element $+\infty$ adjoigned, play a key role in the polarity theory.
Also, the study of sublinear operators is essential for the duality theory and therefore for all problems related to convex mathematical programming. Indeed we need new techniques to extend well-known scalar results, because separation theorem cannot be applied in that framework.
The present paper consist of three sections. In the first one we give new results about sublinears operators. In particular, by using a method founded both on the theory of ordered topological vector spaces and on infimal convolution techniques, we are able to state a topological sandwich theorem that involves those previously known. (see B. ANGER and J. LEMBCKE or J. ZOWE for instance).
In a second part, we give an application to subdifferential calculus closely related to KUTATELADZE's work for instance. The two main operations considered are the addition of convex mappings, the composition of a convex mapping with a linear mapping whose domain is dense and whose graph is closed. Then, we give some applications to polarity theory and under suitable assumptions we establish a connection between the polar of the sum of two convex operators and the infimal convolution of their polars.
We also try to answer the question wether the infimal convolution $f\nabla g$ of two convex mappings belongs to $\Gamma(E\ ,\ F)$ (i.e. : $f\nabla g$ is the upper-envelope of affine continuous operators), when f and g are both in $\Gamma E,F)$.
In the final section, this work is related to the perturbation theory with all the applications we can look to convex mathematical programming.

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Variable Aggregation in Convex Programming

G. Huberman, Yale University, New Haven, Ct., U.S.A.

Applied mathematical programming problems are often approximations of larger, more detailed problems. One criterion to evaluate an approximated program is the magnitude of the difference between the objective values of the original and the approximated program.
The approximation we consider is variable aggregation in a convex program. Bounds are derived on the difference between the two objective values.
Previous results of Geoffrion and Zipkin are obtained by specializing our results to linear programming. Also we apply our bounds to a convex transportation problem.

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Optimal Design and Operation of Central Energy Systems by Bender's Decomposition

P. Nanda, D.E. Cullen, W. Price, Mathtech Inc., Princeton, N.J., U.S.A.

With DOE funding, a model has been developed for determining optimal designs and operating rules for community total energy systems. Using a system approach, we considered the production, transmission, storage, distribution, and end use utilization of multiple energy forms. The resulting model is a large-scale mixed integer programming model for the minimization of total system cost or maximization of efficiency. Bender's decomposition of the model leads to large, continuous, linear problems in which many constraints are superfluous, yet binding, at each iteration. Methods of suppressing these constraints and implicitly generating the corresponding dual variables have been incorporated into the solution procedure along with a special branch and bound algorithm for solution of equipment selection problems arising in the integer problem.

The model has been applied to the energy system at Starrett City, Brooklyn, New York, a residential apartment community with a housing capacity for 22,000 people. The existing energy system provides heating, cooling, domestic hot water, and electricity for the entire community using a central plant with thermal distribution of heating, cooling, and hot water. Optimal operating rules for existing equipment as well as optimal equipment configurations allowing different types of energy production were determined. Designs obtained with the model result in an annual cost savings of 25%, and a savings in fuel oil of 2.5 million gallons.

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The Optimization of the Energy Consumptions in the Heating of Buildings by Nonlinear Programming

F. Archetti, C. Vercellis, Universita di Milano, Milan, Italy

The steep increase in the cost of energy has led in the last years to a closer analysis of the process of heating (cooling) of a building.
Some complex models and the related computer programs have been developed, by which one can evaluate the "energy performance" of different designs. These programs could also be exploited in a search for optimum performance design: the search could be ranged by a simulation process, selecting a set of control variables and trying different allowable configurations.
The feasibility of a simulation approach is severely restricted by the excessive computer time required to scan the search domain accurately enough.
In order to overcome these limitations the authors have been considering the use of mathematical programming techniques in order to identify optimum performance designs.
A method based on recursive quadratic programming has been used in the computations; the results of the optimization algorithm has been subsequently controlled, analyzing the sensitivity of the solution to perturbations in the constraints and in the parameters of the problem.
It's important to remark that the optimization approach enables to consider a wider set of control variables than simulation: this results in a better sensitivity of the model to weather data.
The computational evidence, obtained by the authors in some real problems, supports the numerical feasibility of mathematical programming techniques in this problem.

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Optimum Sizing of Heat Exchangers in a Network

J.H. Bryant, M.M. Gutterman, Standard Oil Company, Chicago, Il., U.S.A.

In any oil refinery and in many chemical plants, there are fluid streams at many different temperatures. A stream which leaves a process unit at a particular temperature usually must be cooled or heated before entering another process unit. Such temperature changes are accomplished through the use of furnaces, coolers, and heat exchangers. Use of heat exchangers is very desirable, when it is possible, for energy efficiency reasons. The relationships between the temperatures, heat contents, and flows of the various streams in a complex of such heating and cooling devices are given by non-linear functions and the cost of the configuration is also given by a non-linear function. This paper describes a design system which we have developed which uses a non-linear optimization procedure to determine the parameters for heat exchangers, furnaces, and coolers to be built for a given process and temperature configuration and, optionally, given existing units. The system actually changes the objective function during the course of the optimization. We describe both the optimization techniques used and the appearance of the system to the user, who is usually an engineer with no particular background in non-linear optimization.

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The Application of Nonlinear Programming to the Control of Automobile Engines

A.I. Cohen, H.S. Rao, Systems Control Inc., Palo Alto, Ca., U.S.A.
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A procedure has been developed to determine the control settings for a fuel-injected internal combustion engine under warmed-up operating conditions. The controls found maximize fuel economy while ensuring that the exhaust emissions remain below the federal standards. The procedure uses engine models that have been developed and validated using an engine dynomometer. The method of multipliers is used to solve the resultant nonlinear programming problem. The controls have been implemented and tested in a microprocessor based control system in an automobile.

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Optimization of a Complex Chemical Process Using an Equation Oriented Model

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G.V. Reklaitis, Purdue University, West Lafayette, In., U.S.A.

Meaningful evaluation of the utility and performance of available nonlinear programming techniques for engineering design optimization requires the availability of realistic design models. Such models have been notably lacking in the process design area. In this paper a detailed equation oriented model of a process for the production of ethylene oxide and ethylene glycol is developed to fill this void. The model involves a large number of variables and equality constraints which are by equation sequencing reduced to 23 independent variables, twelve primary constraints, as well as bounds. The salient features of such models, their usefulness and limitations are presented. The applicability of available NLP algorithms and inherent difficulties in their application are discussed. Test results are reported with representative codes. GRG codes are found to provide the only practical approach to large scale design optimization.

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An Alternative Branch and Bound Strategy for Ordered Sets

J.J.H. Forrest, Advanced Mathematical Software Ltd., Milton Keynes, England

The Branch and Bound algorithm for solving problems with 0-1 variables involves choosing a variable taking a non-integral value at a node and creating two subproblems which correspond to the variable taking the value 0.0 and the value 1.0. This binary choice naturally extends to general integer variables and was also used by Beale and Tomlin when they extended the scope of Branch and Bound to include Special Ordered Sets. However, especially when using some more recent heuristics, a more natural strategy may involve the creation of three or more sub-problems at each node. This paper examines the consequences of such a strategy especially when applied to sets of variables whose natural order varies throughout the search - 'Dynamically Ordered Sets'. Computational experience with a new Mathematical Programming code for large mini-computers.

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A Bounding Technique for Integer Linear Programming with Binary Variables

F.S. Hillier, N.E. Jacqmin, Stanford University, Stanford, Ca., U.S.A.

We present a bounding technique for use in implicit enumeration algorithms for solving the integer linear programming problem with binary variables. The main assumptions used by this technique are (1) the associated linear program, obtained by dropping the integrality constraints on the variables, possesses a unique optimal solution, (2) this optimal solution is not binary, and (3) a good feasible solution to the original problem is available. An alternative to assumption (3) which is weaker is also presented.
We show that "joint" bounds can be obtained on the values of a subset of the variables. In addition we give an efficient method to implement this bounding technique. Finally, a class of problems particularly well-suited to this bounding procedure is specified. Computational results will be presented.

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On Computational Techniques for Parametric and Postoptimality Analysis in Integer Linear Programming

H.H. Hoc, Ecole Polytechnique, Montreal, Canada

We are interested in solving a general finite family of integer linear programs

$$\{ (P)^1, (P)^2, \ldots, (P)^k, \ldots, (P)^K \}$$

where $(P)^k$: minimize $c^k x$ subject to $A^k x \geq b^k$, x_j integer, $j \in J$ $x \geq 0$

and the problem data A^k, b^k, and c^k vary with the index k in a systematic way, e.g., continuously with respect to a single scalar parameter.
The purpose of this paper is to take stock of computational techniques for parametric and postoptimality analysis in integer linear programming and to evaluate their performances, experimentally. We consider the techniques currently available that are build on (A) a natural generalization of the branch-and-bound/cutting-plane approach from a single problem to an entire family of problems, and (b) the notion of checking and reestablishing by further computations, if necessary, a set of sufficient conditions for optimality (these sufficiency conditions can be expressed in terms of a relaxation or an equivalent problem representation or a partition of the solution space satisfying certain properties). To perform proper computational tests on large and difficult problems, particular attention is given to the data structures used to support computational implementation.

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Implicit Enumeration Approach to Integer Linear Programs

R. de Fco, J. Larraneta, Escuela Superior de Ingenieros Industriales, Sevilla, Spain

Any linear integer program with bounded admissible region can be equivalently modelled as a positive linear integer program with only plus signs and positive coefficients. Concentrating on positive integer linear programs of n variables, we derive a sequence P(i) of n problems of the same type which are successive relaxations of the original, obtained by disregarding less promising variables. We characterize the relation between the admissible regions of two problems of the sequence, obtaining admissibility and optimality criteria with respect to the original problem.
We present an implicit enumeration algorithm, starting with the lexicographically greatest admissible solution to the original problem, based on a particular ordering of the variables. Working upward through the sequence of the P(i) problems testing for dominance and optimality, it successively generates lexicographically smaller solutions. We report computational experiences with the algorithm in positive integer linear programs of several sizes, whose data is randomly generated.

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Parametric Integer Linear Programming Using a Standard Package

L. Jenkins, University of Windsor, Windsor, Canada

A procedure is described for performing parametric analysis on a mixed integer linear program (MILP) using a commercially available computer code (e.g. APEX-III, MPSX-MIP). A large fixed-charge facility location problem is used as an example. The procedure requires solving MILP problems at point values of the parameter of variation θ. The solutions in the intervals between these point values are obtained by "pegging" the integer variables and performing routine LP parametric analyses. The major computing burden lies in the branch-and-bound searches which must be performed at different values of θ. The judicious selection of these values is an important part of minimizing cost of computation. Methods are discussed for proving that the global optimum has been found for every value of θ in the range considered. A heuristic rule is presented which may greatly decrease the total cost of computation. The rule is particularly applicable to any MILP problem in which a robust decision is dependent primarily on the values of the integer variables. The procedure is illustrated by application to a problem of recovering resources from solid waste in Ontario.

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Heuristic Methods for Balancing Assembly Lines: Some Preliminary Results

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In this paper, we describe an integer programming algorithm for solving the assembly line balancing problem. The algorithm developed takes special advantage of the resulting problem structure of this important integer programming problem. Then, we compare the resulting optimal solutions with several heuristic procedures for solving the same problem, and describe how resulting problem structure also influences heuristic performance. Our paper differs from earlier published works in comparing heuristic assembly line balancing procedures in that we solve for a minimum line length given a fixed output rate (cycle time), and we compare heuristic performance relative to the optimal solution, obtained from the integer programming algorithm.

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Container Stacking - An Application of Mathematical Programming

D.B. Taylor, Ford Motor Company, Dearborn, Mi., U.S.A.

The Metal Stamping Division of Ford Motor Company stacks containers of automotive body panels and loads these stacks into railcars that are shipped throughout North America. The containers come in various widths and heights and the requirement for each container type is known. The objective is to stack all required containers such that the unused space above and around the containers is minimized. A solution technique, which is similar to the class of <u>Stock Cutting</u> problems, is presented that solves this <u>combinational</u> problem. This paper includes overall formulation as well as a description of the vector generating <u>knapsack</u> problem.

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HSTRIM - A Program for Scheduling Cutting Stock Operations

M. Fieldhouse, F.W. Tromans, Haverly Systems Europe Ltd., St. Albans, England

The theoretical problems underlying the scheduling of cutting stock operations with trim loss in the paper industry contain many of the complexities of advanced mathematical programming. These include fractional objective functions, integer variables, semi-continuous variables and dual-integer variables. The use of general-purpose LP packages to handle these complexities is uneconomic. The paper describes how the major difficulty of combining fractional programming techniques with integer programming techniques in the same model was overcome by using a recursive technique. A relatively simple special-purpose program was written for solving the problems. The details of computed examples show that this program can solve such problems economically.
The model may also be of interest for teaching or research purposes. Few published models contain semi-continuous variables. This is the first model that the author can recall which is explicity stated to contain dual-integer variables.

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Construction of Optimal Synthetic Weather Data by Convex Combination

G. Silverman, D. Low, IBM Scientific Center, Los Angeles, Ca., U.S.A.

Energy analysis computer programs use hourly weather data observations to compute building energy consumption. A problem is to reduce these hourly data from one year to a few weeks of hourly synthetic data. The problem is formulated by combining each week of hourly building energy demands for both heating and cooling by convex combination. In this formulation the objective is to come as close as possible to weighting each week of weather equally. The problem of duplicating the total energy demands of the original weather can be enforced on the synthetic weather. A large non-convex quadratic program results from this formulation. It is solved by converting it into a sequence of smaller strictly convex quadratic programs. Numerical results on an actual building with Los Angeles weather are presented. It is shown that one year of actual weather data can be accurately represented by a few weeks of synthetic weather.

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A Plant Location Problem for the Optimal Use of Byproducts in Slaughtering Industry

U. Bertele, F. Brioschi, P. Dalla Favera, S. Poggi, Politecnico di Milano, Milan, Italy

The extremely relevant balance of payment deficit in the agricultural and food sector in Italy has, recently, stimulated the Italian National Research Council to perform a detailed examination of all unemployed resources in the area.
Among such resources a minor but not irrelevant one is the blood associated to the slaughtering process which, in Italy, is not totally used (the most common use is the preparation of feeding stuff for animals) with negative effects of both economical and ecological type.
A portion of the work done, which this paper summarizes, consists in verifying whether the partial or total use of the presently unused blood is economically feasible from the point of view of a private enterprise i.e. whether the revenues associated with the final product obtained (blood flour) exceeds the transportation and manufacturing costs.
For doing that a mathematical model has been constructed which is based on the knowledge of the unused raw material availabilities in 37 different origins, of the (non linear) processing costs in 9 a priori possible plants, of the transportation costs and of the revenues associated to the final product obtained. The task of the model is selecting the value of the blood available in origin i $(i=1,\dots,37)$ and processed in plant j $(j=1,\dots,9)$ which maximizes total profit.
The result is that, also from the point of view of private interest, there is a (slight) convenience of processsing the presently non processed blood.
For solving the non linear problem which has 37 x 9 variables, a two level procedure has been adopted which allows considering at the first level a 9 - variable problem (which is solved combining the Hooke and Jeeves pattern

search algorithm with a penalty function) and at the second level a standard transportation problem.

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Planar Variable Metric Methods (PVMM) for Unconstrained Saddlepoint Problems: The Quadratic Case

S.S. Oren, Xerox Palo Alto Research Center, Palo Alto, Ca., U.S.A.

A new class of Variable Metric Methods is introduced that can locate a unique stationary point of an n-dimensional quadratic function in at most n steps. When applied to positive or negative definite quadratic functions, the new class is identical to Huang's symmetric family of Variable Metric methods. Unlike the latter however, the new family can also handle indefinite quadratic forms and is therefore capable of solving saddlepoint problems that arise, for instance, in constrained optimization.
The novel feature of the new class is a planar iteration which is activated whenever the algorithm encounters a singular direction of search, along which the objective function has zero curvature. In such iterations, the next point is selected as the stationary point of the objective function, over a plane that is conjugate to all previous search directions and contains the last singular direction. The inverse Hessian approximation is then updated with respect to that plane via a new four parameter family of rank-three updates.
The new class possesses properties that are similar to or that generalize the properties of Huang's family. In particular, it is shown that the new class generates Planar Conjugate directions of search, i.e., every two search directions are conjugate unless they result from the same planar iteration. It is also shown that if the initial inverse Hessian approximation is the identify matrix, then the new class is equivalent to a generalization, introduced by Luenberger, of the Conjugate Gradient method.

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When to Stop Making Quasi-Newton Updates

P. Barrera, J.E. Dennis, Cornell University, Ithaca, N.Y., U.S.A.

A method, based on the least change secant principle, is proposed for deciding when to stop updating the approximate Jacobian, or Hessian, in a quasi-Newton method. This method is applied to judge the noise in a finite difference Jacobian. A rule is proposed for making a choice of the step size when computing the finite difference Jacobian. The numerical results are encouraging.

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Design of Newton-like Methods for Solving Systems of Nonlinear Equations

J.C.P. Bus, Mathematical Centre, Amsterdam, The Netherlands

A class of Newton-like methods for solving systems of nonlinear equations will be described which includes most well-known methods (Newton with step size control, quasi-Newton etc.).
Global convergence results for such methods with step size control lead to special techniques for a priori estimating of the step length and other modifications to well-known methods in this field, as there is conditional use of quasi-Newton approximation to the jacobian matrix or a priori estimating of the optimal difference step if the jacobian is approximated by divided differences.
Conclusions about the usefulness of the various methods in this field will be given, based both on theoretical arguments as well as on a thorough analysis of the practical behaviour of the methods.

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Why is the BFGS Method So Good?

L. Nazareth, Universidad Nacional Autonoma de Mexico, Mexico, Mexico

There is a growing body of evidence, both theoretical and computational, which favours the BFGS update, in its standard version (Broyden) or its projected version (Davidon), over other members of the one parameter family of updates to which it belongs. We briefly summarize some of this evidence, and prove new theoretical results which give added support to the BFGS method.

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Numerical Comparison of Self Scaling Variable Metric Algorithms

G. Van der Hoek, Erasmus University, Rotterdam, The Netherlands

The results of a numerical comparison of Self Scaling Variable Metric algorithms will be presented. The following algorithms were implemented: Oren and Luenberger (1974) (25 parameter combinations)
Oren and Spedicato (1976) (4 switches of optimally conditioned self-scaling updates)
Shanno and Phua (1978) (2 initial scalings for Broyden's update) and the classical Davidon-Fletcher-Powell and Broyden-Fletcher-Goldfarb-Shanno algorithms.
An efficient implementation of the Oren-Spedicato switches was obtained by applying a computationally cheap switch criterium.
Attention will be paid to the following points:
1. performance criteria / termination criteria.
2. required accuracy of the line search.
3. special requirements for test functions for these algorithms such as:
a. the influence of initial ill conditioning (a sequence of increasing ill conditioned problems was used).
b. the influence of the number of variables.

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Cutting Planes in Integer Programming: Are They Useful?

E. Balas, Carnegie-Mellon University, Pittsburgh, Pa., U.S.A.

The traditional cutting plane methods for integer programming, that use the simplex method on a constraint set successively amended with inequalities implied by the integrality conditions, have proved inefficient except for a few special cases. This has led some workers in the field to conclude that cutting planes are useless. We claim, to the contrary, that while cutting planes by themselves can rarely solve the problem, if properly selected and used in the proper context, they can provide, in combination with other techniques, some highly efficient procedures. We discuss several computationally successful algorithms for solving certain classes of integer programs (set covering, traveling salesman problem, etc.), which use cutting planes in new ways. The new features include: the cuts themselves; their use in conjunction with various heuristics and subgradient optimization, to generate sharp bounds; new Lagrangean relaxation schemes; and, finally, the use of branching as a last resort.

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Disjunctive Programs and Sequences of Cutting-Planes

C.E. Blair III, University of Illinois, Urbana-Champaign, Il., U.S.A.

A disjunctive program maximizes a linear functional on a polytope subject to constraints restricting the feasible set to unions of certain faces of the polytope. Special cases include bivalent integer programs and linear complementarity problems. We continue the study of cutting plane methods for disjunctive programs, begun by Balas and Jeroslow. A finitely-convergent cutting plane method is presented which allows considerable freedom in generating the cut at each step. Some comments on finiteness of cutting-plane methods on discrete problems in general will also be given.

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Expanded Group Knapsack Networks and Integer Programming

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E.V. Denardo, Yale University, New Haven, Ct., U.S.A.

Gomory (1965) recasts pure integer programs in group-theoretic terms. He develops a relaxation, phrased as a (cyclic) group knapsack network, that guarantees integrality of all variables but not nonnegativity of basic variables. Building on Gomory's fundamental construct but pursuing a line sharply different form those taken in the literature, we propose an "expanded" network that greatly tightens Gomory's relaxation and that lends itself to nonredundant implicit enumeration. Our network dovetails acyclic portions that model 0-1 variables with Gomory-like subnetworks that model unbounded variables, if any. We handle generalized upper bounds, variable upper bounds, and certain other constraints simply by reconnecting arcs. All this teams up to raise simultaneously the lower bound and the chance of finding a (new) feasible solution (and of lowering the global upper bound) at any node of the implicit enumeration tree. It sets the stage for a (likely) substantial pruning of that tree. In location-distribution problem, an arc can carry flow only if the corresponding location is open. Viewing such problems as partitioned pure IPs rather than as mixed IPs, we tailor the network to enforce those constraints whose variables are nonbasic.

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Integer Programming Duality: Price Functions and Sensitivity Analysis

L.A. Wolsey, CORE, Louvain, Belgium

Recently a duality theory for integer programming has been developed. Here we examine some of the economic implications of this theory, in particular the necessity of using price functions in place of prices, and the possibility of carrying out sensitivity analysis of optimal solutions. In addition we consider the form of price functions that are generated by known algorithms for integer programming.

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On the Number of Feasible Solutions of an Integer Programming Problem

M.A. Frumkin, Central Economics Mathematical Institute, Moscow, USSR

The number $v(b)$ of feasible solutions of integer programming problem

$$\min cx, \quad Ax \geq b, \quad c, x \in Z^n, \quad b \in Z^m, \quad A \in M(m \times n, Z)$$

may be considered as a measure of complexity of the problem.

Theorem 1. There is a partition $Z^m = \bigcup_{i=1}^{N} C_i$, where C_i is a cone with its vertex in the origin of coordinates and with integer generators, $\dim C_i \cap C_j \leq m-1$, $i=j$ and $v(b)$ is a quasipolynomial for any $b \in C_i$.

Theorem 2. If all minors of the matrix A are equal to 0,1,-1 then $v(b)$ is a polynomial for any $b \in C_i$.

Theorem 3. If $m=n+1$ and $\{x \in Z^n | Ax \geq b\}$ is a simplex then $N=n+1$ and $v(b)$ is sum of a polynomial and a periodic function for any $b \in C_i$.

Similar results are true for the optimal value of objective function.

Relation between the computational complexity of integer programming problem and the complexity of function $v(b)$ is discussed.

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Optimization Under Complex Indeterminacies

B. Bereanu, Bucharest, Romania

The purpose of this paper is to present some recent approaches to optimization in linear models under complex indeterminacies.
Such "complex indeterminacies" include the following (or their combinations): "objective indeterminacies" due to the inherent "state of nature" and represented by the presence of some stochastic processes (controllable or not) among the coefficients of the decision variables; "conventional indeterminacies" resulting from the feedback of discarded connections in the simplified model used; lack of knowledge or partial knowledge of the underlying probability distribution function of the "objective indeterminacies".
Some applications, including large group decision making with multiple objectives are discussed. Numerical methods for some structured models are given. A class of stochastic processes with special properties of their trajectories appearing in the context of this paper is also briefly discussed.

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Controllability of Parabolic Systems

E. Sachs, Technische Universitat Berlin, Berlin, West Germany

We consider a linear parabolic equation with controls in the boundary or in the differential equation as distributed controls. We give a general theorem on controllability which allows to show that states with Fourier coefficients decaying at a certain rate are controllable. Since the decay is not as fast as in known theorems this is an extension of the results of Russell and Fattorini. In addition, we are able to conclude from our theorem also Gal'chuk's result that stationary states are controllable. Furthermore, we give explicitly a set of functions which are exactly controllable and in some cases it can be shown that they form a dense set in the range space.

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Optimal Control Strategies for the Discrete Time Stochastic Systems with Nonlinear Measurements

K. Uchida, E. Shimemura, Waseda University, Tokyo, Japan

The objective of this paper is to derive optimal control strategies in the framework of the control problems with nonlinear measurement systems and stochastic uncertainties. As the types of nonlinear measurement systems we define an additive type, a multiplicative type and a data processor. The uncertainties of the systems are modeled as random variables, however any specific assumptions of stochastic properties (e.g., Gaussianness, whiteness, independence, etc.) are not imposed on such random variables. When the nonlinear measurement systems and uncertainties of the systems are thus formulated, the optimal strategies are determined for the linear process models with a quadratic cost criterion. At the same time it is shown that these optimal strategies have so-called certainty equivalence property. This property implies that the optimal control strategy can be realized as a tandem connection of the state estimator and the deterministic optimal controller. It is well known that the linear-quadratic-Gaussian problem has this property. Our results say that this property holds in spite of the presence of some nonlinearities of measurement systems and without any particular assumptions on the stochastic uncertainties. In the course of discussions several examples are presented, and their estimation problems are also solved.

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Time-Optimal Control Problem with Limitations for the Derivatives of Control

T. Gicev, V. Vel'ov, Institute of Mathematics, Sofia, Bulgaria

Consider the control system $\dot{x} = A(t)x + B(t)u$; $x \in R^n$, $u \in R^r$. The admissible set of controls consists of all r-dimensional vector-functions u(t) defined and absolutely continuous in some interval $[t_0, t_1]$ and satisfying the following conditions:
1) $u(t) \in U$ for every $t \in [t_0, t_1]$;
2) $u(t) \in V$ for almost every $t \in [t_o, t_1]$;
3) $u(t_0) = u(t_1) = 0$
where U and V are convex compact polyhedrons, containing the origin. Let H(U,V) be the time-optimal control problem with as target point the origin in R^n and let H(U) be the time-optimal control problem with admissible controls - all mesurable r-dimensional vector functions u(t), for which $u(t) \in U$ for almost every t. Let $\{V_N\}_{N=1}^{\infty}$ be a sequence of convex compact polyhedrons, such that V_N contains a ball with a center - the origin in R^r and radius N. If the problems $H(U,V_N)$ satisfy some assumptions and if initial state x_0 can be driven into the origin for the problem H(U), then for every $\varepsilon > 0$ there exist integer numbers N_0 and α such that for every $N \geq N_0$ the following statements are true:
1) x_0 can be driven into the origin for the problem $H(U,V_N)$;
2) If u_N^* and u_0^* are optimal controls for the problems $H(U,V_N)$ and H(U), respectively and T_N and T_0 are the corresponding optimal times, then $0 \leq T_N - T_0 \leq \varepsilon$ and there exist d intervals $\Delta_N^1, \ldots, \Delta_N^\infty$ such that mes $(\bigcup_{i=1}^{\alpha} \Delta_N^i) \leq \varepsilon$ and $u_N^*(t) = u_0^*(t)$ for every $t \in [t_0, T_0] \setminus \bigcup_{i=1}^{\alpha} \Delta_N^i$.
The dependence of the optimal solution on the initial state and parameter is also studied for the problem H(U,V).

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Stability, Duality and Necessary Conditions for Optimality in Infinite-Dimensional Non-Linear Programming and Optimal Control

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Stability results of ROBINSON for linear, differentiable and convex systems of inequalities can be generalized to systems of the form

(P_ω) Find $x \in C$ with $g(x,\omega) \in K$! ,

where C resp. K are closed convex subsets of real BANACH spaces X resp. Y, Ω is a real topological linear vector space, $\omega \in \Omega$ is a perturbation parameter, and $g : X \times \Omega \to Y$ is a mapping with suitable properties.

By means of these stability results strong duality theorems for convex optimization problems, necessary conditions for optimality for differentiable optimization problems and stability properties of the extremal value function for families of perturbed infinite-dimensional nonlinear programming problems can be proved.
A brief survey of these results is given, some non-standard perturbations of finite and infinite-dimensional optimization problems are analysed, and applications to optimal control problems with phase constraints and distributed parameters are presented.

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Design and Implementation of a New Large Scale Optimization Algorithm and System

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G.W. Graves, University of California, Los Angeles, Ca., U.S.A.

A new Large Scale Optimization system of advanced design has been in use over the past several years to solve a wide variety of challenging contemporary problems. Design and implementation of this system has included not only consideration of the optimization module ('XS'), but also preprocessing ('PREP') and post processing ('POST') analysis modules and other service functions necessary to conveniently deal with problems having many thousands of rows and columns. Effective integration of these functions in a Large Scale Optimization system is profoundly affected by design effort far removed from the central solution routines. The activity leading to this new prototype for optimization systems will be reviewed and performance statistics revealed for extremely difficult (MIP) benchmark problems.

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Large Scale Nonlinear Programming

J. Denel, Universite de Lille 1, Villeneuve d'ASCQ, France

The aim of the paper is to describe numerical experiments in large scale non-linear programming. The problems that are considered are up to 3,000 upper and lower bounded variables, 2,000 non linear constraints, 2,500 linear constraints.
The linearized method of centers (introduced by Huard) is used with its particular adaptations.
The computational problems involved by large scale problems are described, mainly the solving at each iteration of the method, of a linear program whose scale is just the one of the problem considered, and both the search of the supremum and the zero of the F-distance on a line, (the F-distance used in the infimum of m functions, with m up to 4,500).
The solving of the linear programs by the simplex and dual-simplex methods uses direct calculations at each step instead of calculating the inverse of the basic matrix. The method used to solve linear systems has been introduced by P. Huard in 1976, and is particularly well-adapted for large sparse linear systems.
The particular attractive features of the linearized method of centers are completely summarized in order to present a method well adapted to large scale non-linear problems.
A Fortran code has been written to solve the electrical economic dispatching model in collaboration with Y. Hauw and numerical results, obtained on a CII IRIS 80 computer with a 92 K 32-bits-words memory, are given.

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Implementing Large Models in Nonlinear Optimization Codes Using Automatic Formula Manipulation

A. Drud, The Technical University of Denmark, Lyngby, Denmark

The computer costs can in most cases be decreased substantially if the user can supply the optimization code with detailed information on the model such as analytic derivatives of the constraints and the sparsety pattern of the Jacobian. However, the manual costs of producing this detailed information is often substantial, especially in large models without any special structure.
The paper describes some attempts that have been made in producing the detailed information automatically from a simple representation of the model. Both attempts are based on automatic formula manipulation. In one case the formula manipulator creates Fortran subroutines that can be used by the optimization code, in the other the computational expressions for the constraints and their derivatives are stored as numerical data.
Some experiences with automatic formula manipulation used on large econometric models will be given.

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Optimization of Dynamic Econometric Models Using the GRG-Algorithm

A. Drud, The Technical University of Denmark, Lyngby, Denmark

The paper will describe a GRG-code, CONOPT, designed to optimize large econometric models. First, a description of the mathematical properties of econometric models are given followed by a list of the computational procedures used by a GRG-algorithm. Second, the paper discusses the data structures that are chosen to store the information such as the model equations, their derivatives in all time periods, the variable metric matrices, inverse matrices, etc., and we discuss the interaction of these data structures with the computational procedures. Finally some computational results with some danish models are given.

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A Survey of Computational Advances in Dynamic Programming

T-L. Morin, Purdue University, Lafayette, In., U.S.A.

This paper critically surveys and provides a unifying framework of research on reducing the "curse of dimensionality" in dynamic programming. Computational experience is cited and existing computer codes are reviewed. The paper concludes with a prognosis for future work.

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Separable Programming, Nonlinear Networks and Duality

R.T. Rockafellar, University of Washington, Seattle, Wa., U.S.A.

Abstract not available

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Extension of Lipschitz Functions. Applications to the Minimization of Nonsmooth Functions

J.B. Hiriart-Urruty, Universite de Clermont II, Aubiere, France

A function satisfying a Lipschitz property on an arbitrary set S is extended to the whole space E preserving the Lipschitz condition. This extension is obtained by performing the infimal convolution of two functions associated with the data of the problem. Comparison results between the generalized gradient of the extended function and that of the given function are provided. In view of applications, problems dealing with optimization and approximation of the extended function are studied; in particular, it will be proved that the search of global or local minima of f on S is equivalent to the same problem on E with the extension as objective function.

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Compactly Lipschitzian Mappings and Mathematical Programming

L. Thibault, Universite de Pau, Pau, France

We consider mathematical programming problem

$$(P) \quad : \quad \{f(x) \ \mid \ g(x) \leq 0 \ , \ \ell(x) = 0\}$$

where the objective and inequality functions f and g take their values in ordered topological vector spaces and are compactly lipschitzian and the function ℓ takes its values in a Banach space and is strictly differentiable (in Bourbaki's sense). When a constraint qualification of Slater's type is verified we have necessary conditions with Kuhn-Tucker multipliers and without constraint qualification we have necessary conditions with Fritz John multipliers. These necessary conditions are given in terms of subdifferentials of a compactly lipschitzian mapping, a notion which extends to the vector case the subdifferential of Clarke of a locally Lipschitz real-valued function.

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A New Approach for Complete Characterization of Optimality in Nonsmooth Programming

J.J. Strodiot, V. Hien Nguyen, Facultes Universitaires de Namur, Namur, Belgium

In this paper, we are concerned with the problem of minimizing a nondifferentiable function subject to nondifferentiable constraints. Recently, optimality conditions for these problems have been derived by several authors but these theories do not give a complete characterization of the optimal solution unless a certain "constraint qualification" is imposed on the problem (see, for example, J.-B. Hiriart-Urruty, "On Optimality Conditions in Nondifferentiable Programming", Mathematical Programming, Vol. 14, pp. 73-86, 1978).
In the first part of the paper, necessary conditions for optimality of Kuhn-Tucker type are given for locally Lispchitz problems which do not satisfy a constraint qualification at the optimal solution. First we characterize subsets of constraints which satisfy a constraint qualification and, for regular problems, the largest subset of constraints which satisfies this property (in the convex case, this subset is independent of the optimal solution). Then we prove that the sets of multipliers corresponding to these subsets are compact.
In the second part of the paper, we prove that, in the convex case, our necessary conditions are also sufficient and are equivalent to the inconsistency of a system of convex inequalities and cone relations. So we recover, by an entirely different technique, a result due to Ben-Israël, Ben-Tal, Zlobec ("Optimality Conditions in Convex Programming", Proc. IXth International Symposium Mathematical Programming, 1976, Budapest).

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Submonotone Mappings in Nondifferentiable Optimization

J. Spingarn, Georgia Institute of Technology, Atlanta, Ga., U.S.A.

A property of multifunctions, which we call "submonotonicity", will be discussed. Submonotonicity is a property which is similar to, but weaker than, monotonicity in the sense of Minty. The class of locally Lipschitz functions whose generalized subdifferentials (Clarke) are submonotone will be characterized, some of its properties explored, and relationships with other classes, e.g. quasi-differentiable (Pshenichnyi), regular (Clarke), semi-smooth (Mifflin), and lower semi-differentiable functions (Rockafellar), will be discussed.

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An Optimization Problem with a Max-min Constraint

A.A. de Oliveira, Coppe, Universidade Federal do Rio de Janeiro, Rio de Janeiro, Brasil

The purpose of this paper is to present some results which lead to a necessary optimality criterion for the problem:

(1) $\min \sum_{i=1}^{k} f(y_i)$

subject to:

$\max\{\min\{g(x, y_i):i=1,\dots,k\}: x\in X\}\leq 0$, where $y_i \in \mathbb{R}^m$,

$X = \{x \in \mathbb{R}^m / A.x \leq b\}$, X compact, A is a mxn matrix,

$b \in \mathbb{R}^n$, g is convex and differentiable and f is differentiable. For the obtention of that criterion is employed an expression for the directional derivatives of the function $\phi(y) = \max\{\min g(x, y_i):i=1,\dots,k\}: x\in X\}$, which is also given in the article.

An additional result assures that the computations demanded by the proposed optimality test need to be made only at a finite number of points $x \in X$. This allows the use of the criterion in algorithms.

Consider f nonnegative. Then a condition is indicated which guarantees the existence of finite optimal solutions for problem (1) assuming that the number of facilities k is a variable which should be considered in the optimization process. This condition is verified if the zero set of f satisfies a special property.

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Low Piece-Linear Approximation and Minimum Conditions in Non-Smooth Mathematical Programming

M.S. Dubson, Academy of Science of the USSR, Moscow, USSR

The necessary condition of minimum for the mathematical programming problem with Lipschitzian functions was studied by F. Clarke in terms of the upper generalized gradients. Naturally this condition is very weak.

The sufficient condition of minimum in the same problem suggested here is based on the low generalized gradients and their piece-linear approximating minorants. Some special properties of non-smooth problem enable to get in certain cases a sufficient condition of minimum which is in a sense close to the necessary one. This sufficient condition is easy to check-up effectively.

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Network Planning Using Two-Stage Programming Under Uncertainty

A. Prekopa, Computer and Automation Institute of the Hungarian Academy of Sciences, Budapest, Hungary

The network is defined as the collection of some nodes and arcs where the letters have nonnegative capacities not necessarily the same in both directions. Let N designate the set of nodes and y(a,b), $(a,b)\varepsilon N\times N$ the capacity function. A flow is a function f(a,b) on NxN such that f(a,b) + f(b,a) = 0 and $f(a,b) \leq y(a,b)$ for every $(a,b)\varepsilon N\times N$. We define further a demand function d(a), $a\varepsilon N$ that is said to be feasible if there exists a flow f such that f(N,b) = $\Sigma(f(a,b):\ a\varepsilon N) \geq d(b)$ for every $b\varepsilon N$. If d(a) < 0 then -d(a) is interpreted as a supply.

We assume that in the network the demand function is of the following form: $d(a) = \xi_a - x_a$, $a\varepsilon N$, where the ξ_a are random variables whereas the x_a are deterministic variables to be determined by the network planning model. The x_a may be electric power generating capacities or water reservoir capacities etc. in practice. Due to the randomness, the d(a) demands may take on negative, positive and zero values and by the time it occurs we solve a "dispatching problem" and transmit flows on the arcs to meet all the demands, if possible. This type of problem is the second stage problem that we solve many times. The first stage problem is the network planning problem where we seek optimal x(a), $a\varepsilon N$ and y(a,b), $(a,b)\varepsilon N\times N$ values such that 1./ the demand should be feasible by a large probability and 2./ the cost of these capacities plus the long term average dispatching cost should be minimum.

Among the various applications we will describe the reservoir system design and the electric power system design problems. The algorithmic solution will be discussed and numerical example will be presented.

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Network Flow Models for Scheduling Problems

M. Segal, Bell Laboratories, Holmdel, N.J., U.S.A.

In many situations, the requirements for employees such as telephone operators or service representatives vary with the time of day and day of the week. In this talk we will illustrate by examples how network flow models can be used to obtain schedules that meet these requirements and additional scheduling constraints.

These network models exhibit common elements of structure. Time epochs are represented by nodes and time intervals by arcs. The commodity flowing in these networks is measured in units such as employee-day or employee-hour and they represent either on-duty or off-duty time.

Arc capacities and costs as well as the network's underlying graph are used to represent a specific scheduling problem. In some cases additional linear constraints on subsets of flows are needed. These constraints may induce a fractional solution which has to be changed to an integral one in a proper way.

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Classification of Network Flow Models and Their Solutions in Engineering Applications

D. Panagiotakopoulos, A.A. Pathak, Canadian National Railways, Montreal, Canada

This paper analyzes the various aspects of network flow problems arising in practice, presents a scheme for classifying these problems, examines the state-of-the-art capabilities to solve the corresponding models, and surveys and classifies the available solution techniques. The areas of application of these techniques are illustrated with typical practical examples while the problem classes which lack efficient solution techniques are identified, thus establishing possible future research directions.

Network flow problems may be characterized and classified according to aspects which relate to the nature of the network, the transported commodities, the flow characteristics, the cost structure, the optimization criterion, etc.

For each class of problems the most efficient algorithm(s) are identified. A boundary of problem classes is identifiable such that efficient solution techniques are available within it, this boundary is traversed by increasing the level of complexity of some aspect of the problem. Problems are thus generated which, although practically important, either lack any solution methodology or rely upon heuristic approaches. Practical ideas for possible research directions are supplied both for engineers and network analysts.

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A Network Flow Approach for Capacity Expansion Problems with Two Facility Types

H. Luss, Bell Laboratories, Holmdel, New Jersey, U.S.A.

A deterministic capacity expansion model for two facility types with a finite number of discrete time periods is described. The model generalizes previous work by allowing for capacity disposals in addition to capacity expansions, and conversions from one facility type to the other. Furthermore, shortages of capacity are allowed, and upper bounds on idle capacities and shortages can be imposed. The demand increments for additional capacity of any type in any time period can be negative. All cost functions are assumed to be nondecreasing and concave. The model is formulated as a shortest path problem for an acyclic network, and most of the computational effort is spent on determining the costs associated with the links of this network.

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Periodic Networks and Cyclic Capacity Scheduling

J.B. Orlin, Stanford University, Stanford, Ca., U.S.A.

Polynomial algorithms are presented for an infinite horizon variant of the transshipment problem. By associating each of the vertices v_1, v_2, v_3, ... with a period, we may view a feasible flow as satisfying demands in each period over an infinite horizon.
The objective is to minimize the average cost per period of satisfying these demands which are assumed to repeat every m periods. The network itself is periodic in that if (v_i, v_j) is an arc, then (v_{i+m}, v_{j+m}) is also an arc and with the same associated cost and capacity; however, we allow for some variation in the demands and arcs associated with the first m periods for purposes of initialization. Under suitable initializing conditions there exists a periodic fractional optimal flow which repeats every m periods. This flow may be determined in polynomial time as the solution of a transshipment problem with a side constraint on the arc flows. Furthermore, this fractional flow may be appropriately rounded off in polynomial time so as to create an integral optimal flow which repeats every qm periods for some q less than m.
A special case of this infinite horizon transshipment problem is the cyclic capacity scheduling problem. The objective here is to minimize the average cost per period of satisfying cyclic demands with integral amounts of capacity available in blocks of consecutive periods. The polynomial algorithm can also be used to determine optimal assignments of workers to shifts in a round-the clock schedule.

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On a Continuous Time Simplex Method

A.F. Perold, IBM Thomas J. Watson Research Centre, Yorktown Heights, N.Y., U.S.A.

This paper considers generalizations of the techniques of linear programming to solve linear optimal control problems with mixed state and control variable inequality constraints. It is shown (1) how the notion of a "basic feasible solution" can be made rigorous in continuous time, and (2) how one can use dual prices to make a basis change yielding an improved solution.

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Linear Programming Solution of the Discrete Stationary Linear Predictor Problem

E.A. Galperin, Universite du Quebec, Montreal, Canada

The problem of model building in the form of a system of linear stationary differential or difference equations is ubiquitous in many areas of science. We assume that the observations (measurements) are discreet. If one model exists, there always exist infinity of models equivalent with respect to observations. These models form the equivalence class that contains continuous and discreet systems of various orders and different dynamics which all are informationally equivalent. Given the observations, we look for a minimum order linear stationary difference equation (predictor) which reproduces data in ε-neighborhood of the observations. Linear programming solution of this problem is presented. The problem is formulated as the Chebyshev approximation problem wich can be reduced to a particular linear program convenient for solution by the dual simplex method.

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Optimization of Piecewise Linear Dynamical Systems

M. Kaltenbach, Bishop's University, Lennoxville, Canada

The numerical optimization of large dynamical systems often requires some form of decomposition; i.e. the problem is decomposed into optimization subproblems to be solved in sequence. An illustration involving the sequential solution of linear programming problems has recently been described by Wierzbicki for the solution of:

$$\text{Maximize:}\quad J = \sum_{k=o}^{N-1} C_{k+1}^T x_{k+1} + d_k^T u_k \tag{1}$$

Subject to dynamic constraint:

$$x_{k+1} = A_k x_k + B_k u_k + a_k \tag{2}$$

$$x_o \text{ given}$$

and static constraints:

$$D_{k+1} x_{k+1} + E_k u_k \leq e_k$$

$$x_{k+1} \geq o,\ u_k \geq o \quad \text{for } k=o,\dots,N-1$$

where $x_{k+1} \in \mathbb{R}^n$, $u_k \in \mathbb{R}^m$.

In the presentation we extend this approach to the case

where the right-hand-side in (2) is a piecewise linear function in $x_o, x_1, \ldots, x_k$; u_o, u_1, $u_2, \ldots, u_k$; $a_o, a_1, a_2, \ldots$, a_k that was found to occur in the modelling of time varying flows in networks. The main idea to be developed is that of recursively generating bounds on the variables involved at any stage of the optimization so that the active constraints of the problem can be ascertained for some ranges of the variables. From the solution obtained, an improved active constraints set is generated or a new stage considered. The proposed procedure achieves a global optimum solution in a finite number of iterations. A distinct advantage is that the solution is obtained with great numerical accuracy. This approach may be combined with a gradient type approach in order to make the algorithm computationally efficient.

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Project Management by Means of Optimal Control Theory

S. Hansen, The Technical University of Denmark, Lyngby, Denmark

The purpose of the paper is to develop dynamic optimization techniques for use in project management. A project in general is a task limited in time and space leading from an existing production or service level to another - the target state. Through this definition the problem of project management is seen to be of the same nature as optimal control problems with known final states. The control variables are resource allocation intensities to different activities, and the state variables are the amount of work actually performed. The precedence order of the activities is expressed by means of a PERT network.
The first control model analyzed is the parallel activity problem in which the n project activities can be performed simultaneously. An analytical solution to the minimum time problem is given, and this is the basic tool in solving the general project control problem which can be regarded as a series of parallel problems each with an undetermined performance period. A two-level method using both linear programming and the maximum principle seems suitable for solving this problem.

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Numerical Solution of State Constrained Optimal Control Problems by Semi-Infinite Mathematical Programming

L. Paquette, College Militaire Royal, Saint-Jean, Canada

The usual approach to state constrained optimal control problems is to use an iterative process to determine the time interval where the state constraints are binding and, at each step, to solve a two-point boundary value problem. Here, we present a more direct approach based on recent results in semi-infinite programming. It is proposed to approximate the state function and the control function by piecewise polynomials and to solve the resulting problem which is a finite dimensional optimization problem with an infinite number of constraints (a semi-infinite problem).
First, the special case of a quadratic objective, linear constrained control problem is considered. After substitution of the polynomial approximations and using the gradient inequality, we obtain a semi-infinite linear problem. Then, this problem is solved by a simplex-like algorithm which was proposed recently by Hoffmann-Klostermair and, independently, by Carasso.
Next, to solve nonlinear control problems, we propose an iterative quasi-linearization algorithm. In this case, Taylor expansions are used to transform the original problem into a simpler problem solvable with the preceding technique.
Finally, we have solved on the computer some problems found in the literature. The numerical results suggest that the above algorithms are highly stable. This important property follows from the fact that, in our primal approach, the behavior of the original control problem is modelled almost perfectly.

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A Reduced Gradient Method for Quadratic Programs with Quadratic Constraints and Lp-Constrained Lp-Approximation Problems

F. Cole, Katholieke Universiteit, Leuven, Belgium
J.G. Ecker, Rennsselaer Polytechnic Institute, Troy, N.Y., U.S.A.
W. Gochet, Catholic University of Louvain, Louvain, Belgium

A dual algorithm is developed for solving a general class of nonlinear programs that properly contains all convex quadratic programs with quadratic constraints and ℓ_p-constrained ℓ_p-approximation problems. The general dual program to be solved has essentially linear constraints but the objective function is nondifferentiable when certain variables equal zero. Modifications to the reduced gradient method for linearly constrained problems are presented that overcome the numerical difficulties associated with the nondifferentiable objective function. These modifications permit "blocks" of variables to move to and away from zero on certain iterations even though the objective function is nondifferentiable at points having a block of variables equal to zero.

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Computing Errors Detection and Control in LP and NLP Codes

P. Tolla, Universite P. et M. Curie, Paris, France

The LP and NLP codes using Dantzig Simplex method or Wolfe Reduced Gradient or Abadie Generalized Reduced Gradient, often are disturbed by computing errors in the solution of several linear algebraic systems in each iteration. The first codes used the Gauss-Jordan elimination form in order to update the basic matrix inverse; but this method was not very accurate: in LP, for instance, the inversion pivots are imposed by the second Dantzig rule necessary for obtaining a new feasible solution. Bartels and Golub suggested an LU gaussian decomposition and a partial pivot selection in order to obtain a good numerical accuracy. I proposed a simple rule for computing errors detection which consists in performing the basic reduced coasts (LP), reduced gradient or generalized reduced gradient (NLP). An efficient way for controling computing errors in the solution of linear algebraic systems resids in the statistical control of La Porte and Vignes. This method being very expensive, I propose a new reinversion algorithm where the statistic control is performed after several different pivot choices.

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Reduction Methods in Nonlinear Programming
G. Van der Hoek, Erasmus University, Rotterdam, The Netherlands

Reduction methods solve constrained nonlinear programming problems by reducing them to one or more simpler problems, where 'simpler' refers to a decrease in the degree of nonlinearity of the problem or a decrease in the number of constraints.
We shall discuss the following reduction methods:
1. the minimizing trajectory of the exterior penalty functions is approximated by minimizing the quadratic approximation of the objective function subject to the linearization of the first order Kuhn-Tucker-conditions of the penalty functions. Self Scaling Variable Metric update formulae are applied to the approximated inverse Hessian.
2. An auxiliary objective function is defined by adding a penalty term to the objective function. However, only the linearized active constraints occur in the reduced problem.
The linearly constrained problems are solved using decomposed representations of the problem matrices.
Newly developed update formulae will be presented.

294
A Hybrid Variable Penalty Method for Constrained Optimization
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The paper describes a hybrid variable penalty method for solving the general nonlinear programming problems. The method combines two classes of variable penalty functions to obtain a hybrid formulation in such a way that the error in the approximation of the Hessian matrix resulting from using only the first derivatives, are minimized both in the feasible and infeasible domains.
Variable penalty functions are the sequence of piecewise continuous penalty functions defined over the entire space where the function f(x) to be minimized and the constraints are defined. The extension of the variable penalty function into the infeasible space is accomplished by defining a transition or cut off point at which some characteristics of the extended part are matched with the part of the penalty function defined strictly in the feasible space. The hybrid algorithm poses a sequence of unconstrained optimization problems with mechanism to control the quality of the approximation for the Hessian matrix. Several possibilities, for choosing penalty parameters, which in a posed constraint situation best control the error in the approximation of the Hessian matrix, are examined. The unconstrained problems are solved using a modified Newton's algorithm. The hybrid method is found to exhibit the following characteristics.
(i) Admit small values of the penalty parameter r to start SUMT (i.e. $r=1.x10^{-3}$ or smaller) with no apparent illconditioning.
(ii) Stimulate faster rate of convergence for a single r.
(iii) Permit several initial feasible and infeasible starting points.
The numerical effectiveness of the algorithm is demonstrated on a relatively large set of test problems. The capability shows up in obtaining a converged minimum in a small number of iterations even when the initial starting point is extremely different from the final one.

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L1 and Chebychev Estimators
G.M. Appa, Middlessex Polytechnic, Enfield, England

This paper looks at some new applications and generalizations of the properties of least absolute sum (L_1) and least absolute maximum deviation (Chebychev) estimates and approximations derived from linear and parametric programming models. In particular, three new developments are described.
1) The computational problem of sorting N numbers to find the t^{th} largest, is a special case of parametric programming applied to the L_1 problem.
2) The dual constraints of the LP formulation of L_1 and Chebychev problems have common links with the necessary conditions for best approximations for any norm, giving new statistical information.
3) Special algorithms for L_1 and Chebychev estimates and approximations derived by econometricians and numerical analysts, are special cases of the simplex method.

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An Application of Stochastic Duality to Estimation Problems
R.J-B. Wets, University of Kentucky, Lexington, Ky., U.S.A.

We consider a system evolving with time which is disturbed by stochastic perturbations. The rate of the system can be partially observed at discrete time intervals. These observations may be themselves perturbed by noise. We seek the best estimate of the state of the system at time t on the basis of the information collected so far, e.g. in terms of least squares. This estimation problem can be formulated as a stochastic optimization problem. It can be shown that an appropriate choice of duality relations leads to a dual problem which is non-stochastic which can be solved by deterministic techniques. This leads to a new derivation of some classical results in filtering theory as well as a new approach to nonlinear estimation problems.

297
On the Numerical Solution of the Stochastic Programming Problem with Complete Fixed Recourse
P. Kall, University of Zurich, Zurich, Switzerland

There are various proposals how to solve the two stage problem of stochastic programming with complete fixed recourse. After a short presentation of the different approaches, their advantages, disadvantages and limits will be discussed on the basis of theoretical investigations as well as computational experiences.

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Feasible Direction Methods for Stochastic Programming Problems

A. Ruszczynski, Technical University of Warsaw, Warsaw, Poland

In the paper the following nonlinear stochastic programming problem will be considered: minimize the mathematical expectation of a function f(x,w), dependent on random w, subject to nonlinear deterministic inequality constraints. Since it is usually difficult to compute the mathematical expectation of f(x,w) at a given point x, the problem cannot be reduced to the ordinary nonlinear programming problem. Thus, for the numerical solution of the problem a direct stochastic method, consisting in simultaneous optimization and simulation, must be used. In the paper a class of direct methods will be described, which resemble the classical feasible direction methods. At a given feasible point x a random vector g, corresponding to the gradient of the objective function, is computed. Next, a direction - finding problem is solved, which produces a random feasible direction d. Finally, a step in the direction d is made and the iteration continues. In the paper convergence of these methods will be studied. Conditions on the stochastic point-to-set maps which describe the algorithm will be given. It will be proved that under these conditions the algorithm produces a sequence of points which converges with probability one to the solution of the problem. Finally, on the base of the general theory, stochastic analogues of the methods of h hypercubes and the quadratic approximation methods will be defined and studied.

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Stochastic Subgradient Methods

Y. Ermoliev, Institut Kibernetici Teremki, Kiev, USSR

We will present a review of stochastic methods for solving smooth, nondifferentiable and discontinuous optimization problems without calculating exact values of the objective function, constraints and their differential characteristics (gradients, subgradients). Applications to the stochastic programming problems and large scale optimization are discussed.

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On the Statistical Models of the Multimodal Functions and Their Application for the Construction of the Minimization Algorithms

A. Zilinskas, Academy of Sciences of the Lithuanian SSR, Vilnius, USSR

The minimization algorithms usually are constructed on the base of a certain model of an objective function. One of the mostly useful local models is a quadratic one however the global model of a multimodal function f(x), $x \in A \subset R^n$, may be addequate to more uncertain behavior of f(.). In many practical problems the objective information on f(.) is only $f(x_i)$, x_i where $x_i \in A$, i=1,k. Besides we have subjective information (for example, the experience of the solution of similar problems in the past) about the complexity and multi-modality of f(.). Constructing a model of f(.) it is reasonable to make use of objective information as well as of subjective information on f(.). The minimal considerably subjective information on f(.) must contain, it seems, the possibility to compare (taking into account $f(x_i)$, x_i) any two intervals of the values f(x), $x \in A$, $x \neq x_i$, according to their probabilities. Supposing that the available information on f(.) generates a binary relation of comparative probability (CP) at the intervals of the possible values of f(x) which satisfies some natural axioms, it may be proved that a family of random variables ξ_x with density functions $p_x(.)$ represent CP and therefore ξ_x may be interpreted as a statistical model of f(.). The problem of further characterization of $p_x(.)$ is considered axiomatically as a problem of extrapolation under uncertaity. An optimization algorithm may be constructed as an optimal procedure in respect to the statistical model. The algorithm for one-dimensional multimodal minimization constructed using a statistical model of an objective function is more efficient in comparison to the other algorithms of analogous destination. The one-dimensional multimodal minimization algorithm in the presence of noise may be constructed on the base of similar considerations. The statistical models for the multidimensional multimodal minimization are discussed in the paper too.

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Generalized Arcwise Connected Functions and Characterizations of Local-Global Minimum Properties

M. Avriel, Israel Institute of Technology, Haifa, Israel
I. Zang, Tel-Aviv University, Tel-Aviv, Israel

In this paper new classes of generalized convex functions are introduced, extending the concepts of quasi-convexity, pseudo-convexity and their associate subclasses. Functions belonging to these classes satisfy certain local-global minimum properties. Conversely, it is shown that under some mild regularity conditions, functions for which the local-global minimum properties hold must belong to one of classes of functions introduced.

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Generalized Convex Quadratic Functions - A Unified Approach

S. Schaible, University of Alberta, Edmonton, Canada

In this paper we demonstrate that all criteria for quasiconvex and pseudoconvex quadratic functions proved by various authors can be derived from a characterization earlier obtained by the author. This criterion essentially shows convexifiability of quasiconvex quadratic functions. First we derive various characterizations of the maximal domain of quasiconvexity and pseudoconvexity from which we also see the equivalence to Ferland's results. We then obtain the finite criteria by Martos, Cottle and Ferland proved for quadratic functions in nonnegative variables. In contrast to these authors we do not make use of the concept of positive subdefinite matrices. Finally in our paper we also obtain new criteria for generalized convex functions.
In particular finite characterizations of strictly pseudoconvex quadratic functions will be presented.

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Necessary and Sufficient Conditions for Quasi-Convexity and Pseudo-Convexity

J.A. Ferland, Universite de Montreal, Montreal, Canada

Necessary and sufficient conditions for a real-valued twice continuously differentiable function to be quasi-convex and pseudo-convex on an open convex set are established. Some characteristics of quasi-convex functions and functions with Hessien having exactly one negative eigenvalue are also given. The paper concludes with new necessary and sufficient conditions for the quasi-convexity and the pseudo-convexity of quadratic functions. These are equivalent to conditions earlier established by the author.

304

Nine Kinds of Concavity and Quasiconcavity Revisited

W.E. Diewert, University of British Columbia, Vancouver, Canada

The paper provides alternative characterizations of various kinds of quasiconcavity and concavity, including cases where the functions are not necessarily differentiable (or even continuous). An attempt is made to provide characterizations of the various types of quasiconcavity in terms of the local properties of the function. A generalization of the Mean Value Theorem to nondifferentiable functions is established in an appendix, and is used frequently in the main part of the paper.

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About Differentiability of Quasiconvex Functions

J.P. Crouzeix, Universite de Clermont II, Clermont, France

It is a well known fact that a convex function on R^n which is Gâteaux-differentiable at a point x is also Frechet-differentiable at this same point. We prove that this property is also true for quasiconvex functions on R^n. Denote by f'(x,h) the one sided directional derivative of f at x along h, it is also well known that for a convex function, the function f'(x,.) can be expressed as a support function, for quasiconvex function and under certain conditions of existence and continuity on f'(x,.) we can also express f'(x,.) as the minimum of two support functions. We also prove that if for a quasiconvex function the directional derivative is two sided at x, f is differentiable at x.

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Linked Ordered Sets: A New Formulation for Product Terms

E.M.L. Beale, R.C. Daniel, Scicon Computer Services Limited, Milton Keynes, England

Linked Ordered Sets consist of Chains of Special Ordered Sets that are constrained to branch at the same point. They are useful in various otherwise linear, or integer, programming problems. For example they were used by Beale and Forrest to find global optima to problems containing product terms.
The original formulation used two Linked Ordered Sets to represent the convex hull of all possible combinations of values of x_j, y and z_j such that $z_j = x_j f_j(y)$.
But experience has shown that the branching strategy can be improved if one uses some arbitrary (small) number L_j (≥ 2) of Linked Ordered Sets to represent the convex hull of all possible combinations of values of x_j, y and z_j such that $z_j = g_j(x_j) f_j(y)$. Ordinary Special Ordered Sets are used to represent the functions $g_j(x_j)$. As in the original formulation, a single chain can be used to control the value of the nonlinear variable y in several product terms of this type occurring in different constraints.
Experiences with Linked Ordered Sets as implemented in the general Mathematical Programming System SCICONIC will be reviewed.

307

Comparative Study of Some Branch and Bound Algorithms in Integer Nonlinear Optimization

J. Akoka, Essec, Cergy Pontoise, France
H. Dayan, Ministere de l'Industrie, Paris, France

In this paper we present three algorithms for integer nonlinear programming problems. The three branch and bound methods are evaluated and compared using several discrete nonlinear programming problems, some of them being nonconvex.
Our results show that the method which limits a priori the number of nodes to be stored in the main memory of the computer is the most efficient. It is also shown that the overall efficiency of the algorithms are heavily dependent on the criteria used to select the nodes and branching variables.
In this paper, we will rank these criteria and the algorithms. We will show that the combined use of pseudo-costs criteria and the Bounded Branch and Bound method lead to efficient solutions for integer nonlinear programming problems.

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Algorithms for a Variant of the Resource Allocation Problem

N. Katoh, Osaka Center for Adult Diseases, Osaka, Japan
T. Ibaraki, H. Mine, Kyoto University, Kyoto, Japan

We consider the following simple integer programming problem: Maximize Σx_i subject to $\Sigma f_i(x_i) \leq R$ and x_i: nonnegative integers, where f_i's are nondecreasing convex functions defined over $[0, \infty)$ and satisfy $\Sigma f_i(0) \leq R$ and $\lim_{x_i \to \infty} f_i(x_i) = \infty$. R is a real number. This is a variant

of the well known resource allocation problem: Minimize $\Sigma f_i(x_i)$ subject to $\Sigma x_i = N$ and x_i: nonnegative integers. Three algorithms are presented for this problem, which are respectively based on the incremental method and two versions of the Lagrange multiplier method. Under the assumption that each evaluation of $f_i(x_i)$ is done in constant time, the first algorithm requires $O(N^* \log n + n)$ time, where N^* denotes the optimal objective value. The second one requires $O(n^2(\log \bar{N})^2)$ time, where $\bar{N}$ denotes a given upper bound of the optimal value. The third one requires $O(b(n,R) + n \log n)$ time, where $b(n,R)$ denotes the computational time required to solve the continuous problem obtained from the original one by dropping the integrality condition on x_i's.

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Postoptimality Analysis in Nonlinear Integer Programming the Right-Hand-Side Case

M.W. Cooper, Southern Methodist University, Dallas, Tx., U.S.A.

An algorithm is presented to gain postoptimality data about the family of nonlinear pure integer programming problems in which the objective function and constraints remain the same except for variations in the right hand side vector. It is possible to solve members of such families simultaneously to give optimum answers for each problem of the family.

310

Best Linear Relaxations for Quadratic 0-1 Optimization

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P. Hansen, Faculte Universitaire Catholique de Mons, Mons, Belgium
B. Simeone, Istituto per le Applicazioni del Calcolo, Roma, Italy

Linear bounds (TORs) of a given quadratic function f(x) in 0-1 variables, generalizing several previously considered bounds, can be obtained by termwise optimal relaxation. All "best" TORs (i.e. those which minimize a certain "duality gap") are shown to be optimal also in the larger class of all linear functions ℓ, such that ℓ-f is a quadratic posynomial in the variables x_i and $\bar{x}_i=1-x_i$. It is proved that the coefficients of such best linear bounds can be obtained by solving a continuous vertex-packing problem. The non-zero coefficients of a best linear bound fix the values of the corresponding components in every maximizing point of f. For certain functions f, in particular for the supermodular ones, there is no duality gap.

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On the Solution of Some Large, Specially Structured Linear Complementarity Problems with Applications

J-S. Pang, Carnegie-Mellon University, Pittsburg, Pa., U.S.A.

In this talk, we shall discuss the application of a simplified version of the parametric principal pivoting algorithm for solving some large specially structured linear complementarity problems arising from portfolio selection and quadratic networks. We shall also report computational results which include comparisons with Lemke's algorithm.

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Quadratic Complementarity Problems and Optimal Design of Submarine Pipelines

L. Jurina, Technical University of Milan, Milan, Italy
C. Lupetti, University of Pisa, Pisa, Italy

The paper considers a kind of problems which arise in the field of structural engineering and shows how they can be formulated as quadratic (or linear) complementarity problems, i.e. as minimum of a convex quadratic form under linear inequalities and complementarity eqs. A decomposition method, based on dual properties, is described and it is shown that it enables one to define flexible solving algorithms as required in the engineering context. The main problem is concerned with the optimal design of a submarine pipeline freely resting on the seabed. Some application to real problems and some computational aspects are discussed.

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Software for a Class of Complementarity and Quadratic Problems. Implementation, Evaluation and Applications in the Field of Structural Engineering

C. Lupetti, University of Pisa, Pisa, Italy

This paper aims to describe a set of computer programs for solving linear and quadratic complementarity problems (CP), i.e. for finding the minimum of a convex quadratic (or linear) form, subject to linear inequalities and complementarity equalities.
The underlaying solving methods are based on a decomposition method which reduces the given CP to a sequence of convex quadratic (or linear) programs. This method has been conceived to handle design problems in the field of structural engineering; these problems are of large-scale and exhibit a nonconvex structure in an essential way. These difficulty have been overcome by defining an interactive software which combine the human skillness and the mathematical properties of the model.
Computational experience both on real problems and on random data complete the paper.

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Khacian's Algorithm for Linear Programming

P. Gacs, L. Lovasz, Stanford University, Stanford, Ca., U.S.A.

L.G. Khachian's algorithm to check the solvability of a system of linear inequalities with integral coefficients is described. The running time of the algorithm is polynomial in the number of digits of the coefficients. It can be applied to solve linear programs in polynomial time.

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Solving Large Scale Linear Programs Without Structure

P. Huard, Electricite de France, Clamart, France

The standard Simplex method, using the explicit inverse of the basis, has a memory requirement proportional to the square of the number of rows. Apart from the linear programs with particular structures (as block angular structure) for which the decomposition of the calculations is possible, the current computers are unable to solve problems of more than two or three hundred constraints. For sparse but without nice structure linear programs, we propose here a variant of the Simplex method, not using the inverse of the basis, and needing only a number of memories proportional to the number of rows (instead of the square). At each step, the vectors required for changing the basis (that is the right hand member and the candidate column of the Simplex tableau, and the reduced costs) are obtained by solving directly three linear systems, whose matrices are the basic matrix or its transposed. Then the sparsity is kept. By simple permutations of the rows and columns, the basic matrix is transformed into a triangular-band matrix of little thickness, and the solving of the systems is very fast with the method of parameters. For problems of about two hundred constraints numerical experiments gave a mean iteration time for the proposed variant twice faster than for the standard simplex method.

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On the Solution of a Linear Fractional Programming Problem

B. Mond, La Trobe University, Melbourne, Australia

Consider the linear fractional programming problem: Maximize $(c^t x + \alpha)/(d^t x + \beta)$ subject to $Ax \le b$, $x \ge 0$. Three different techniques for solving this problem have been given by Charnes and Cooper, Isbell and Marlow, and Martos. Wagner and Yuan have shown that the first and third methods are algorithmically equivalent. Here it is shown that this equivalence may break down for a problem with an unbounded constraint set. Also, a number of examples are given to demonstrate that the Isbell-Marlow method is not algorithmically equivalent to the other two methods even for a problem with a bounded constraint set. Some extensions to pseudomonotonic functions are also discussed.

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Morphological Analysis of Linear Programming Algorithms

H. Muller-Merbach, Technische Hochschule, Darmstadt, West Germany

The design of algorithms (not only in linear programming) requires quite a few decisions on highly complex "systems". The decisions become more transparent if (i) the complex system is broken down into elements and (ii) all the decision alternatives are made explicit. A technique that serves both purposes is "morphological analysis" (developed by Zwicky). It is an efficient tool for the design of high quality algorithms.
The author's experience with morphological analysis in the design of combinatorial optimization algorithms was encouraging. And he sees promising applications of morphological analysis in the field of linear programming algorithms. Morphological analysis can contribute to (i) an extended insight into the order of existing algorithms in linear programming and (ii) the development of new versions of the (revised) simplex algorithm which are not as yet investigated. In this paper, the author will emphasize these two aspects.

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The First Algorithm for Linear Programming: An Analysis of Kantorovich's Method

C. Van de Panne, The University of Calgary, Calgary, Canada

An analysis is given of Kantorovich's method of resolving multipliers. It is shown that the method is equivalent to a parametric method but that it is also equivalent with the simplex method with a special rule for the choice of the new basic variable.

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Solving Staircase-Structured Linear Programs by Adaptation of the Simplex Method

R. Fourer, Stanford University, Stanford, Ca., U.S.A.

Certain planning and control models give rise to multi-period linear programs that have a natural "staircase" structure. Such structure may be exploited to design more efficient versions of the simplex method. These new versions retain the fundamental concepts of bases, pricing and pivoting; they gain their advantage by adapting many of the specialized algorithms that are required for large-scale computing.
For example, standard algorithms to perform sparse Gaussian elimination (LU factorization) of the basis are ineffective for many staircase-structured bases; much faster alternatives can be devised. These alternatives lead in turn to economies in the algorithms that solve linear systems of equations at each iteration. Algorithms for partial pricing can take advantage of the staircase structure as well.
Detailed computational experiments are described for several practical test problems.

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The Solution of 100-City Travelling Salesman Problems

A. Land, London School of Economics, London, England

A simplex-based Fortran code, working entirely in integer arithmetic, has been developed for the exact solution of travelling-salesman problems. The code adds tour-barring constraints as they are found to be violated. It deals with fractional solutions by adding two-matching constraints and as a last resort by 'Gomory' cutting plane constraints of the method of integer forms. Most of the calculations are carried out on only a subset of the variables, with only occasional passes through the whole set of possible variables. Computational experience on some 100-city problems will be reported.

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On the Complexity of the Monotone Asymmetric Travelling Salesman Polytope

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Y. Wakabayashi, Universidade de Sao Paolo, Sao Paolo, Brazil

The monotone asymmetric travelling salesman polytope $\widetilde{P}^n_T$ is defined to be the convex hull of the incidence vectors of all hamiltonian circuits and all subsets of these in a complete digraph of order n. We prove that certain hypohamiltonian resp. hypotraceable diagraphs $G = (V,E)$, i.e. diagraphs which are not hamiltonian resp. traceable but $G - v$ is hamiltonian resp. traceable for all $v \in V$, induce facets $x(E) \leq n-1$ resp. $x(E) \leq n-2$ of $\widetilde{P}^n_T$.
These results show that the asymmetric travelling salesman polytope has very complicated facets and that it is rather unlikely that an explicit complete characterization of $\widetilde{P}^n_T$ can ever be given. These results also indicate that for (practically successful) LP-approaches to solve the ATSP, at least when they are based merely on facetial cutting planes, a convergence proof can hardly be obtained.

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A Procedure for Obtaining Optimal Itineraries for Search and Rescue Vehicles

J.A. Smith, US Coast Guard, Groton, Ct., U.S.A.

The U.S. Coast Guard is often faced with the problem of determining the best route for a vehicle to transit to a distress location and return to its home base in the most expedient manner. This problem is complicated by the fact that the vehicle may need to refuel at one or more locations. The number of these locations can be quite large and distributed in space in a variety of patterns. Unlike a flight plan for commercial air traffic which is designed to avoid bad weather, the search and rescue aircraft must respond as rapidly as possible irrespective of weather and other constraints. Priorities of the distressed unit, uncertain times on the scene, type of service required and the possibility of preempting the vehicle, necessitate balancing the effects of an immediate decision against the effects of continuation of the plan. This places unique constraints on the optimal routing of a vehicle. This is a type of the General Shortest-Path problem with cyclic networks. The solution technique uses Dynamic Programming as the core mechanism for calculating the optimal path, where the stopover points and additional nodes that account for time lapses, such as refueling and servicing, are the stages. The length of the arcs are weighted times which are constrained by fuel and crew endurance limits with occasional diversion forcing the readjustment of nodes.
This procedure was used in a large-scale simulation model to assign the best routes for refueling type vehicles.
The algorithms developed can be applied to similar traffic flow situations where the vehicles exhibit deterministic behavior.

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Determining Extremal Paths Through Probability Fields

R.G. Brusch, General Dynamics, San Diego, Ca., U.S.A.

Many problems of practical interest involve determining extremal paths through probability fields. For example determining the safest route between two points in continuously variable dangers is often required. The path with a given length which maximizes the probability of detecting some event is also frequently sought. This paper presents a computationally efficient algorithm for determining the solution to such problems for two dimensional probability fields.
Standard methods are available for solving this problem. This paper presents a new approach which we have found computationally more efficient than previously published approaches. The efficiency is realized primarily through a reduction in the number of accesses required to slow bulk storage. The algorithm permits the efficient segmenting of large networks into subproblems with small intermediate storage requirements. The algorithm is based on the principle of optimality from dynamic programming. The algorithm determines globally optimal routes through the probability network.
A method for determining an optimal route from the starting node to the end node which is constrained to pass through a given intermediate node is also presented. These solutions can be used in turn to determine an optimal route corridor about an optimal extremal. A route exists from the starting point to the ending point through each point in the corridor having a total probability within a predetermined factor (say 1.01) of the optimal probability.
These optimal route corridors are frequently of greater value than the optimal routes themselves.

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General Quadratic Programming: An Overview and Evaluation

G. Van de Panne, The University of Calgary, Calgary, Canada

An overview is given of methods for general quadratic programming. Methods proposed by Ritter, Balas, Candler and Townsley, Konno, Swart and van de Panne are compared and their computation of efficiency is evaluated for a number of test problems.
Keywords: quadratic programming, general quadratic programming, cutting planes.

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Gradient Projection Approach for Quadratic Programming with One Constraint

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B. Lemaire, Universite des Sciences et Techniques du Languedoc, Montpellier, France

An iterative solution procedure is introduced to solve convex quadratic programming problems with bounded variables and only one equality constraint. It can be shown to be a special application of the gradient projection method. At each iteration, the descent direction is an optimal solution of a quadratic programming problem with bounded variables and only one equality constraint where the quadratic form is diagonal. Analytic solution can be determined for such problems. Numerical experiences are presented.

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A Numerical Algorithm for Solving Non-Positive-Definite Quadratic Programming Problems

J.R. Bunch, University of California, San Diego, Ca., U.S.A.
L. Kaufman, Bell Laboratories, Murray Hill, N.J., U.S.A.

A numerical algorithm is presented for solving the quadratic programming problem of determining a local minimum of $f(X) = \frac{1}{2}X^TQX+C^TX$ such that $A^TX\geq b$ where Q is a symmetric matrix which may not be positive definite. Our method combines the active constraint strategy with an algorithm for the stable decomposition of a symmetric matrix. Under the active constraint strategy one solves a sequence of equality constrained problems, the equality constraints being chosen from the inequality constraints defining the original problem. The sequence is chosen so that f(X) continues to decrease and X remains feasible. Each equality constrained subproblem required the solution of a linear system with the projected hessian matrix, which is symmetric but not necessarily positive definite; the stable decomposition for symmetric matrices facilitates the stable determination of the solution to the linear system. The heart of this paper is a set of algorithms for updating the decomposition as the method progresses through the sequence of equality constrained problems. The algorithm has been implemented in a FORTRAN program and numerical examples are given.

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Construction of the Optimal Dual Solution of Quadratic Programmes with Equality Constraints

R. Caron, University of Waterloo, Waterloo, Canada

This paper presents a new method for constructing the optimal dual solution of quadratic programmes (QP's) with equality constraints. The QP is assumed to have n variables, r equality constraints and m inequality constraints. This new method can cause significant savings in both storage and execution time.
The method is demonstrated via the QP algorithm of Gill and Murray (1977). Inherent in their algorithm is a QR factorization of a matrix A. This matrix contains the gradients of the active constraints. It is shown that a certain block of the matrix R, previously used in the computation of the optimal dual variables for the equality constraints, is no longer necessary for these computations. In fact, the entries in this block, need not be stored or computed.
A QP algorithm due to Best and Ritter is also discussed.
The computational work required by this algorithm can be reduced by (4n-1)r floating point operations at each iteration. Computational results from ten test problems are presented.
The results of this paper are also applicable to more general algorithms. Two such algorithms will be discussed.

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Bilinear Programming

I. Czochralska, The Central School of Planning and Statistics, Warsaw, Poland

The bilinear programming problem (BLP) belongs to a class of mathematical programming problems having many local maximums. It may be stated as Maximize $F(x,y) = a^Tx + b^Ty + y^TQx$, subject to $x\varepsilon X = \{x\varepsilon R^n | Ax = c, x \geq 0\}$ and $y\varepsilon Y = \{y\varepsilon R^m | By = d, y \geq 0\}$.
The methods for solving the problem proposed in the literature work under the assumption that the set X×Y of feasible solutions is bounded which ought to be verified before applying them. The boundness of X×Y implies the existence of an optimal solution, but such a solution may exist though X×Y is unbounded.
This paper addresses itself to the general problem BLP as defined above without any additional assumptions.
Some general properties of the problem, theoretical foundations of an algorithm and the algorithm for finding an optimal solution is presented.
A class of nonconvex quadratic programming problem with a nonnegative definite matrices of the objective function can be solved by some modification of the algorithm solving problem BLP.

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Some Recent Results on Preference-Ordered Markov Decision Processes

N. Furukawa, Kyushu University, Fukuoka, Japan

Discrete time Markov decision processes with countable state space and vector-valued utilities are considered. Optimization criterion is defined by a partial-order preference relation. The preference relation is required to satisfy some conditions related to the existence of maximal points and a kind of monotonicity. Certain assumptions concerning the topological structure of the underlying decision model are made. Under this problem setting, the main results presented in this paper are the following:
(i) Dealing with a vector-valued additive utility, a recurrence formula of finite-stage optimal return is given.
(ii) For an infinite horizon decision process with a vector-valued discounted additive utility, a policy improvement algorithm with respect to the chosen preference order is given.
(iii) For the same decision process as in (ii), the relation between an optimal policy and an optimality equation is solved.
(iv) Dealing with a vector-valued average utility in the infinite horizon case, a policy improvement algorithm is given.
(v) Several examples are illustrated.

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A Two Level Algorithm for Multiobjective Linear Programming

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An algorithm is described for finding the faces of all dimensions of the set Z(eff) of nondominated values of the objective functions in a multiple objective linear programming (MOLP) problem. With the possible exception of certain degenerate problems, all nondominated faces are identified from a single tableau without recourse to auxillary problems to test for nondominance and/or dimension. The enumeration of faces is based on the usual "vertex following" procedure, with all facets of Z(eff) which contain a nondominated vertex being identified as the vertex is "processed." This is accomplished by solving alternating sequences of linear programming problems. The "upper level" LP problem, which constrains the values of all but one objective function in the MOLP problem, is used to find facets of Z(eff). The "lower level" LP problem relaxes the constraints on the MOLP objective function values, and is used to identify vertices in Z(eff).
If the MOLP problem has k objective functions, the "cost" of performing all computations in a single tableau is approximately 2k pivot operations for each vertex in Z(eff). Here comparison is made to an algorithm utilizing identical logic but maintaining separate tableaux for upper and lower level subproblems.

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An Overview of Subjective Programming in Multi-Criterion Decision Making

H. Nakayama, Konan University, Kobe, Japan

In multi-criterion decision making, the best alternative is usually selected from a number of alternatives as the one that gives the best effect with respect to the preference of the decision maker. Subjective value judgment of the decision maker, therefore, plays an important role in these situations. In 1964 Aumann published the first results available on programming based on the subjective judgment of the decision maker, which he named subjective programming.
Following his work, first in this paper, some duality theorems for subjective programming with additive (linear) and incomplete preference are derived by defining vector-valued Lagrangean.
Next, subjective programming with convex preferences is considered: some theoretical results regarding necessary and/or sufficient conditions are proved on the basis of the information only about the marginal rate of substitution in order to exclude the necessity of any numerical evaluation of the preference of the decision maker. A kind of interactive optimization method, namely, the interactive relaxation method (abbreviated as IRM) is suggested from the point of view of subjective programming. Several applications are introduced for proving the efficiency of the method.
Finally, some conditions for optimality in group decision making are presented. The obtained results for subjective programming with a single decision maker are embedded in those of group decision making problems.

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Interactive Programming Procedure to Solve Multiple Criteria Linear Programming Problem

W. Michalowski, Research Institute for the Management Organization and Development, Warsaw, Poland

Multiple criteria linear programming /MCLP/ problem is considered. One has assumed a possibility of cooperation with the decision maker in order to get a final solution of MCLP problem. In such a case the best approach is to use one of the interactive programming methods. We propose new interactive procedure leading to efficient /i.e. Pareto - optimal/ solution of MCLP. Lousily speaking, it belongs to a general class of methods initiated by STEM method of Benayoun et al./Math.Progr.1/1971, pp.366-375/. As a reference point we take "the worst compromise", i.e. the solution which decision maker perceives as the worst possible outcome. Taking into account some aspects of psychological theory of Atkinson, we propose an interactive procedure providing "the best improvement" in reference to the worst compromise. MCLP problem stated in this manner we solve using the following metric:

$$\max_{x\in X}\left\{\min_{1\le i\le \ell}\left(\frac{f^i/x/ - \mu^i}{h_i}\right)\right\}$$ where μ represents a vector

of the worst compromise. This MAXMIN problem which forms one step of our procedure can be solved by one of standard LP computer systems. Some possible areas of application are discussed as well.

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The Use of Reference Objective or Displaced Ideals in Group Assessment of Solutions of Multiobjective Optimization or Cooperative Game Problems

A.P. Wierzbicki, IIASA, Laxenburg, Austria

Two basic approaches to the choice of a solution of a multiobjective optimization or a cooperative game problem by a group of experts (called here decision makers) can be distinguished. The first, the more classical one, is based on the use of weighting coefficients in relation to utility and value functions, identifying individual or group preferences, etc. It is difficult, however, to make this approach operational, since the identification of decision makers' preferences and of the corresponding weighting coefficients requires quite an extensive amount of information. The second approach is based on the assumption that the decision-makers express their goals only in terms of desirable points in objective space. For example, in a dynamic economic model a decision-maker is very likely to express his goals in terms of an ideal gross national product and an ideal energy consumption as functions of time, but is not likely to state explicitly his preferences between a limitation of energy use and the overall growth. To make the second approach operational, some mathematical background related to the use of objective reference points (or displaced ideals in multiobjective optimization via penalty functions and scalarizing function techniques was developed. The paper presents a further development of the necessary mathematical background and computational techniques needed when several decision makers specify their goals by objective reference points (or displaced ideals) and a cooperative solution should be found. It is assumed that the objective space might as well be infinite-dimensional as finite-dimensional, that is, objective reference points in the form of time-trajectories are admissible. The bargainning between decision-makers has as a goal a point in the objective space which is both extended Pareto-optimal (polyoptimal) and acceptable to all decision-makers. Procedures for finding such a point are described and analysed. Their convergence is investigated in relation to basic properties of the underlying cooperative game. The existence and uniqueness of the

resulting compromise is analysed in relation to the theory of utility and value functions. Examples of applications are given.

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On a Model and a Resolution Approach for the Deterministic Short Term Generation Planning Problem of a Large Hydro-Thermal System

L. Lafond, I.R.E.Q., Varennes, Canada

The generation planning problem over a one week horizon (divided in K periods) of a large system comprising a few thermal plants, and many hydraulic reservoirs and plants spread along a number of valleys, will be considered. Given forecasts for the natural inflows to the reservoirs, the problem is to determine the required generation in order to meet the predicted load demand, at a minimum cost while respecting some fundamental operation constraints. These constraints include the hydraulic constraints for the various valleys, i.e. lower and upper bounds on the storages and outflows of the reservoirs, and also some elementary energy transmission limitations.
The generation cost structure and the generation characteristics of the hydro plants will be described. The problem will then be formulated as a large nonlinear program and it will be seen how, under certain hypotheses, this program can be decomposed by allocation of the demand (i.e. of the right-hand-side) to the valleys: a master problem (MP) which computes successive improved allocations to the valleys and one subproblem $SP_i(y_i)$ for each valley i, depending on the allocated demand y_i, are then obtained.
The resolution approach retained for $SP_i(y_i)$, involving partial duality and gradient reduction and projection methods, will be briefly sketched. It will also be seen how (MP) can, in its turn, be decomposed once its domain and objective have been linearized with the well-known procedures of inner linearization and tangential approximation.

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Modelling and Resolution of an Energy Generation Planning Problem for a Hydro Electric System

M. Hanscom, I.R.E.Q., Varennes, Canada

This paper is concerned with the modelling and resolution of the energy generation planning problem of a large hydroelectric system for a horizon up to two years.
Hydro plant generation, and objective function modelling have been emphasized. The problem has been formulated as a deterministic discrete-time optimal control problem. All the transition equations are linear, the objective is convex and there are bounds on both the state and the control variables.
The solution algorithm is of the reduced gradient type, with the control variables nonbasic. This provides good scaling. Because of the linear relationship between the state and control variables, the set of feasible directions at a point is a polyhedral convex cone and the bounds both on the state and control variables can be accounted for by means of an algorithm for the orthogonal projection of the gradient on the cone.
Successive projected gradients lying in the same linear subspace are then conjugated according to various conjugate gradient algorithms. Their performance on this large-scale non-linear program is compared.

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Mathematical Programming and Planning in the Electric Utility Industry

J.S. Graves, Resource Planning Associates, Inc., Washington, D.C., U.S.A.

This paper surveys several of the models and methods used by electric utilities in the U.S. to conduct their planning. The discussion includes commercial software, some recent theoretical work, and some questions for future research. Attention is paid throughout to the practical applicability of the models.

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Optimal Operation of a Multireservoir Hydroelectric Power System

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A. Turgeon, I.R.E.Q., Varennes, Canada

In this paper we propose an approach to the problem of the most economical operation of a hydroelectric power system composed of reservoirs in cascade. Due to the fact of the highly random nature of some of the problem variables (e.g., water inflow and energy load) an optimal policy over a very large horizon is of no use. It is shown that, under some realistic assumptions, this problem can be broken into two sub-problems:
- an inter-weekly operating problem (long-term), the solution of which gives the strategical behaviour of the system,
- an intra-weekly operating problem (short-term) which gives day-by-day control taking into account the strategy conditions of the long-term problem.
The coupling between the two problems is introduced through the marginal expected profit with respect to each state variable.
The long-term study, which is a large stochastic problem, is solved by performing first the aggregation of the variables. The resulting reduced problem is solved by dynamic programming. In order to restore the above marginal expectation, a solution of the non-aggregated problem is needed: an heuristic method is proposed for that purpose. The short-term deterministic problem is then solved by means of the maximum principle and gradient methods. Finally, the results of numerical experiments are given for a system constituted by three reservoirs series.

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Nonlinear Programming Applied to the Optimal Operation of a Hydroelectric Power System

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The optimal operation of the Alcan hydroelectric system, located in the Saguenay region of Northern Quebec, is explored, using some nonlinear programming techniques. This involves the development of some new methodologies as well as the application of well-known techniques.
The Alcan system consists of four reservoirs and six power stations. The objectives of its operation can be divided into long-term and short-term objectives. In the long-term, the objective is to maximize the operational profits, while in the short-term, the objective is to meet the

"firm load" requirement while minimizing the loss of "generating potential," using a novel definition of "generating potential," both the short-term and long-term operating problems are incorporated into a single nonlinear programming formulation.
In this paper, we discuss some of the distinctive features of the Alcan problem, and the computational techniques used to solve them. These results represent a "real life" application of large-scale nonlinear programming.

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A Suggested Extension of Special Ordered Sets to Non-Separable Non-Convex Programming Problems

J.A. Tomlin, Institute for Advanced Computation, Sunnyvale, Ca., U.S.A.

This paper suggests a branch and bound method for solving non-separable non-convex programming problems where the nonlinearities are piecewise linearly approximated using the standard simplicial subdivision of the hypercube. The method is based on the algorithm for Special Ordered Sets, used with separable problems, but involves using two different types of branches to achieve valid approximations.

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A B-Conjugate Method for Nonlinear Optimization Without Derivatives

M.L. Lenard, Boston University, Boston, Ma., U.S.A.

A method for nonlinear optimization without using derivatives is described. The method belongs to the class of B-conjugate methods proposed by Powell (1975) in which search directions are chosen to be conjugate to a matrix B, an approximation to the matrix of second partial derivatives of the objective function. The need for accurate one-dimensional optimization along the search directions is overcome by periodic use of an accelerating, or quasi-Newton, step. The rate of convergence is shown to be (n + 1)-step superlinear for a function of n variables. The results of some computational experience with the method are reported.

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A Dual Simplex Method for Function Minimization

M. Sato, Tokyo Institute of Technology, Tokyo, Japan

A new direct search method is proposed for function minimization, which is based on cooperative movements of a pair of simplexes. The two simplexes are first made, so that the function value at any vertex of the first simplex is smaller than that of any vertex of the second simplex. Then the second simplex is moved toward the centroid of the first simplex. The movement is achieved at all the vertices of the second simplex by three basic operations: reflection, contraction, and expansion. By iterating the above movements, all the vertices of the two simplexes converge gradually on the minimum point of the objective function.
The method is shown to be effective and compact as well as the simplex method by Nelder and Mead. Though the number of function evaluation of this method is a little larger than that of the simplex method, the basic operations at all the vertices of a simplex can be done independently, which shows the good adaptability of this method for parallel computation.

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Improving Hooke and Jeeves's Pattern Search Method

G. Barabino, M. Marchesi, C.N.R., Genova, Italy

Among optimization methods to minimize a real unconstrained function of n variables, one of the simplest and most suited to be implemented also on small computers is Hooke and Jeeve's pattern method.
A study was made to improve this algorithm in order to use it on mini-computers (word precision of 32 bits).
The algorithm was tested varying the factor R of reduction of the steps after a failure along all the n research directions, and introducing a new percentage factor Q, by which any single research step is increased or decreased after respectively a success or a failure of the exploration move in that direction.

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On the Control of Numerical Accuracy in Two-Level Optimization Methods

J.M. Szymanowski, Technical University of Warsaw, Warsaw, Poland

Analysis of two-level algorithms was carried out under the assumption that lower-level problems are solved inaccurately. A general scheme of two-level methods covering a wide class of existing methods was used. This approach has a number of advantages.
1) Convergence of these methods was proved under general assumptions about stop criteria in upper and lower-level algorithms. Optionally chosen minimization methods may be used in the scheme.
2) An accuracy selection algorithm was described and investigated theoretically and numerically. It is not a new two-level method but an algorithm organizing cooperation of minimization methods within the frame of a two-level method.
3) Propose algorithm is numerically valid i.e. each its step requires finite number of computations. Moreover, proper equilibration of stop criteria results in substantial savings of computations. Numerical experiments confirmed theoretical considerations.
4) Applications of the idea suggested during the lecture to
- some mathematical programming methods such as: the penalty methods, the gradient projection methods in minimax problem,
- on-line hierarchical control for steady control for steady-state systems.
5) The results of numerical experiments confirm the theoritical considerations and they show that the accuracy selection algorithm may be useful in various optimization methods.

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Computational Implications of Degeneracy in Large Scale Mathematical Programming

G.W. Graves, University of California, Los Angeles, Ca., U.S.A.
G.G. Brown, Naval Postgraduate School, Monterey, Ca., U.S.A.

The computational implications of degeneracy in the solution of large linear programming problems is an almost completely neglected subject in the literature. Although it has always been recognized that theoretically the bases can cycle in extreme point algorithms, it has become fashionable to categorically assert that this is of no practical significance. Our recent experience with large sparse systems indicates that 50% - 90% of all pivots are degenerate. Most computer codes that do have anti-degeneracy procedures almost invariably rely on completely arbitrary lexicographic ordering. The question of the efficiency of various lexicographic orderings is completely ignored. Efficiency cannot be measured in raw number of pivots alone. Design considerations, measures of effectiveness, supporting data structure, and computational experience will be presented.

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Accelerating Degenerate Simplex Method Iterations by Means of a Basis factorization Algorithm

A.F. Perold, IBM Thomas J. Watson Research Centre, Yorktown Heights, U.S.A.

The simplex method usually performs many degenerate iterations when used to solve models encountered in practice. We show how the time spent on such iterations can be significantly reduced by means of a basis factorization algorithm. This algorithm can be reinterpreted as one that takes only nondegenerate steps, with the direction at each step determined by an auxilliary homogeneous linear program having as many rows as there are basic variables at zero level in the current solution.

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A Primal Simplex Network Approach Which Circumvents Degeneracy

D.L. Adolphson, University of Washington, Seattle, Wa., U.S.A.

This paper is an extension of some previous work of Fong and Srinivasan on real (or non-degenerate) shadow prices for transportation problems. Real shadow prices are distinguished from ordinary shadow prices because they are always valid over a strictly positive amount of change even if the optimal solution is degenerate. This is true because real shadow prices are a property of the extreme point solution rather than a particular basis representation of the extreme point solution. The simplex approach presented in this paper computes real improvement potentials at each iteration in contrast to ordinary improvement potentials. The distinction between real and ordinary improvement potentials is the same as the distinction between real and ordinary shadow prices; namely, the real improvement potentials are a property of the current extreme point solution rather than a particular basis representation of the solution. Like real shadow prices, the real improvement potentials are always valid over a strictly positive range of change. The result of using real improvement potentials is a simplex approach in which each iteration corresponds to a different extreme point solution. The cost of eliminating degenerate pivot steps is the maintenance of a shortest path matrix used to compute real improvement potentials.

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An L.P. 'Crash' Procedure Developed Geometrically

A.J. Pryor, London School of Economics, London, England

In an L.P. problem geometric considerations indicate that the constraints and non-negativities whose hyperplanes make the smallest angles with the objective hyperplane are likely to be the binding constraints in the eventual solution.
A description will be given of the steps necessary to define a smaller L.P. problem, whose final basis is probably a good starting basis to the original L.P. problem.
Some computational evidence is included.

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Some Remarks About Degeneracy in Optimal Solution of Linear Programming Model

J. Merkel, University of Maria Curie-Sklodowska, Lublin, Poland

In a dual pair of linear problems values of the maximized objective function are less or equal to values of the minimized objective function. If these problems are uncontradictory, optimal values of both objective functions are equal. On may write:

$$\sum_j / \sum_i a_{ij} y_i^o - c_j / x_j^o = \sum_i / b_i - \sum_j a_{ij} x_j^o / y_i^o = 0$$

/the upper index "o" means the optimal solution /.
As a implication of this it is well known in economy that fully utilized production factors have positive shadow prices, partialy utilized - equal zero shadow prices / with a greater-than or less-than condition of primal problem associated dual variable shall be positive when above mentioned condition is equal zero or shall be equal zero when this condition has positive right hand-side /.
It seems that to this time was not as object of deeper interest the fact that fully utilized production factors may have zero equal shadow prices. This is characteristicaly for the dual pair of degenerated linear problems - with their alternative optimal solutions and shadow prices.

The case $b_i - \sum_j a_{ij} x_j^o = 0$ and simultaneously $y_i^o = 0$ is treated by author as a border case of inequality $b_i - \sum_j a_{ij} x_j^o \geq 0$, implicated $y_i^o = 0$. One may say about "tendency" of utilization the production factor at less level as the resource of that factor".
Conclusion: Fully utilized production factors have nonnegative shadow prices.

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Computational Experimentation with Primal Geometric Programming Algorithms

M.J. Rijckaert, Katholieke Universiteit Leuven, Leuven, Belgium

Recently, Dembo (1978) and Rijckaert et al. (1978) published results of two extensive numerical investigations into the effectiveness of different algorithmic approaches for geometric programming problems. The conclusions of the above mentionned studies were quite parallel. In fact they both stressed the excellent performance of primal codes.

The present talk will communicate the results of additional and complementary investigations in this area, without however strictly limiting itself to the geometric programming approach.

The topics discussed will mainly deal with

- the different alternative methods for estimating parameters and/or multipliers and their numerical efficiency.
- the possibility of hybrid codes (combination of different approaches in one code.)
- the problem of dealing with active constraints in primal geometric programming.

The conclusions of these numerical investigations will not only help potential users of geometric programming in selecting a code from among various available ones and in avoiding numerical difficulties while implementing them, but might also affect the strategy used for comparing different algorithms and might therefore be of interest in the more general non-linear programming context.

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Globally Convergent Algorithms for Convex Programming with Applications to Geometric Programming

E. Rosenberg, Stanford University, Stanford, Ca., U.S.A.

We consider solving a (minimization) convex program by sequentially solving a (minimization) convex approximating subproblem and then executing a line search. Each subproblem is constructed from the current estimate of a solution of the given problem, possibly together with other information. Under mild conditions, solving the current subproblem generates a descent direction for an exact penalty function. Minimizing the exact penalty function along the current descent direction provides a new estimate of a solution, and a new subproblem is formed. For any arbitrary starting estimate, this scheme generates a sequence of estimates that converges to a solution of the given problem. Moreover, it is not necessary to require that the functions defining the given problem and each subproblem be differentiable.

As an application, we propose a new globally convergent algorithm for geometric programming. The algorithm is particularly effective for solving large geometric programs: given a geometric program with degree of difficulty dd, where $dd \gg 0$, we take each approximating subproblem to be a geometric program with a common degree of difficulty $\overline{dd}$, where $\overline{dd} \ll dd$. When each subproblem is solved with a dual geometric programming code, our scheme is faster than directly solving the given geometric program with the same code.

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Fractional Programming - State of the Art

S. Schaible, University of Alberta, Edmonton, Canada

The paper deals with nonlinear programming problems where the objective function is the ratio of two given functions. These problems are commonly called fractional programs. Since the early paper by Charnes and Cooper on linear fractional programming a good number of papers has appeared. It is attempted to survey some of the results and to relate them to each other.

We start out with discussing some of the major areas of applications where fractional programs come up. From this we see that linear, quadratic and concave-convex fractional programs are of special interest. We then survey some of the major theoretical results including duality relations. The last part is devoted to algorithms in linear and nonlinear fractional programming. In particular parametric and nonparametric as well as dual methods are considered.

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Decomposition Methods for a Class of Non-Linear Fractional Programs

A. Cambini, L. Martein, L. Pellegrini, University of Pisa, Pisa, Italy

We have studied the problem $\min \{z(x) = f(x)(D(x))^p\}: x \varepsilon S\}$ where $f : R^n \to R$ is a C^1-convex function, $D(x)$ is an affine linear form, $p \varepsilon Z$ and S is a polytope of R^n.

We state some theoretical properties relative to the objective function and to the set of optimal solutions.

If $D(x) = \xi_k$, it becomes a convex problem, in which the objective function assumes as minimum value z_k.

An appropriate use of Farkas' lemma allows us to determine $\overline{x} \varepsilon S$ such that $f(\overline{x}) < z_k$. A sequential method is proposed for the problem when f is a linear or a convex quadratic function.

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Are Adobe Walls Optimal Phase Shift Filters?

C.V. Coffman, R.J. Duffin, G.P. Knowles, Carnegie-Mellon University, Pittsburg, Pa., U.S.A.

The adobe house construction gives an automatic, air conditioning effect because the rooms tend to be cool at midday and warm at night. Presumably this is brought about by the walls acting as a heat filter so that there is nearly a twelve hour phase lag. This raises the question of how to optimize the adobe phenomena by a suitable design of the walls. In this study it is supposed possible to make the walls of layered construction with layers having different thermal resistivity. Such a layered wall can be modeled electrically as ladder filler of capacitors and resistors. The input to the ladder is a sinusoidal voltage. Then the following question arises. If the filter capacitors have given values how should the resistors be choosen so that the output voltage has a given phase lag but least attenuation? It is found possible to answer this question by use of a special variational principle. Applying this analysis to building construction shows how to maximize fuctuation of interior temperature with a phase lag of twelve hours.

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Application of the GRECO Algorithm to the Optimal Generation Scheduling for Electric Power Systems

J.L.D. Faco, Pontificia Universidade Catolica do Rio de Janeiro, Rio de Janeiro, Brasil

The problem of the determination of the short range optimal generation scheduling for a complex hydrothermic electric power system, in a deterministic context, can be formulated as a Nonlinear Discrete Optimal Control problem with time lag and inequality constraints on the state and control variables, and solved by the algorithm GRECO-Gradient Reduit pour la Commande Optimale -- an extension of the Generalized Reduced Gradient Method (GRG) of Abadie and Carpentier for constrained Nonlinear Programming.
This extension consists mainly of keeping the basis matrix in a factored representation using specific sparsity techniques that take advantage of the block triangularity of the jacobian.
The choice of the independent variables in GRECO considers the constraints of the problem, resulting in different combinations of state and control variables along some parts of the trajectory. Moreover the use of LU decomposition (GAUSS) of the diagonal blocks and some heuristics in the re-inversions of the basis give better numerical accuracy and less growth of nonzeros in the factorization.
Some experiments with brazilian energy systems have been made using the code GRECO with successful results.

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Management Strategies for an Ecological System with Time Delays

S. Reed, W. Kilmer, University of Massachusetts, Amherst, Ma., U.S.A.

We consider a model for the control of a two-level ecosystem containing time delays. The system is composed of a grazer population which is harvested by man and a plant food base which renews itself according to a fixed time schedule following depletion by the grazers. Our model is a pair of equations, a nonlinear differential equation describing the grazer population dynamics, and a nonlinear integral equation describing the proportion of the food base which is depleted at each point in time. The grazer population is modelled by an augmented logistic equation:

$$X(t) = rX(t)\left[1 - \frac{X(t)}{K\ (1 - D(t))}\right] - U(t)X(t)$$

where r is the "intrinsic growth rate", K is the food base's maximum "carrying capacity" and U(t) is harvesting effort. The food depletion equation is:

$$D(t) = \int_{t-L}^{t} F\ [X(\tau),D(\tau)]\ P(t - \tau)d\tau$$

where $F(\tau)$ describes the depletion incurred at time τ and $P(\sigma)$ is the proportion of depletion incurred σ time units in the past which is still not renewed at the present time.
We have derived optimal control policies with economic objectives for a limited class of systems. Solutions to several optimization problems have been obtained by approximation, dynamic programming and heuristic search techniques. Some adaptive control schemes which operate when the parameters of the model are uncertain have also been developed.

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Long Term Decision, Optimal Control and Decision: The Breeder Case

E. Bringuier, Cournon d'Auvergne, France

The object of this communication is the study of the decomposition of an optimal control problem, in an uncertain future, the uncertainty being taken into account thanks to subjective probabilities. Within the period examined the uncertainty will progressively be lessened so the directing scheme is an eventuality-tree enabling to constrain the control vector. The reasoning will be made in terms of mean which will entail a linear structure in relation to subjective probabilities and so make possible a decomposition which will be effected through the theory of coordination.
The concrete theme serving as a basis for the study is the problem of the choice of investments for equipments to produce electricity, this problem being more acute since the possible introduction of breeders.

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Mathematical Programming and Complementarity Via Generalized Equations

S.M. Robinson, University of Wisconsin, Madison, Wi., U.S.A.

Generalized equations are point-to-set relations, involving a function and a normal-cone operator, which extend the usual framework of nonlinear equations. By using this extension, optimality conditions for nonlinear programming, as well as linear and nonlinear complementarity problems and more general problems of equilibrium, can be easily and efficiently expressed. In this expository lecture we describe some properties of generalized equations and relate these to the applications from which the generalized equations are derived (such as nonlinear programming). By using these techniques one can prove results, for example about the existence and stability of solutions, which are stronger than those previously known. We shall describe some of these results, and indicate directions in which work is currently progressing.

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Penalty Methods for Computing Points that Satisfy Second Order Necessary Conditions

A. Auslender, Universite de Clermont, Clermont, France

The goal of this paper is to compute points that satisfy the classical second order necessary conditions for the constrained case in nonlinear programming. The functions that define the nonlinear program are twice continuously differentiable.
A general convergence theorem is first presented. Then penalty techniques are given for solving the problem and convergence of these methods is proved with the use of the former theorem.

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A Unifying Framework for Duality Theory in Mathematical Programming

J. Tind, University of Aarhus, Aarhus, Denmark
L.A. Wolsey, CORE, Louvain-la-Neuve, Belgium

It has been shown by Gould that if one allows dual functions in place of dual variables, one can construct a dual of any mathematical program, so that weak and strong duality results hold for the primal dual pair. We show that interpreting the dual functions as price functions there is an underlying hierarchy of primal and dual problems which have a simple (traditional) economic interpretation. This hierarchy permits us to make precise the relationship between primal, dual and saddlepoint results in both the traditional Lagrangean and the more general theory, and to see the implications of passing from prices to price functions. Results for problems with special structure, such as convex and integer programs, follow very simply from the observation that the perturbation function of the primal problem is the natural optimal solution to the dual problem.

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Pseudo Duality in Mathematical Programming

U. Passy, S. Yutav, Israel Institute of Technology, Haifa, Israel

In this work non-convex programs are analyzed vie Legendre transformation. The first part includes definitions and the classification of programs that can be handled by the transformation. It is shown that differentiable functions that are represented as a sum of strictly concave and convex functions belong to this class. Conditions under which a function may have such representation are given. Pseudo duality is defined and the pseudo duality theorem for non linear programs with equality and inequality constraints is proved.
The techniques described are constructive ones, and they enable to calculate explicitly a pseudo dual once the primal program is given. Several examples are included. In the convex case these techniques enable the explicit calculation of a dual even in cases where the conjugate function can not explicitly be calculated.

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Minimizing a Differentiable Function Over a Differentiable Manifold

D. Gabay, Laboria-Iria, Le Chesnay, France

In this paper we consider the problem of minimizing a real valued function f of n variables on a constraint set G defined by $m < n$ non-linear equality constraints. Under the assumption of transversality of the mapping g: $R^n \to R^m$ defining C to the origin of R^n, C is an (n-m) sub-manifold of R^n, and we can construct a tubular neighborhood C_ε of C.
Once C is endowed with a Riemannian structure we can define the gradient of f_c, restriction of f to C, and gradient methods for the solution of the problem; they consist of a sequence of descent steps along arcs of geodesics of C.
If the Riemannian structure of C is the one induced by the orginary Euclidean metric of R^n, we obtain an idealized gradient projection method, while an idealized reduced gradient method is also included and corresponds to the natural Riemannian structure associated to the charts of C based upon the independent variables. This framework also allows to define Quasi-Newton methods when the variable Riemannian metric is based on an estimate of the Hession of f_c, of dimension (n-m). For practical implementation the arcs of geodesics are approximated by a step in the tangent space followed by a restoration phase to enforce feasibility; the convergence of such processes is analyzed using a metric in the tubular neighborhood C_ε of R^n extending the Riemannian metric on C.
This metric can then be used to prove (under generic assumptions) global convergence of algorithms combining at each step a move in the tangent space and a correction phase to only improve feasibility; they are thus simpler to implement than the previous class. In particular a Quasi-Newton type method is proposed and its superlinear convergence established; it can be interpreted as the solution of the original problem by a recursive sequence of quadratic programs in the spirit of Han-Powell, requiring only the updating of a (n-m) dimensional approximate Hessian. The relationship with multiplier methods for Augmented Lagrangians is also investigated.
This framework is particularly useful for optimal control of discrete time nonlinear dynamic systems. While the idealized reduced gradient corresponds to the classical gradient methods for the control problem, our Reduced Quasi-Newton method with feasibility improvement gives a new efficient method for the solution of such problems; since it does not require the solution of the system equations at each step, it seems particularly interesting for systems described by implicit dynamics, arising for instance, in many economic models.

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A Report on Future Directions in Routing and Scheduling of Vehicles and Crews

M. Ball, L. Bodin, S. Gass, B. Golden, University of Maryland, College Park, Md., U.S.A.

In June 1979, an international workshop was held at the University of Maryland to investigate future research directions in the routing and scheduling of vehicles and crews. The workshop was jointly sponsored by the National Science Foundation and the University of Maryland. The five major problem areas discussed at this meeting were as follows:

1. Testing and Evaluation of Heuristics;
2. Exact Mathematical Programming Solution of Routing and Scheduling Problems;
3. Heuristic Techniques for Routing and Scheduling;
4. Formulation and Structure of More Complex/Realistic Routing and Scheduling Problems;
5. Compilation and Review of NP-Complete Routing and Scheduling Problems.

The objective of the workshop was to provide an opportunity to focus on these and other issues relating to the theory and practice of Routing and Scheduling. Several papers presented at the workshop will appear in a special issue of Networks. In this paper, we report on progress made at this recent workshop and identify promising new directions for research.

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A Decomposition Algorithm for Vehicle Routing

M.L. Fisher, R. Jaikumar, University of Pennsylvania, Philadelphia, Pa., U.S.A.

We consider the problem of scheduling a fleet of delivery vehicles to meet fixed demands at minimum cost. We propose an algorithm based on decomposition of the problem into a generalized assignment problem to assign demand to vehicles and a travelling salesman problem for each vehicle to sequence the assigned demand. The algorithm iterates between solving these two problems. Bender's Decomposition is used to pass information about the cost of various vehicle routes from the travelling salesman problem to the generalized assignment problem.
The basic method can easily incorporate a variety of problem features including vehicle capacity constraints, time window constraints, and the possibility of incurring a penalty cost by not servicing particular demand points. There are also an enormous number of options possible in the basic method. The method can be used either as a heuristic or as an optimizing algorithm. It is not necessary to solve either the generalized assignment problem or the travelling salesman problems to optimality on every iteration. Finally, the entire procedure could be embedded within Branch and Bound. We will present the results of our computational experiments with these various options.

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A School Bus Scheduling System

J.A. Ferland, J-M. Rousseau, J. Desrosiers, Universite de Montreal, Montreal, Canada

Bus scheduling for urban and rural school systems often gives rise to large scale planning for more than 50 schools and 10,000 students. We describe a computer system for generating and scheduling bus routes for each school in a region. The system combines vehicle routing techniques and a heuristic for a large scale integer programming scheduling problem.
Our approach to these problems is hierarchical in nature. The first stage generates the routes to be used for each school by applying vehicle routing procedures as modified differently for urban and rural settings. The second stage schedules buses to the routes determined in the first stage with an objective of reducing the number of buses used. The system will fix the starting time of each school within a given interval. Computational results are presented.

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Model of Mail Distribution in Sweden

B. Bjorklund, Scandinavian Airlines System, Bromma, Sweden
G. Lundgren, Royal Institute of Technology, Stockholm, Sweden

First class mail distribution in Sweden handles some 4.5 million letters and parcels per day. The principal objective is to achieve overnight delivery between most pairs of cities and towns, at most about 2000 km apart. The present handling involves mail collection from 2000 peripheral post offices, sorting at some 80 regional centers, transportation from each center to each of some 40 other regional centers, renewed sorting to further level of detail and transportation to local post offices for final distribution to offices and private homes. Transportation involves truck, train and air.
A study of the postal distribution system has been carried out by the Contract Research Group of Applied Mathematics at the Institute of Technology in Stockholm. The aim of the study was i) to find and evaluate different structures of regional sorting centers and ii) to find solutions to the suggested and present transportation system between all pairs of regional centers.
A mathematical model of the distribution system has been formed, involving several tools of mathematical programming such as a modified shortest path algorithm, simple plant location and route scheduling. Several additional models of great detail have been formed to evaluate the different structural alternatives regarding total cost and service level.

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Critical Graphs, Matchings and Tours

W.R. Pulleyblank, University of Calgary, Calgary, Canada

Let P be a property possessed by some graphs. We say that a graph G is P-critical if G does not possess P but if whenever we delete a node of G the resulting graph does possess P. For many properties P these P-critical graphs have been shown to be very important. For example if P is the property of possessing a perfect matching then the P-critical graphs essentially determine the matching polytope.
Recently it has been shown that if P is the property of possessing a perfect 2-matching without small cycles, then the P-critical graphs again essentially determine the polytope of feasible solutions. Moreover several classical theorems of Tutte on matchings can be strengthened by expressing them in terms of P-critical graphs.
We discuss these results for various properties P and describe those properties for which P-critical graphs have been seen to be important and for which the P-critical graphs have been characterized.

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Sufficient Conditions for Graphs with Threshold Number Two

T. Ibaraki, Kyoto University, Kyoto, Japan
U. Peled, Columbia University, New York, N.Y., U.S.A.

The threshold number $t(G)$ of a graph $G = (V,E)$ is the smallest number of linear inequalities whose 0-1 solutions are precisely the characteristic vectors of the independent sets of vertices of G. The graph $G^* = (V^*,E^*)$ is such that $V^* = E$, and two vertices (pq) and (rs) of V^* are adjacent when $(pr), (qs) \notin E$ or $(ps), (qr) \notin E$. The chromatic number of G^* is denoted by $\chi(G^*)$. Chvatal and Hammer have shown that $t(G) = 1$ if and only if $\chi(G^*) = 1$, that $t(G) \geq \chi(G^*)$ and that computing $t(G)$ is NP-hard. We prove that $\chi(G^*) = 2$ implies $t(G) = 2$ provided one of the following conditions is satisfied: (i) V can be partitioned into a clique and an independent set; or (ii) G^* has at most two nonsingleton connected components. It is conjectured that no conditions such as (i) or (ii) are required.

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Antiblocker of Spanning and k-Spanning Arborescences of a Directed Graph

G. Calvillo, Banco de Mexico, S.A., Mexico, Mexico

This paper settles a question posed by Fulkerson; namely, which is the antiblocker of spanning arborescences. These results are obtained from the theory of branchings developed by Edmonds and Calvillo.

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Adjacency on the Postman Polyhedron

R. Giles, University of Kentucky, Lexington, Ky., U.S.A.

Let $G=(V,E)$ be a loopless, undirected graph and $C \subseteq V$ have even cardinality. A <u>postman set</u> is a subset $J \subseteq E$ such that for every node $v \in V$, the number of edges of J incident to v is odd if and only if $v \in C$. The <u>postman polyhedron</u> P(G) is the sum of the convex hull of all incidence vectors of postman sets and the nonnegative orthant R_+^E. We give a simple characterization of adjacency for vertices of P(G). An upper bound on the distance between two vertices, and hence the diameter of P(G), is given.

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Blocking Theory in Max-Algebra

L. Superville, The Graduate School of the CUNY, New York, N.Y., U.S.A.

Here, we first introduced an algebra of real numbers extended by $-\infty$ (negative infinity) in which the regular multiplication and addition is replaced by addition and maximum of two numbers respectively. We then define a concept of "blocking" in our new notation that generalizes the concept of blocking for sets described by Edmonds and Fulkerson. They showed that blocking is a dual notion. We generalize their blocking notion and show that the dual notion also holds in our generalization.

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Production Sets with Indivisibilities, a New Approach to Integer Programming

H. Scarf, Yale University, New Haven, Ct., U.S.A.

Abstract not available

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A Class of Methods for Nonlinear Programming

J. Abadie, Electricite de France, Clamart, France

A class of methods for solving the nonlinear programming problem is presented. All functions are assumed twice continuously differentiable. Particular members of this class are: quasi-Newton method for unconstrained optimization; Newton's or Gauss-Newton method for solving a system of nonlinear equations; reduced gradient or projected gradient method for linearly constrained optimization. Numerical experiments are presented.

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Computational Experience with Minos/Augmented

B. Murtagh, University of New South Wales, Kensington, Australia
M.A. Saunders, Stanford University, Stanford, Ca., U.S.A.

MINOS/AUGMENTED is a general-purpose Fortran code implementing various Lagrangian and augmented Lagrangian algorithms for the solution of sparse nonlinear programs. (Each algorithm solves a sequence of sparse linearly-constrained subproblems.)
The system will be described from the user's point of view. Standard matrix generators may be used to assemble the bulk of the data for a problem. Two subroutines are also required to specify nonlinear terms in the objective function and the constraints (along with their gradients). Results will be given for some significant practical problems. Assuming convergence occurs, computation time is typically within a moderate factor of the time required for linear programs of similar size.

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Computation of the Search Direction in Constrained Optimization Algorithms

W. Murray, M.H. Wright, Stanford University, Stanford, Ca., U.S.A.

A number of algorithms for constrained optimization are based on the general idea of choosing the search direction as the solution of a quadratic program. However, there is considerable variation in the precise nature of the quadratic program to be solved. Furthermore, significant differences exist in the procedures advocated to ensure that the search direction is well-defined, so that some algorithms abandon the quadratic programming approach for particular iterations under certain conditions. In this talk, we examine the merits of various proposals for computing the search direction in bounds-constrained, linearly constrained, and nonlinearly constrained optimization, together with an appraisal of their likely effectiveness.

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A Unified Theory for Nonlinear Constrained Optimization Methods by Geometric Approach

K. Tanabe, The Institute of Statistical Mathematics, Tokyo, Japan

This paper is concerned with robust methods which can produce convergence from a very poor initial estimate of the optimal solution of constrained optimization problems A differential geometric method is developed specifically to obtain robust algorithms without resorting to the penalty-type approach. In particular, a generic class of "feasibility-improving gradient acute projection methods" and their Levemberg-Marquardt-type modifications is developed. Each method in this class is an amalgamation of a generalized gradient projection method and a generalized Newton-Raphson method which respectively take care of improving the value of the objective function and satisfying constraint equations at the same time. A unified theory is developed for this class of method by using the concept of various generalized inverses and related projectors, which facilitates geometric interpretation of the existing algorithms. Analysis is also given to the continuous analogues of the methods to obtain robust algorithms, which gives us an insight into the global behavior of the related algorithms. Various new algorithms are derived from the general theory, that use the QR decomposition, the SVD and other decompositions of the Jacobian matrix of the constraint functions. Quasi-Newton algorithms which estimate projected Hessian matrix and in some cases require only approximations of nonnegative definite matrix of size n-m, are developed to enhance the local convergence, where n and m are numbers of variables and constraint equations respectively.

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A Comparison of Some Recent Methods for the Minimization of Unconstrained Nonlinear Functions

G.R. Lindfield, Y. Bhayat, The University of Aston, Birmingham, England

The main aim of this study is a comparison of unconstrained minimisation techniques which attempt either to relinquish entirely with the linear search stage of the minimisation procedure or limit the level of accuracy which has to be achieved.
The methods considered are the method of Davidon (1975), Fletcher (1970) and Norris and Gerkin (1977). The method of Fletcher although earlier than the others is included since it appears one of the most successful of the original attempts to limit the use of linear search stage. In addition the most recently implemented and available technique in the British National Algorithms Group (N.A.G.) library has been used in the comparison. This allows a check on how this widely available N.A.G. routine stands up to comparison with recent developments.
The four methods have been tested on a range of test problems of varying difficulty and numbers of variables. The comparison is made in terms of function and gradient evaluations and in terms of run time to permit as revealing a comparison as possible.

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Smoothing Techniques for Nondifferentiable Optimization

I. Zang, Tel-Aviv University, Tel-Aviv, Israel

We suggest approximations for smoothing out the kinks caused by the presence of 'max' or 'min' operators in many nonsmooth optimizations problems. These approximations replace the original problem in some neighborhoods of the kink points. These neighborhoods could be made arbitrarily small, thus leaving the original objective function unchanged at almost every point of R^n. The possible applications for these approximations include: min max problems, L_1 curve fitting problems, piecewise regression problems, and exact penalty functions for constrained optimization. I some of the above application, it is shown that the maximal possible difference between the optimal values of the approximate problem and the original one, is determined a priori by fixing the value of a single parameter. The approximations introduced preserve convexity properties and make the approximate problem up to four time continuously differentiable provided that the functions composing the original problem have got the same properties. This enables the use of efficient gradient techniques in the solution process. Some numerical results will be presented.

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A Second Order Method to Solve the (Constrained) Minimax Problem

A.R. Conn, University of Waterloo, Waterloo, Canada

A new algorithm is presented for solving the (constrained) nonlinear minimax problem $\min(\max(f_i(x): i=1,\dots,m): x \varepsilon R^n)$ subject to $\phi_i(x) \geq 0$ $i=1,\dots,m'$, $\psi_i(x) = 0$ $i=1,\dots,m''$. The algorithm is a natural generalization of the earlier, first order, technique of Conn and Charalambous. The method is globally convergent with an asymptotic convergence rate that is superlinear.
The proposed implementation is numerically stable and involves the computation of non-orthogonal projections using QR and LTL^T factorizations and rank-one updates.
Some numerical results will be included.

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Minimax Optimization Combining LP Methods and Quasi-Newton Methods

J. Hald, K. Madsen, Technical University of Denmark, Lyngby, Denmark

We present 2-stage algorithms for nonlinear minimax optimization, which require first order partial derivatives. Let $\underline{x}^*$ be a solution of our problem. If the minimax objective function is non-differentiable in $\underline{x}^*$ along any direction through $\underline{x}^*$, then a quadratic final rate of convergence can be obtained using only first derivatives without approximating second derivatives (stage-1 iteration). However, if the objective function is smooth along some directions, then some second derivative information is required in order to obtain a fast final convergence. Under some regularity conditions a quasi-Newton algorithm (stage-2) approximating the Hessian-matrix of the minimax Lagrangian function has been shown to yield superlinear

final convergence. Our algorithms always start in stage-1, and if a slow final rate of convergence is detected, then a switch to stage-2 is made. In case stage-2 does not converge then stage-1 is re-entered. Several switches are allowed. The algorithms have been shown to possess sure convergence properties. We have considered different strategies for updating the Hessian matrix. The algorithms are illustrated by some numerical examples.

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Quasi-Newton Methods for Minimax Optimization

C. Charalambous, O. Moharram,
University of Waterloo, Waterloo, Canada

The minimax optimization problem may be stated as minimize $F(x) = \max(f_i(x):\ i \in M)$ where $[M] = [1, 2, \ldots, m]$, $x = [x_1, x_2, \ldots, x_n]^T$ and $f_1(x), \ldots, f_m(x)$ are in general nonlinear functions with respect to the variables $x_1, \ldots, x_n$.
The objective function F(x) is in general a non-differentiable function. Thus we cannot use directly the well-known gradient methods to minimize F(x). The purpose of the present paper is to introduce some ideas of how to solve the minimax problem using the wealth of information which is already available on the Newton or Quasi-Newton methods for unconstrained optimization. Numerical results will also be presented.

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Projected Lagrangian Algorithms for the Nonlinear Minimax and L1 Problems

W. Murray, M.L. Overton, Stanford University, Stanford, Ca., U.S.A.

The minimax and ℓ_1 problems are both unconstrained optimization problems whose objective functions are not differentiable everywhere. It is well known that both problems can be transformed into constrained optimization problems whose objective and constraint functions are differentiable. These equivalent problems (EP) do, however, have special properties which are ignored if solved by applying a general purpose routine. In this talk we present algorithms which solve the minimax and ℓ_1 problems via EP but which do so in a manner that fully utilizes the special properties of EP. The algorithms are based on projected Lagrangian methods and incorporate a special linear search for nondifferentiable functions. One of the concerns in designing the algorithms is that satisfactory performance should not be limited to a small neighborhood around the minimum.

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Minimization Methods for the Sum of a Convex Function and a Continuously Differentiable Function

M. Fukushima, Kyoto University, Kyoto, Japan

Two algorithms are presented to minimize a general function which is the sum of a convex function and a continuously differentiable function. Specifically, the problem to be considered is

$$\text{minimize} \quad \phi(x) \triangleq f(x) + g(x) \quad \text{over} \quad x \in R^n,$$

where $g : R^n \to (-\infty, +\infty]$ is a closed proper convex function and $f : R^n \to (-\infty, +\infty]$ is a function continuously differentiable on an open set containing dom g. The objective function ϕ is thus in general neither convex nor differentiable. It is noted that this class of problems contains, as a special case, the problem of minimizing a continuously differentiable function over a closed convex set. In view of this, the first proposed algorithm is regarded as a natural extension of the Frank-Wolfe method to the more general problems. On the other hand, the second algorithm is a generalized version of the proximal minimization algorithm for convex functions. Convergence of the algorithms is proved and the rate of convergence is discussed.

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A Secant Approximation Method for Convex Optimization

R.R. Meyer, The University of Wisconsin, Madison, Wi., U.S.A.

A secant approximation method for convex programming is described. This method employs at each iteration a convex piecewise-linear approximation of the objective determined by the objective function values at (n+1) points for a function of n variables. If the constraints of the original problem are linear, then each approximating problem is an LP. The algorithm may be considered to be a generalization of an iterative separable programming method to the non-separable case.

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Uses of a Minimax Model for Nondifferentiable Optimization

R.S. Womersley, University of Dundee, Dundee, Scotland, U.K.

A nondifferentiable function is one with discontinuities in its gradient. For such functions the smooth quadratic model which is used by most of the recognized optimization methods is inappropriate. A suitable local model for a nondifferentiable function is the minimax function. The method to be considered attempts to follow surfaces of nondifferentiability and use a quasi-Newton approximation to the curvature within this surface. These methods have applications to the minimization of exact penalty functions for nonlinear programming problems, the minimax solution of systems of equations and to the minimization of more general nondifferentiable functions.

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Two-Segment Separable Programming

R.R. Meyer, The University of Wisconsin, Madison, Wi., U.S.A.

A new iterative separable programming method for convex optimization is described. The method differs significantly from existing separable programming techniques in that it employs a piecewise-linear approximation of at most two segments for each objective function term at each iteration. The optimal values of the approximating problems can be shown to converge to the optimal value of the original problem. Computational experience with this method on a variety of problems (including a model of an actual water supply system that involves about 500 variables and 500 constraints) has shown it to be very efficient and rapidly convergent.

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On the Dynamics of Concave Input/Output Processes

J.J.M. Evers, Twente University of Technology, Enschede, The Netherlands

Formally we define an I/O-process as a function $\mu: S \subset R^m \times R^n \to R$ satisfying the axioms:

i) $S \subset R^m_+ \times R^n$ (R^m_+ refers to the nonnegative orthant of R^m

ii) For all $(x,y) \in S$: $x \in R^m$, $y \in R^n$: for all $\bar{x} \in R^m$, $\bar{x} \geq x$: $(\bar{x},y) \in S$, $\mu(\bar{x};y) \geq \mu(x;y)$

iii) $\mu: S \to R$ is closed concave (i.e. its hypograph is a closed convex set).

In an economic context, an I/O-process can be taken as the description of a process (or a complex of processes) where S represents the set of feasible input/output combinations $(x,y) \in R^m \times R^n$ and where $\mu(x;y)$ represents the corresponding utility value. Then, (i) postulates nonnegativity of the inputs, and (ii) postulates free disposal concerning inputs.

A so-called dual I/O-process ${}^*\mu: {}^*S \subset R^m \times R^n \to R$ assigned by:

$\mu^*(u;v) := \sup_{(x,y) \in S} \mu(x;y) - u.x + v.y$,

${}^*S := \{(u,v) \in R^m \times R^n \mid \mu^*(u;v) < +\infty\}$.

In the expression: $\mu(x.y) - u.x + v.y$. the vectors u and v can be conceived as vectors of input prices and output prices resp., and consequently $\mu(x.y) - u.x. - v.y$ represents the value of an input/output combination (x.y). In case $\mu: S \to R^1$ is an I/O-process it appears that $(-{}^*\mu): {}^*S \to R$ is an I/O-process.

Concerning the dynamics we restrict ourselves to maximization problems generated by one single I/O-process $\mu: S \subset R^m \times R^m \to R$, where trajectories $(x^t,y^t)_1^\infty := (x^1,y^1,x^2,y^2,\ldots) \in S \times S \times S \times \ldots$ are considered satisfying: $x^1 = r$, $\rho x^{t+1} \leq y^t - s^t$, $t = 1,q,\ldots,$ given $\rho \in R$, $\rho > 0$ representing a scaling factor, $r \in R^m$ representing the initial state, and given the vectors $s^t \in R^m$, $t = 1,2,\ldots$. The corresponding sets of these "feasible" trajectories are denoted $F(\rho,r,s(s^t)_1^\infty)$, i.e.:

(1) $F(\rho,r,(s^t)_1^\infty) := \{(x^t,y^t)_1^\infty \mid (x^t,y^t) \in S,\ x^1 = r,$

$\rho x^{t+1} \leq y^t - s^t,\ t = 1,2,\ldots)$.

Given an exponential time discount factor $\pi \in]0,1/\rho[$ and "valuation" vectors ${}^*s^t \in R^m$, $t = 1,2,\ldots$ a trajectory $(\hat{x}^t,\hat{y}^t)_1^\infty \in F(\rho,r,(s^t)_1^\infty)$ is called optimal is there ne $(x^t,t^t)_1^\infty \in F(\rho,r,(s^t)_1^\infty)$ such that for some period T and some $\varepsilon \in R$, $\varepsilon > o$: $\sum_{t=1}^h (\pi\rho)^t (\mu(x^t;y^t) + {}^*s^t.y^t) \geq \varepsilon + \sum_{t+1}^h (\pi\rho)^t (\mu(\hat{x}^t;\hat{y}^t) + {}^*s^t.\hat{y}^t)$, $h = T, t+1, \ldots$. Clearly, in this "not worse on the long run" criterion, convergency of the series $(\sum_{t=1}^h (\pi\rho)^t (\mu(x^t;y^t) + {}^*s^t.y^t)\}_{h=1}^\infty$ is not required.

Because of the symmetry between an I/O-process and its dual, it is possible to assign a dual problem possessing the same structure as the original dynamic sustem. Under simple and weak "strictly" primal and dual deasibility assumptions the following results are deduced: (i) boundedness of trajectories and converfency of the series of objective functions, as a necessary condition for optimality, (ii) the existence of optimal trajectories, both for the primal and dual problem, (iii) normality, (iv) finite horizon approximation, (v) upper semi-continuity of the sets of optimal trajectories with respect to the vectors r, $(s^t)_1$, $({}^*s^1)_1$, (vi) stability of optimal trajectories, and finally (vii) under convergency of $\{s^t\}_1^\infty$, $\{{}^*s^t\}^\infty$, vectors (x,y), (u,v) exist such that primal and dual optimal trajectories $(x^t,y^t)_1^\infty$, $(u^t,v^t)_1^\infty$ converge in the sense of $(x^t,y^t) \to (x,y)$, $(u^t,v^t) \to (u,v)$ if $t \to \infty$.

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Equilibria in Non-Cooperative Games and Competitive Economies

S.D. Flam, Universitetet i Bergen, Bergen, Norway

We demonstrate the existence of an equilibrium in certain non-cooperative games with an arbitrary set of players. The results are applied to the two-players, zero-sum case and give a kind of generalized Farkas lemma. Finally we give an application to production economies where individual choice and welfare depends on actions taken by the other agents. No explicit demand functions are used and we do not assume non-satiation.

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Linear-Quadratic Stochastic Games

G.J. Olsder, Twente University of Technology, Enschede, The Netherlands

A two-person differential game, not necessarily zero-sum, is considered in which the state is affected by noise. Both players, P1 and P2, know the system characteristics and have access to noisy measurements which are not necessarily the same. Player P1 does not know P2's measurements and vice versa.

The differential game we consider has linear dynamics (continuous time) and quadratic pay-offs. Optimal Nash strategies are calculated within the class of linear functionals of the available measurements. Sufficiency conditions will be given for the existence of this solution, which turns out -- for zero-sum games -- to be the unique saddlepoint in a much broader class of strategies.

The method of solution essentially contains the following steps. First the state space (R^n) is extended to an infinite dimensional Hilbert space. Subsequently two one-sided infinite dimensional optimal control problems are solved. The existence of solution is proved by application of a contraction theorem. In order to be able to work exclusively in a Hilbert space setting, L_2-white noise is used, as opposed to the conventional Wiener process approach.

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Dubovitskii-Milyutin Theory for Differential Games

M. Brokate, Freie Universitat Berlin, Berlin, West Germany

We discuss the derivation of maximum principles in Hamiltonian form for deterministic nonlinear closed-loop differential games via the state space approach of Dubovitskii-Milyutin.
If we treat the feedback law as an additional equality constraint, then (except in the simple C^1-case) this constraint is not differentiable in any reasonable strategy space topology. We formulate auxiliary problems where the singular surfaces enter as side conditions and the optimal strategies are C^1, and apply the usual function space multiplier rules. Transforming back to the original problem we get the looked-for global maximum principle for the original discontinuous optimal strategies where as expected the normals of the singular surfaces enter the adjoint equation. The technical difficulties associated with Lyusternik's theorem and uniqueness of Lagrange multipliers (needed for the minimax principle in the 2-person-zero-sum case) are handled by suitably chosen additional control functions in the auxiliary problems. For single and double transition surfaces (which usually are the first object of study) we get the full classical results.

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On Repeated Games with Incomplete Information Played by non- Bayesian Players

N. Megiddo, Northwestern University, Evanston, Il., U.S.A.

The following model is considered. A two-person zero-sum game G in the matrix form is repeatedly played infinitely many times. A player may be absolutely uninformed as to what game G is being played. The only information revealed to him at the end of each stage, is the payoff. The central question is: Can an uninformed player guarantee that his long-run average payoff be the value of the game? This question is answered in the affirmative and a specific strategy is described, such that with probability one, for every game G, the liminf, as n tends to infinity, of the average payoff for the first n stages will be not less than the value of the game G. The probability measure regards only the internal randomization of the strategy. No probabilistic assumptions are made with respect to the space of games.

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Strategic Planning by Mathematical Programming

P. Bod, Hugarian Academy of Sciences, Budapest, Hungary

Experts of the Mathematical Institute of the Hungarian Academy of Sciences and of the Hungarian Aluminium Trust are dealing for a long time with the elaboration of a mathematical programming type model for the preparation of structural decisions concerning the long run development of the Hungarian aluminium industry.
The model building has been finished under completely specified form last year. The quantification of the parameters should be done in this year and the first experimental computations are provided for 1980.
The basic ideas of the model will be expounded. Role and place of the model in the system of Hungarian long run national planning models will be outlined.
The long run economic and technological interdependences prevailing in the aluminium industry are highly non linear and often non convex. It will be shown how we try to describe these in the framework of a large scale, multiperiodic, mixed-integer linear programming system.

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Allocation and Control of Radio and TV Production Via Quantitative Management Methods

F.A. Lootsma, University of Technology, Delft, The Netherlands

The presentation is concerned with a series of projects carried out in the largest Product Division of the Philips Concern (a multinational electronics firm with its headquarters in Eindhoven, Netherlands) to support the production allocation and the production-level control in the European factories. The presentation covers a period of some twenty years, and shows the particular strengths and weaknesses of the methods that have been employed: (a) (a) cost minimization via mathematical programming in order to draw up long-term plans, (b) industrial dynamics, information systems, and dynamic programming for smooth medium-term control of the production level, and (c) an organizational approach to support the introduction of new products and a new production technology. The failures of these projects are analyzed in depth, and particular attention is given to the European and the American attitude with respect to mathematical modelling and quantitative management methods in the area of medium-term, long-term, and strategic planning. A discussion of E.F. Schuhmacher's and J. von Neumann's viewpoints on the role of mathematics will conclude the presentation.

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A Planning Model for a Vertically Integrated Forest Industry

O. Barros, A. Weintraub, Universidad de Chile, Santiago, Chile

The tactical planning problem for a vertically integrated forest industry is analyzed. The industry jointly manages forest lands, a pulp plant and a sawing mill. Basic planning decisions include the managing (thinning, harvesting, planting, etc.) of timber lands, selling (exporting) of logs, assignment of timber to and production level of the plant and mill. Due to the slow growth of timber, these decisions should be made considering a long planning horizon.
A Linear Programming model for this tactical planning problem is presented, in which strategic, financial, silvicultural, access and technological characteristics are considered. The model is supported by a sophisticated computational system that, starting with very basic and simple timber land, plant characteristics and economic data, produces the L.P. model matrix in MPSX format. The system also includes a report writer.
Results and experience with the model at a large forest industry in Chile, which is using it on a routine basis, are reported. Besides normal tactical planning, the model has proved to be a useful tool for strategic planning such as timber land adquisition or plant expansion.

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Use of Linear Programming in Integrated Planning - A Case Study from the Egyptian Industry

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This paper presents an LP model intended for planning the annual activities of a coking coal manufacturing company in Egypt. The model is a multi-time period model which produces the optimum level of the followings: components of the blend in the batteries and supply of the blended components. The supply of the blended components includes the following activities: placing orders for importing coal, arrival of coal to Alexandria and temporary storage there, transport of coal to the factory by several modes of known available capacities and costs, and finally storage on the factory premises until its use in the batteries.
The model was run, expost, for the three past years to assess its possible contribution to cost reduction as well as to smoothing inventories and avoidance of bottlenecks. Subsequently, it was used for planning the 1979 activities. In the paper we describe the various stages of data collection, formulation and programming and computer implementation. The Matrix Generator, Report Writer and the Revised Simplex Code were all done in FORTRAN IV, we briefly describe some of their characteristics. We also demonstrate the practical lessons gained by the company's management through implementing the model.

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An LP-Model for Natural Processes with Seasonal Fluctuations in Production and/or Marketing Possibilities

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The present model was developed, originally, for the purpose of planning greenhouse operations. The solution of this planning problem calls for a model which, among other things, takes the following into consideration: 1. it should be able to handle the horizon problem caused by the fact that a given plan will tie up resources for a long future period (perhaps several years); 2. it should exploit the known seasonal changes in production patterns and expected fluctuations in market prices; 3. it should facilitate the adaptation to the emergence of new products and new production technology. For natural processes a full seasonal cycle is usually one year and we use an activity analysis model where, for a given product, a corresponding production activity is defined for each week of the year. For each activity we specify its expected economic contribution as well as its input requirements, week per week. The concept of an equilibrium model is introduced. The solution of the equilibrium model maximizes average return per year over an infinite horizon. However, the main contribution of the equilibrium solution is that it provides us with the proper terminal conditions for the solution of the finite horizon problem. The model presented is now being used by several of the major firms in the Danish greenhouse industry.

399

Capacitated Plant Location and Network Problems with Integer Variables

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The difficult capacitated plant location problem, with possible additional constraints on the integer variables (supplied or generated), has been found to be quite tractable via a direct dual method. We resolve the dual of a strong relaxed form of the problem. The approach constructs dual feasible solutions in a number of ascent or exchange steps, possibly iterated when the situation warrants, and possibly followed by standard dual improvement and relaxation methods. An interesting feature is an imbedded linear program, which is often found to be necessary and is easy to resolve. It avoids the dimensionality difficulties of the original relaxed problem, and it accomodates the integer side constraints well.
Primal solutions are obtained from the orthogonality relations and a greedy enumerative phase. We have solved a variety of problems (up to 201 plants and 201 destinations) to within 5-10% of optimality, even with a slow APL implementation. Much larger problems appear to be within reach.
We have extended our work to a class of quite general network problems with integer variables. This necessitates substantially enlarged primal and dual formulations which are interesting in their own rights in that they could be employed as starting points for various relaxation schemes. Preliminary computations suggest that our direct dual method can be extended to treat the larger class of problems. An interesting result is that certain bounds on the shipments will have to be obtained in a preprocessing of the network if the subsequent application of the algorithm is to be maximally effective.

400

A Canonical Representation of Uncapacitated Plant Location Problems and Its Applications

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We consider a location problem whose mathematical formulation is $\max \{v(S) : |S| = K\}$ where $v(S) = \sum_{i \in I} \max_{j \in S} c_{ij}$ and $C = (c_{ij})$ is any real matrix with row index set $I = \{1,\dots,m\}$ and column index set $N = \{1,\dots,n\}$ and $S \subseteq N$. A set function of the form v(S) is called a location function. We give a constructive proof that a set function w is a location function if and only if it can be represented in the canonical form $w(S) = \sum(r_T : T \cap S \neq \phi)$ with $r_T \geq 0$ for all $T \subset N$. The proof is also a polynomial algorithm for reducing the matrix C to the canonical form r_T, $T \subseteq N$. Using this canonical representation we perform a probabilistic analysis of the location problem. In particular we show that for some simple probability distributions several popular heuristics nearly always solve the location problem optimally. Finally we give a worst-case result. We show that any procedure that uses matrix C only to calculate values of the function v cannot with a number of values polynomial in n guarantee to find one of the best n^{δ} solutions of the location problem for any $\delta < 1/2$.

401

An Exact Algorithm for Minimizing Delivery and Operating Costs in Depot Location

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The problem of locating a single depot among n points is considered. The objective is to minimize the sum of depot operating cost and delivery cost. The best depot location is found by means of an exact algorithm that determines simultaneously both the best depot location and the associated optimal delivery routes. A global integer programming formulation of the problem is given; the model is solved by relaxing most of its constraints and by introducing them only when they are violated. Two versions of the algorithm have been applied successfully to random problems ranging from 20 to 50 points.

402

Variations on a Theme of Weber - 1. Theorems

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The well-known Weber problem consists in locating one or several facilities with respect to fixed supply and demand points in order to minimize total transportation cost. For the unconstrained case, it is shown that:
i) a solution to the problem with several facilities and a single norm can be found in the convex hull H of the supply and demand points;
ii) the octogonal hull 0 of the supply and demand points contains one solution to the problem with one or several facilities when different l_p - norms are used.
For the constrained case, we prove that:
i) a solution to the problem with one facility and one norm exists in the set constituted by the intersection of the convex hull H and the feasible set S and by the points of S - H visible from H - S;
ii) this set can be further reduced in the Euclidean and rectilinear cases.

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Variations on a Theme of Weber - 2. Algorithms

D. Peeters, Universite Catholique de Louvain, Louvain, Belgium

The simple Weber problem consists in locating one facility with respect to fixed supply and demand points in order to minimize the total transportation cost. An efficient algorithm is proposed to solve this problem when transportation costs are given by convex functions of the distance covered and in the case where the set of possible locations for the facility is limited to some regions of R^2. An algorithm is also presented for the case where the transportation costs are non-decreasing with the distance but not necessarily convex.

404

Matroid Matching and Applications

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Given a matroid and a partition of its elements into pairs, we would like to find the maximum number of these pairs whose union is in the matroid. This problem, proposed by Edmonds and Lawler, has many interesting special cases and applications, like the matroid intersection problem and the matching problem. The main results presented in this talk are: (1) Every algorithm which solves the matroid matching problem for every matroid has exponential running time in the worse case; (2) If the matroid is given by the columns of a matrix, then there is a polynomial-bounded algorithm to solve it (although the exponent is about 10, so the algorithm is too slow in practice). (3) Again if the matroid is given by the columns of a matrix, then there is an algorithm involving random steps, which gives the correct answer with probability arbitrarily close to 1 fast (in $O(n^3)$ simple arithmetic operations).

405

Integer Rounding for Polymatroid and Branching Optimization Problems

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Where matrix $M \geq 0$ and vector $w \geq 0$ have rational entries, define $r^*(w) = \max\{1 \cdot y: yM \leq w, y \geq 0\}$, $z^*(w) = \max\{1 \cdot y: yM \leq w, y \geq 0, y \text{ integral}\}$. Integer round-down holds for M if, for all integral $w \geq 0$, $\lfloor r^*(w) \rfloor = z^*(w)$. Similarly, when $\lceil r_*(w) \rceil = z_*(w)$ for all integral $w \geq 0$, where $r_*(w) = \min\{1 \cdot y: yM \geq w, y \geq 0\}$, $z_*(w) = \min\{1 \cdot y: yM \geq w, y \geq 0, y \text{ integral}\}$, integer round-up holds for M. The integer round-down and round-up properties are shown to hold for certain matrices related to integral polymatroids and branchings in directed graphs. In particular, a "covering" analogue for Edmonds' theorem concerning edge-disjoint packings of rooted branchings is given, and polymatroid generalizations of combinatorial min-max and max-min theorems for matroids (involving integer rounding) are developed.

406

On the Number of Complementary Bases in a Matroid

J. Fonlupt, Universite de Grenoble, Grenoble, France

Let $\mathcal{M} = (E, \mathcal{F})$ be a matroid having elements labelled by:

$$E = \{a_1, a_2, \ldots, a_n, \bar{a}_1, \bar{a}_2, \ldots \bar{a}_n\}.$$

Suppose that $B_0 = \{a_1, a_2, \ldots, a_n\}$ is a basis of $\mathcal{M}$
A basis of $\mathcal{M}$ is called a complementary basis if it contains one element of each pair $\{a_i, \bar{a}_i\}$ $1 \leq i \leq n$.
$\mathcal{M}$ will be called irreducible if for any set

$$\{a_{i_1}, a_{i_2}, \ldots, a_{i_k}\} \subset \{a_1, a_2, \ldots, a_n\}$$

$$\{\overline{a_{i_1}}, \overline{a_{i_2}}, \ldots, \overline{a_{i_k}}\} \not\subset \text{closure } \{a_{i_1}, a_{i_2}, \ldots, a_{i_k}\}$$

Suppose that $\mathcal{M}$ is irreducible.

Suppose at least that $M=(E,F)$ is the direct sum of exactly α matroids $(\alpha \geq 1)$

$$= \sum_{i=1}^{\alpha} M_i.$$

With these conditions the following result will be established.
The number of complementary bases of M is at least equal to $n+\alpha$. Moreover this bound is the best possible.
This gives a positive answer to the conjecture of D.J. KLEITMAN on the number of complementary trees in a graph.

407
Leontief Substitution Systems and Matroid Complexes
J.S. Provan, SUNY, Stony Brook, N.Y., U.S.A.
L.J. Billera, Cornell University, Ithaca, N.Y., U.S.A.

A pre-Leontief substitution system is the collection of nonnegative variables x satisfying Ax = b, where b is a nonnegative vector and A is a matrix containing at most one positive element in each column. We investigate the face structure of a pre-Leontief substitution system whose underlying polyhedron is simple. A characterization of such a polyhedron is given for the bounded case (called a totally pre-Leontief substitution system by Dantzig and Veinott), and it is shown that the dual simplicial complex always comprises the independent sets of a matroid. The number of such (isomorphically different) polyhedra is given, as a function of the dimension and the number of facets. Conversely, a characterization is given for the class of matroids whose independent sets form the dual complex to a polyhedron, and it is shown that this polyhedron can always be chosen to be a (not necessarily bounded) pre-Leontief substitution system. The number of such matroids is also given, as a function of the rank and the cardinality of the base set.

408
Some Remarks Concerning Combinatorial Optimization Problems
M.W. Padberg, City University of New York, New York, N.Y., U.S.A.

Abstract not available

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Path Following in Fixed Points and Solutions
W.I. Zangwill, University of Chicago, Chicago, Il., U.S.A.

Traditionally, much work in the fixed point and equation solution field has been done via simplices, triangulations, and piecewise linear approximations. A new book by C.B. Garcia and myself, now in draft form, suggests an approach by path following, the path being a differential curve. This new approach is conceptually easier, more elegant and yields more powerful results. The two approaches will be compared and some recent results summarized. For example, an algorithm which combines ideas from both approaches will be presented. This algorithm, the flex-simplicial, converges by a different theoretical process than previously; that is, it does not utilize a complementarity argument at all. Moreover, the contrast between simplicial and differential equation techniques is clarified by the path following approach. Plus greater orientation information is obtained. These and other observations about the path following methodology in the book will be presented.

410
A Constructive Proof of Tucker's Combinatorial Lemma
M.J. Todd, Cornell University, Ithaca, N.Y., U.S.A.

Tucker's combinatorial lemma is concerned with certain labellings of the vertices of a triangulation of the n-ball. It can be used as a basis for the proof of antipodal fixed-point theorems in the same way that Sperner's lemma yields Brouwer's theorem. Here we give a constructive proof, which thereby yields algorithms for antipodal fixed-point problems Our method is based on an algorithm of Reiser.

411
Monotone Complementarity Problems
L. Mclinden, University of Illinois, Urbana, Il., U.S.A.

A number of results are presented for a broad class of nonlinear complementarity problems, namely, those problems involving a maximal monotone multifunction. The only assumption is that the image of the positive orthant under the multifunction intersects the positive orthant. The solution set is then nonempty compact convex, and a certain parametrized class of approximate complementarity problems possesses unique solutions. Varying the parameters enables one to specify uniquely natural trajectories of solutions to these approximate problems. These trajectories are both continuous and "monotone" in the parameters, and when the original problem is nondegenerate they converge to some particular solution (even though there may in general be many solutions). This limiting solution is characterized in terms of its additional special properties. The nature of the convergence is analyzed in detail, and the results suggest that numerical approaches based on the theory may not be unreasonable. Stability and generic properties are also developed.

412
A Unification and Generalization of the Eaves and Kojima Fixed Point Representations of the Complementarity Problem
S-C. Fang, E.L. Peterson, Northwestern University, Evanston, Il., U.S.A.

Although the mappings used in the Eaves and Kojima fixed point representations appear to be different, this paper

shows that they are essentially the same -- a unification that is accomplished via a geometric programming argument in the more general setting of the "geometric complementarity problem".

413

Graphs Related to the Fixed Point Algorithms

L. Filus, Central School of Planning and Statistics, Warsaw, Poland

We treat the fixed point algorithms of H. Scarf, H.W. Kuhn, "the homotopy method" and "the sandwich method" as the procedures to obtain finite sequences of almost complete/with respect to some label function/sets which ends up with a complete set in some family P of sets.
In this way we can look at the various procedures used in those algorithms as at a property of the replacement relation defined on the family P. This relation is symmetric and irreflexive which makes it possible to treat members of P as vertices of some non-oriented graph and the replacement relation as the set of arcs of the graph. In this way we are able not only to obtain the results which can be used in proofs of generalization of theorems on sequences of sets appointed by the algorithms mentioned above, but also to answer some graphtheoretic questions.

414

An Algorithm for Solving Bilinear Knapsack Problems

H. Konno, University of Tsukuba, Tsukuba, Japan

This paper introduces 0-1 bilinear knapsack problems (BK) and proposes a finitely convergent cutting plane algorithm for solving them. (BK) is a special type of integer quadratic programming problem to be defined below:

$$\begin{cases} \text{maximize} & \sum_{i=1}^{m} c_i x_i + \sum_{j=1}^{n} d_j y_j + \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_i y_j \\ \text{subject to} & \sum_{i=1}^{m} a_i x_i \le a_o, \quad x_i = 0 \text{ or } 1, \quad i=1,\dots,m. \\ & \sum_{j=1}^{n} b_j y_j \le b_o, \quad y_j = 0 \text{ or } 1, \quad j=1,\dots,n. \end{cases}$$

where a_i's and b_j's are positive integers and c_i's, d_j's are integers. This problem has applications in bipartite matching problems, cutting stock problems, multi-attribute utility analysis to name only a few. The main purpose of this paper is to develop a finitely convergent cutting plane algorithm which parallels the one proposed for bilinear-linear programming problems by the present author. We show that we have to solve a sequence of knapsack problems and 0-1 integer programs to obtain local maxima, but that we do not have to generate expensive cuts to guarantee finite convergence. We will show in addition, that an efficient dynamic programming algorithm can be constructed if $m=n$ and $c_{ij}=0$ for $i\neq j$. Computational results will also be reported.

415

A Stability Concept of 0-1 Programming Problems and its Implications

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O. Oguz, Middle East Technical University, Ankara, Turkey

A stability concept for 0-1 multiple knapsack problems is introduced. The concept measures those subsets of variables which can be fixed at their current values in some optimal solution, extending the reduction concepts of single variables. The implications of stability on the computational difficulty of enumeration techniques is discussed along with its use in improving the performance bounds of approximation algorithms.

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On the Collapsing 0-1 Knapsack Problem

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G. Plateau, Universite de Lille I, Villeneuve d'Ascq, France

This non-linear 0-1 knapsack problem -denoted by (P)- (the right-hand side of the constraint is a monotone non-increasing function of the sum of the variables) is solved by using a branch and bound algorithm which starts with an efficient reduction phase; it includes
-the determination of lower and upper bounds for the sum of the n variables (the four associated methods which do not need any systematic data sorting algorithm, have a time complexity O(n))
-the construction of a convex hull of points in Z x Z which allows the introduction of two types of linearized 0-1 knapsack problems:
(i) the lower one leads to greedy algorithms for a lower bound for v(P) (optimal value of (P))
(ii) the solving of the relaxation -complexity O(n)- of the upper one gives an upper bound for v(P) and leads to the fixing of variables by using a reduction scheme based on lagrangean relaxations and constraints aggregations.
Extensive computational results terminate the communication.

417

Probabilistic Analysis of the Knapsack Problem

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C. Puech, Universite R. Descartes, Paris, France

This paper deals with the knapsack problem under probabilistic assumptions about the values of coefficients.
By the "Knapsack Problem" (KP) we mean the selection of a subset of n couples (weight;profit) in order to maximize total profit, without exceeding a given capacity.
We analyze the probabilistic behaviour of some algorithms based on the following basic steps: 1- obtain the greedy solution. 2- improve the greedy solution by adjoining one further couple. 3- reduce the size of the problem by lagrangean techniques. 4- perform a partial enumeration of the reduced problem solutions.
We make three different probabilistic assumptions, all considering uniform distribution on an integer interval (d,..,C) for the coefficients and on (1,....,nC) for the capacity: a- weights proportional to profits and d=1;

b- weights and profits independent, d=1; c- weights and profits independent, d≠1.
Then, we study the distribution of the absolute error of the solutions given by the greedy algorithm and by the linear time algorithm A (steps 1 and 2). From this we derive that A almost surely finds an optimal solution to (KP) when C is constant.
Further, we show that algorithms (based on steps 1,2,3 and 4) exist solving (KP) almost surely in time $O(n^{1+a}.C)$, for any a greater than 0.
Most of the results are not only asymptotic but precise evaluations of probabilities; so, our analysis explains the behaviour of real algorithms on usual test problems used for performance evaluations.

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On Multiconstrained Knapsack Problem

Y.P. Aneja, V. Aggarwal, K.P.K. Nair, University of New Brunswick, Fredericton, Canada

Multiconstrained knapsack problem is of considerable interest in mathematical programming besides the fact that it has several interesting applications. In a recent work a modified branch and bound technique has been provided for solving this problem. In this paper first it is shown that the problem is equivalent to shortest chain subject several additional linear constraints, and incidentally this proves the NP - completeness of the latter problem. Then a labelling scheme developed by the authors for solving the multiconstrained shortest chain problem is utilised for the multiconstrained knapsack problem. Computational experiences are included.

419

Reducing Micro-Data Files by Optimization or by Stratified Sampling

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This report considers the problem of constructing micro-data files that portray an accurate image of a very large population database, such as the population of taxpayers in the U.S., without all of the atomic details for each individual record. The primary goal is building a computerized file large enough to capture the basic structure of the population so that accurate policy analyses involving this data can be conducted, but small enough for practical considerations. This problem is called micro-data file reduction.
The typical methods for generating micro-data files are variants of stratified sampling. Herein, an alternative approach is described using a deterministic optimization model. As an objective, the optimization model minimizes the loss of information between the population and the micro-data file; a mathematical objective function is defined to achieve this goal. Although the resulting model is very large indeed, an efficient discrete algorithm known as subgradient optimization is presented for solving problems of this sort. In an investigation of the 1975 Statistics of Income File, it is shown that the optimization approach depicts the original data more closely and with few records than the sampling approach. The results of these experiments are provided.

420

Optimal Location of Files and Programs in Computer Networks

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The problem of Optimal Location of Files and Programs in Computer Networks can be stated as follows. Given a Computer Network with a certain number of nodes (Computer facilities), and a certain number of files and programs, decide how many copies of each file and program have to be stored in the system, and where to locate them. Files are accessed by the users through programs. The traffic requirement from each node to each pair program/file, is given. The communication and storage costs are known. The objective is to minimize the total cost of the system.
A linear Binary Programming model of the problem is formulated, in which two different types of traffic are explicitely considered: query traffic and update traffic. The query traffic corresponds to transactions from an user to one copy of a given file via one copy of a given program. The update traffic corresponds to transactions from an user to all the copies of a given file via one or more copies of a given program.
The problem is decomposed into a Master Problem and a certain number of Subproblems, where the Subproblems are very easy network flow problems. The Master Problem is relaxed into a nonlinear (convex) network flow problem, which is solved through a primal type procedure.
A generalized Benders decomposition approach is used in order to find a suboptimal solution to the problem. The procedure also yields a lower bound for the optimal value of the objective function.
The results of some computational experiments are illustrated.

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Multiprocessor Scheduling Using a Directed Acyclic Graph and a Shortest Path Algorithm

C.C. Price, Stephen F. Austin State University, Nacogdoches, Tx., U.S.A.

The problem of assigning computer program modules to functionally similar processors in a distributed computer network is investigated. The modules of a program must be assigned among processors in such a way as to minimize interprocessor communication while taking advantage of affinities of certain modules to particular processors. This problem is formulated as a zero-one quadratic programming problem, but is more conveniently modeled using a directed acyclic search graph. The model is developed and a backward shortest path algorithm is described which produces an assignment.
The problem has been shown to be NP-complete, and therefore approximate solutions are considered acceptable. When the search graph exhibits certain identifiable structural properties, the labeling algorithm is guaranteed to produce an optimal assignment.
Two alternative approaches to the solution of this scheduling problem are suggested. A non-backtracking branch-and-bound heuristic procedure, using a local neighborhood search at each stage, is described which searches only selected portions of the graph and generally yields a suboptimal assignment. An iterative procedure is defined and a fixed point theorem is proved which guarantees convergence for this method.
Finally, an analysis of the computational complexity of these algorithms is given, and results of performance experiments are reported.

422

Programming Techniques Supportive of Better Communication Between the User and the Analyst in the Construction of Computerized Models

D.M. Homa, Banister Continental Ltd., Edmonton, Canada

The paper describes three programming techniques that encourage better communication between the user and the analyst in the construction of computerized models. In the example given, the techniques are applied to a program for corporate acquisition analysis and mergers; it was successfully used during the summer of 1978 for a multimillion dollar acquisition. First, a section of coding that deals solely with the relationships between the variables of the model must be isolated and coded as simply as possible and in a particular manner. The analyst discusses and shows only this simple routine to the user; the rest of the program is carefully set aside so as not to clutter the dialogue. Second, the program allows the user to assign multiple values to as many of the variables as he wishes without being swamped with output. This is achieved by designating the assigned variables as being either "full-cyclic" or "semi-cyclic" and paying attention to the order of their assignment. And third, the user is able to produce reports that are "tailor-made" to his input specifications. There is complete freedom in specifying the order of printing the selected variables and associated results. Key combinations may be further selected to product auxiliary reports in greater detail such as a consolidated balance sheet.

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Creating Optimal Composite Microdata Files with Large-Scale Mathematical Programming

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J.S. Turner, Oklahoma State University, Stillwater, Ok., U.S.A.

This paper will describe the results of over four years of research, development, implementation, and use of a system which will optimally merge microdata files. The merge process requires solving transportation problems with up to 50,000 constraints and 60 million variables, and is now being performed on a production basis. The resultant statistical data files fuel U.S. tax policy evaluation models which are used today for design and analysis of Federal legislation. Computational experience with this pioneering optimization software will be described as will empirical research into the statistical distortions caused by suboptimization.

424

Average Costs Denumerable State Semi-Markov Decision Problems with Unbounded Costs, Recurrence Conditions and Optimality Results

H.C. Tijms, Free University, Amsterdam, The Netherlands

We consider an average costs denumerable state semi-Markov decision model with compact metric action sets where the one-step costs are unbounded. We first discuss a number of recurrence conditions on the underlying set of denumerable Markov matrices. For the unichained case we establish a rather complete theory for the average costs optimality equation and the existence of an optimal stationary policy.

425

Monte Carlo Simulation of Optimization Algorithms

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This paper discusses the possibility of extending some of the techniques of Mathematical Programming to the case when one deals with random functions.
The task to be performed by the proposed algorithms is that of determining local optima of the expected value of the random function in question. The iterations are produced solely on the grounds of random observations of the function and/or its random gradient. The proposed procedures establish a compromise between the ideas of Stochastic Approximation and those of the classical Mathematical Programming. As opposed to deterministic techniques, the present methodology produces random sequences of minimizers in parameter space. The present work is based upon the idea of Algorithm Model of Polak and its stochastic counterpart on Kushner. The algorithms have been motivated mainly by problems arising in Discrete Time Optimal Control Jacobson & Mayne. It is proved that the produced random sequence converges with probability one to the desirable set (where necessary conditions for optimality hold).

426

Duality in Chance Constrained Programming

L.C. Maclean, Dalhousie University, Halifax, Canada

This paper focuses on the formulation of a duality theory for chance constrained programming models. In the approach here, a chance constrained model is taken as a disjunction of stochastic convex programming problems. Each of these sub-problems has a well defined dual problem. These latter problems are used to define a dual problem to the chance constrained problem as a conjunction of convex programming problems. This conjunctive dual is then represented in the chance constrained format. Applications to price theory in mathematical models of economic growth are considered.

427

Stochastic Approximation Minimization Procedures with Dependent Disturbances

P.J. Szablowski, Technical University of Warsaw, Warsaw, Poland

At first the following problem is considered. Let (Ω, F, P) be a given probability space. Let $F_n : R^k X\Omega \longrightarrow R^u$ be a sequence of functions such that $\forall X \epsilon R^k$, $F_n(x, .)$ is a random variable.
The convergence properties of the following procedure

$$X_{n+1} = X_n - \mu_n F_{n+1} (X_n, \omega); \; n = 0, 1 \qquad (1)$$

$\mu_o = 1, \forall_n \; \mu_n \epsilon \; (0,1)$ are discussed.

It is shown that if (1) converges than it converges to the set of points in R^k defined by

$$\{X \epsilon R^k : \lim_{n \to \infty} \frac{\Sigma a_i F_{i+1}(X,\omega)}{\Sigma a_i} = 0 \text{ a.s}\} (2)$$

where $a_o = 1$, $a_n = \mu_n / \Pi (1 - \mu_i)$

Hence in particular if $X_o \epsilon R^k$ $\forall_n$ $E F_n(X_o,\omega) = 0$ a.s.

then procedure (1) would converge to unique common zero of the functions $G_n(x) = EF_n(X,\omega)$ and sequence $\{F_n(X, \omega) - G_n(x)\}$ would represent measurements corrupting disturbances. Condition in (2) states that noise (at least at the point X_o) must satisfy sort of generalized strong law of large numbers.

After recalling results concerning other conditions which must be satisfied by the functions $\{F_n(x,\omega)\}$ new results concerning the following problems are presented. Given a sequence at random variables $\{Y_n\}$, what are the possible sequences $\{\alpha_i\}$ for which we have

$$\Sigma \alpha_i Y_{i+1} / \Sigma \alpha_i \longrightarrow 0.\text{a.s.} \quad (3)$$

(Thus that problem concerns proper choice of coefficients $\{\mu_n\}$ in (1)).

Given sequence $\{\alpha_n\}$ what conditions must be imposed on the sequence of r. vectors $\{Y_n\}$ in order to satisfy (3). It turns out that not only sequences of independent, uncorrelated or strongly stationary random vectors satisfy (3). Conditions imposed on Y_n are expressed in terms of variance and mutual covariance matrices.

Last part of the paper concerns applications of the developed theory to stochastic minimization problems i.e. to generalizations of Kiefer-Wolfowitz-like stochastic approximation procedures.

428

Service Policies in a Queueing Network with Exponential Service Stations

Z. Rosberg, CORE, Louvain-la-Neuve, Belgium

We consider a queueing network system of the type appearing for example in Jackson (1963) and Baskett et al. (1975), with different classes of customers. No underlying service policy is assumed and the mean cost of the system per unit time is chosen to be the loss function.

In queueing network systems, stationary probability distributions of the multiplicative form under certain service policies and their insensitivity property have been investigated. However, no work has been done as yet for finding optimal or good policies in queueing networks, although an extensive study of this question has been carried out for a single server station. The aim of this paper is to study this question for a queueing network system. The problem of selecting a good service policy arises especially in queueing network models for computer systems.

The following results of the study will be presented:

(1) Two lower bounds for the loss function under any stationary service policy.

(2) An upper bound for the loss function under the optimal service policy.

(3) A heuristic service policy which is found to be a good one in some numerical examples.

(4) A class of service policies which reduce the value of the loss function in comparison with the bounds from (1) and (2). This class is obtained from a special representation of the loss function under any stationary policy. The representation is found by using the technique used in Klimov (1974) for a single server station.

(5) A practical method of reducing the loss function and finding the service policy using the class given in (4) and simulation, is given and programmed for computer use. Also some examples of computer models are given.

429

Entropy in Linear Programs

S. Erlander, Linkoping Institute of Technology, Linkoping, Sweden

Entropy constrained linear programs have applications in transportation planning (the gravity model) and picture reconstruction. Conditions are given for the existence of solutions of the form

$$x_j = \exp\{(\beta^T a_j - c_j)/n\}, \; n > 0$$

to the entropy constrained linear program

$$\min c^T x \quad \text{subject to} \quad Ax = b,$$

$$- x \log x \geq H^0.$$

Newton's method is used to derive an iterative algorithm for obtaining solutions to the problem above.

430

Extremal Programming and the Principal-Agent Problem

G.J. Koehler, A.B. Whinston, Purdue University, Lafayette, In., U.S.A.

A class of problems known as principal-agent problems is currently receiving considerable attention in the economic, financial, and public policy areas. In such problems, a principal gives up his decision making privileges to an agent. The agent acts to maximize his own expected utility. However, a sharing schedule of the output is determined by the principal in advance. The schedule is called a contract. The contract is chosen with the agent's actions anticipated. Variants of the problem arise due to information limitations. While several qualitative properties of solutions to principal-agent problems are known, relatively little has been done on actually solving such problems. In this talk we present an extremal value optimization algorithm for solving a broad class of multi-period principal-agent problems. The procedure is also applicable to certain hierarchical stochastic control problems and some related controlled input-output problems.

431

A Study on an Optimal Allocation Problem Having a Strictly Convex Cost Function

A. Ouchi, I. Kaji, Hokkaido University, Sapporo, Japan

The optimal allocation problem (APC) considered is to minimize a strictly convex cost function subject to a total resource constraint and an upper and lower bound constraint for each resource.
Applying the Lagrange relaxation method, we can derive the dual function associated with the APC and an equation which the optimal dual variable must satisfy. Then we discuss some properties of the equation and perform the sensitivity analysis with respect to the upper and lower bounds on the variables. Finally, we propose three algorithms for solving the derived equation and show some computational results for the APC where cost functions are quadratic. These results reveal the effectiveness of the algorithm.

432

Sylvester's Minimum Circle Problem Revisited

D.W. Hearn, J. Jesunathadas, University of Florida, Gainesville, Fl., U.S.A.

J.J. Sylvester, in 1857, first posed the problem of constructing the minimum circle enclosing a given set of points in the plane. The problem has been shown to be an interesting special case of quadratic programming with applications in location theory. Recently, several primal algorithms have been proposed as alternatives to the dual algorithm of Elzinga and Hearn. In this paper, we compare these methods both theoretically and computationally and give a new finite algorithm for the weighted version of the problem.

433

A Limiting Infisup Theorem

C.E. Blair, R.J. Duffin, Carnegie-Mellon University, Pittsburgh, Pa., U.S.A.
R.G. Jeroslow, Georgia Institute of Technology, Atlanta, Ga., U.S.A.

We show that duality gaps can be closed under broad hypotheses in minimax problems, provided certain changes are made in the maximin part which increase its value. The primary device is to add a linear perturbation to the saddle function, and send it to zero in the limit. Suprema replace maxima, and infima replace minima. In addition to the usual convexity-concavity assumptions on the saddle function and the sets, a form of semi-reflexivity is required for one of the two spaces of the saddle function. A sharpening of our result is possible when one of the spaces is finite-dimensional.

434

Polynomial Algorithms for Computing the Smith and Hermite Normal Forms of an Integer Matrix

A. Bachem, Universitat Bonn, Bonn, West Germany
R. Kannan, Cornell University, Ithaca, N.Y., U.S.A.

Recently, Frumkin pointed out that none of the well-known algorithms that transform an integer matrix into Smith or Hermite normal form is known to be polynomially bounded in its running time. In fact, Blankinship noticed - as an empirical fact - that intermediate numbers may become quite large during standard calculations of these canonical forms. Here we present new algorithms in which both the number of algebraic operations and the number of (binary) digits of all intermediate numbers are bounded by polynomials in the length of the input data (assumed to be encoded in binary). These algorithms also find the multiplier-matrices K, U' and K' such that AK and U'AK' are the Hermite and Smith normal forms of the given matrix A. This provides the first proof that multipliers with small enough entries exist.

435

Inverse Matrix Representation with One Triangular Array

S. Ursic, Madison, Wi., U.S.A.

Let U * A = L, with U upper and L lower triangular arrays of order N. Algorithms are presented that use A and U as input to compute the matrix-vector products A^{-1} * b and b^T * A^{-1} with N^2 multiplications and additions. Array U can be computed with N^3 / 3 multiplications with a technique that avoids the computation of L.
Standard Gaussian Elimination simultaneously computes U and L as follows: start with I * A = A, where I is the identity matrix; perform identical linear combinations of rows on I and on the right-hand side array A; gradually transform I into U and A into L. At an intermediate stage, where A has not yet been fully triangularized, we have L' * A = U'. L' and U' represent one of the pairs of arrays present before performing each linear combination of rows. We only need two elements of L' to compute each linear combination of rows of U', namely the element to be zeroed and the diagonal element on the same column. Compute them with a scalar product of the appropriate row of U' and column of A'. Hence none of the intermediate arrays L' are explicitly needed. I propose calling the technique "IMPLICIT GAUSSIAN ELIMINATION".
The arithmetic performed during an Implicit Gaussian Triangularization has a nice geometric interpretation, similar in character to a Simplex Method search.

436

Invariance and the Product of the Eigenvalues

J. Donato, Syracuse, N.Y., U.S.A.

In economic and econometric theories, the invariances are based on the sum of the eigenvalues. Both theories incorporate surfaces into their foundations. This suggests that

the Fundamental Theorem of Surfaces be presented. Following Carl Gauss's work of 1827 the First and Second Fundamental Forms with their corresponding coefficients are outlined. Gauss's search for the invariance lead him to consider (1) the sum of two eigenvalues (the mean curvature of the surface) and (2) the product of two eigenvalues (the Gaussian curvature of the surface). Gauss concludes that the invariant property is determined by the product of the eigenvalues; the Gaussian curvature is invariant under arbitrary isometric deformation of a curved surface.
Economic and econometric theories are reformulated in terms of the Gaussian curvature. The variational principle is applied to an arc length, the shortest curve (geodesic) between two points on a surface.
As a example an economic model is presented. The space coordinates in dollars represents the microeconomic unit's wealth. The curvature of the unit's utility surface describes the economic interaction. Suppliers and demanders of wealth perform a transformation process in various coordinate systems. The geometric-economic result is that the Gaussian curvature reflects the interest rate of the economic system.
In econometric theory the quadratic form is written as the sum of squares by the application of some orthogonal transformation where the eigenvalues are summed. The geometric influence is suppressed.
In the new formulation the geometric influence is clearly considered. The data points are existing on a surface; they are "co-variant" in a differential geometric view. The "co-variant" matrix is a function of the space coordinates. The dispersion of the data points reflects the existance of the Gaussian curvature.

437

Size of Intermediate Numbers in Certain Numerical Computations

R. Kannan, Cornell University, Ithaca, N.Y., U.S.A.
A. Bachem, Universitat Bonn, Bonn, West Germany

The numerical computations considered are column and row reduction procedures that triangularise or diagonalise a matrix composed of all integers preserving integrality of the entries. These procedures help find the rank, the determinant, the inverse, the Hermite normal form and the Smith normal form of the matrix with complete accuracy (with no rounding). We discuss a couple of known procedures (the only ones found in the literature to our knowledge) and then show how our already published polynomial time bounded algorithms for the Smith and Hermite normal form computations can also be used to advantage to find the other three quantities. The main concern in these algorithms is that all intermediate numbers that are handled must have their number of digits (when written in binary notation) bounded by a polynomial in the length of the input matrix (whose entries are assumed to be coded in binary too). The algorithms discussed in addition to ensuring that the number of arithmetic operations is bounded by a polynomial in the length of the input, also ensure the fact mentioned in the last sentence. This criterion on the size of numbers is generally not proved to be met by most algorithms which is surprising considering that an algorithm is not polynomial unless this is ensured.

438

Theorems of Alternative and Decomposition of Convex and Nonconvex Programs

F. Giannessi, University of Pisa, Pisa, Italy

A theorem of alternative for nonlinear system is considered under general assumptions both for the unknown and for the involved functions. Connections with duality and some convexification aspects are discussed. Applications to (vector) extremum problems are briefly considered from a general point of view.
Particular attention is then devoted to a class of quadratic and complementarity problems. A decomposition approach (based on the preceding theory) is defined, so that the given problem is reduced to a finite sequence of easier problems.

439

Tangential Approximations in Abstract Multiplier Rules

R.J. Gardner, D.H. Martin, NRI, Pretoria, South Africa

Most abstract multiplier rules involve one or more notions of convex tangential approximant to a set in a linear space, and the multiplier rule is essentially a restatement of the fact that these notions of tangential approximant jointly satisfy what we call the Intersection Principle. For example, the fundamental theorem of the method of tents (V.G. Boltyanskii, Russian Math. Surveys 30: 3 (1975, 1-54; Theorem 12) states simply that any number of local tents jointly satisfy the Intersection Principle. The multiplier rules of Dubovitskii-Milyutin (1963), Canon-Cullum-Polak (1966), Hestenes (1966), Neustadt (1966), Halkin (1970), Lasieka (1978) may all be viewed in this way.
Guided by Boltyanskii's definition of local tents, we introduce localization as a general operation in $\mathbb{R}$, and show that if a list of tangential approximation notions jointly satisfies the Intersection Principle, then so does the corresponding localized list. Applying this localization procedure to Neustadt's notion of a first-order convex approximation and to Halkin's k-convex approximations produces new tangential approximation notions in $\mathbb{R}$ which we call indicating (weak indicating) cones respectively, and we prove that any number of indicating cones, one of which may be weak, satisfies the Intersection Principle. This result immediately enables us to strengthen the multiplier rules of Hestenes and of Boltyanskii.

440

On Tents and Other Notions of Tangential Conical Approximants

G.G. Watkins, University of South Africa, Pretoria, South Africa

After 1965 most attempts to formulate and derive necessary conditions for abstract optimization problems involved the local approximation of the constraint sets by convex cones, so as to permit the application of standard separation theorems.
Many such tangential conical approximants occur in the literature, mostly restricted to R^n, and in this paper

we present the full partial ordering by generality for the following notions of tangential approximant: internal cone [INT] (Neustadt, 1965); derived cone [DC] (Hestenes, 1965); linearisation of the second kind [L2] (Canon, Cullum and Polak, 1965); first order convex approximation [FCA] (Neustadt, 1966); k-convex approximation [CA] (Halkin, 1970), tent [T] (Boltyanskii, 1972); and local tent [LT] (Boltyanskii, 1974).
The partial ordering which we establish for sets in $\mathbb{R}^n$ is as follows:

$$\text{INT} \Rightarrow \text{T} \Rightarrow \text{DC} \begin{array}{l} \nearrow \text{L2} \Rightarrow \text{LT} \\ \searrow \text{FCA} \Rightarrow \text{CA} \end{array}$$

where the notation A ⇒ B implies that if a convex cone K is an approximation of type A then it is also an approximation of type B. It is also shown, by means of a series of counter examples, that no implication holds which is not shown, or implied, above. Furthermore all these notions coincide for cones having non-empty interior.
An appraisal of existing multiplier rules will be given, suggesting the overall superiority (in R^n) of local tents.

441

Sufficient Optimality Criteria in Nonlinear Optimization

A. Capozzi, Universita degli Studi di Pisa, Pisa, Italy
C. Resina, Universita degli Studi di Bari, Bari, Italy

In this paper we consider the problem of non-linear programming min $(f(x):\ x \in K)$ with $K = \{x \in R^n:\ g_i(x) \leq 0,\ i=1, \ldots, m\}$, where $f:\ F^n \to R$ is non-linear and continuously differentiable, and $g_i:\ R^n \to R (i=1,\ldots,m)$ are pseudoconvex and continuously differentiable.
We study the problem of generalizing the optimality criteria of Kuhn-Tucker. The work of Resina is extended to the case of any non-linear objective function.
Having the above-mentioned assumptions, in fact, we prove a theorem which establishes sufficient optimality criteria for a given point, which verifies Kuhn-Tucker conditions, to be a solution of the given problem.
Furthermore, we describe an efficient algorithm for the determination of the minimum of the function.
Then, we extend these results to a class of functions defined in infinite-dimensional spaces to resolve the problem in continuous time programming for generalized convex functions.

442

Peruse: An Interactive System for Mathematical Programs

R.P. O'Neill, W. Kurator, Department of Energy, Washington, D.C., U.S.A.

This paper describes the design, implementation, testing, and use of Peruse, an interactive software system for obtaining information from mathematical programs. This system has been found useful for debugging applications of mathematical programming, quickly obtaining small amounts of pertinent information and auditing model content or structure. The command syntax is described and examples are given. The matrix-solution data structure that compactly stores the matrix and solution information is presented. Future enhancements are discussed.

443

A Linear Programming Algorithm for a Parallel Processing Computer

B.L. Stults, D.R. Landolt, Burroughs Corporation, Radnor, Pa., U.S.A.

This paper presents an algorithm for solving large-scale linear programming problems on the Burroughs Scientific Processor - a computer with parallel processing architecture. The unsuitability of the standard implementation of the revised simplex method is indicated and a new approach is given. The approach is a composite of old and new methods which are efficient on the BSP despite an increase in the amount of computation required. Computational results with an initial implementation of the algorithm on a conventional serial processor will be presented.

444

An Equivalence Concept for Establishing Linear Programs

W.R.S. Sutherland, Dalhousie University, Halifax, Canada

There are numerous examples of problems which upon reformulation are solvable as linear programs. This talk presents a general concept of equivalent problems, essentially problems of similar type which have the same optimal solution, and shows how this equivalence concept determines a system of linear constraints. The Simplex method can then be seen as transforming the original problem through a sequence of equivalent problems. When the linear objective is based on ideas from vector maximization, the Simplex method terminates with a particularly simple equivalent problem in which the optimal solution is obvious. The optimal assignment problem and the Markovian dynamic programming problem are presented as examples of this approach.

445

Validity of Simulations: Optimal Simulation Lengths for Means Computation by Various Algorithms

M. Becker, Universite de Paris, Paris, France

Simulations compute temporal means of a large number of random variables in order to estimate their expected values. So, simulations usually need the computations of the means of large sequences of floating numbers. The limited precision of the computer introduces errors, whose propagation causes an important bias.
Two different cases are considered: computers using truncated floating numbers and computers rounding off.
The bias due to the truncation errors, when computing the mean of a sequence of floating variables, is calculated, in two different ways: a first approximation estimate is given, then an exact derivation is made.
An estimation of the bias is given for the rounding off case. A comparison between the two cases is presented.
Various ways are considered that reduce the bias.
An example is given that fits the theory.
A precise calculation of the bias for various algorithms of means computations leads us to the notion of optimal precision and of optimal simulation length. It is proved that it is wrong to assume that the precision is improving

when the simulation length is increasing.
The choice of a convenient algorithm and the choice of the simulation length appear to be most important.

446

New Developments in Matrix / Report Generators

C.A. Haverly, Haverly Systems Inc., Denville, N.J., U.S.A.

A new family of matrix/report generators have been developed. They include systems known as OMNI and PDS. They are based on optimizing compiler theory and usually execute faster than Fortran or PL/1. They have many user-oriented features to facilitate model building. They also utilize data base approaches to manage the voluminous data frequently required for large scale math programming application. Language details and some typical applications will be discussed.

447

Principles of Apportionment

H.P. Young, IIASA, Laxenburg, Austria

Apportionment theory deals with the question: what is the best of way to allocate a given number of indivisible entities proportionally to given fractional claims on the whole? As such it is closely related to integer approximation problems. A particular application is the allocation of seats in a legislature according to the populations of different regions (or according to vote totals of different parties). This talk surveys four basic fairness principles for apportionment: uniformity, house monotonicity, population monotonicity, and satisfying quota.
We discuss the interrelations between these principles, and the allocation procedures they imply. In particular, it will be shown that not all four properties can be satisfied simultaneously, so that some compromise with principle must be made. Fortunately, the compromise is not too serious, since there exists a method (first suggested by Daniel Webster) which satisfies the first three principles absolutely, and satisfies quota "almost all" of the time.

448

Nonlinear Programming Estimation of the Cost of Decent Subsistence

J.L. Balintfy, University of Massachusetts, Amherst, Ma., U.S.A.

Experimental evidence suggests that food preference is a concave unimodal function of eating frequency and consequently preference maximized human diets can be planned by nonlinear programming. The dual solution of such models shows the marginal preference with respect to the constraints applied and it was found in particular that the dual value of the cost constraint is positive within a problem specific range of minimum and maximum cost levels. It was also found that for properly formulated problems there exists a unique cost level within this range where the marginal preference with respect to calories is zero. This cost level is called the cost of decent subsistence and it can be found for preference maximized diet models by parametric search. Since the food preference function parameters are derived from survey ratings, these ratings become indirect estimates of the cost of decent subsistence via the nonlinear programming model. Estimates for several populations in several applications are presented. It appears that the cost of decent subsistence is very sensitive to tastes, prices and dietary needs.

449

Dynamic Water Quality Management Using Nonlinear Ecological Models

Y. Smeers, S. Olivieri, CORE, Louvain-la-Neuve, Belgium

Basin water quality management has been investigated extensively in the literature. The existing models are usually based on the two following assumptions:
(i) the water quality indices are state variables of a linear differential equation river model and (ii) the treatment plants are chosen so as to minimise the overall cost of meeting the quality standards during low flow conditions. In this paper we present a general mathematical programming framework which deals with these two assumptions : non linear ecological models can be taken into account and time varying operation of the treatment plants is allowed. The proposed method is based on a particularised version of the Generalized Reduced Gradient Method and exploits the special structure of the constraints of the problem.

450

Menu Scheduling by Lagrangian Relaxation

J.K. Balintfy, University of Massachusetts, Amherst, Ma., U.S.A.
P. Sinha, University of Georgia, Athens, Ga., U.S.A.

Earlier work has established that menu planning can be modeled as a nonlinear programming problem, whereas menu scheduling is an integer programming problem in which compatibility of items between and within meals becomes an additional criterion of optimization. The approach to scheduling presented in this paper uses the dual values of nutrient constraints for the planning problem to adjust the objective coefficients of the scheduling problem. The scheduling problem is then solved by a sequence of multiple choice integer programs alternating with a sequence of multiple choice knapsack problems.
Computational results from an application in which a minicomputer was used to solve all the mathematical programs, is reported. The approach was successfully able to control attributes such as nutrients that were incorporated in the model through Lagrangian relaxation.

451

Equi-Assigned Sequences and Explicit Solution of an Integer Program. A Proposal for Generating and Recognizing a Particular Language

A. Volpentesta, University of Pisa, Pisa, Italy

First of all the definition of equi-assignment of weights to the vertices of a cycle in a graph is introduced. Then, the equi-assigned sequences are determined by means of a procedure based on the euclidean algorithm for finding the greatest common divisor of two numbers. By using these results a set of optimal solutions of a particular integer program is characterized and explicitily determined. Finally, an application to the field of computer science with the purpose of defining some recursive schemes for the generation and the recognition of a particular language and a generalization of the equi-assignment concept are briefly discussed.

452

A Review on Taha's General Algorithm

R.E. Campello, Furnas, Brazil
N. Maculan, Universidade Federal do Rio de Janeiro, Rio de Janeiro, Brazil

Taha's general algorithm to globally minimize well defined concave functions over a convex polyhedron is reword.
The algorithm is basically a branch and bound procedure which utilizes a cutting plane approach, based on Glover's general theory for constructing legitimate cuts, to find out the optimal solution to the problem.
It is briefly discussed the relationship between convexity cuts and disjunctive cutting planes, described in a recent paper on Disjunctive Programming by Balas.
Formal convergence proof as well as a complete numerical example to illustrate the algorithm are provided.
For the fixed charge problem it is suggested the inclusion of an adjacent extreme point search to improve lower bounds, which seems to be inexpensive and computationaly attractive.
Computer results are also included indicating that the proposed fixed charge simplex algorithm is useful for moderate sized fixed charge problems.

453

A Decomposition Method for Quadratic Fixed-Charge Problems Applications to a Submarine Pipeline Design Problem

F. Arcangeli, University of Verona, Verona, Italy
L. Pellegrini, University of Pisa, Pisa, Italy

First of all the classic (linear) fixed-charge problem is considered and a decomposition solving procedure, based on dual fathoming, is defined. The problem is reduced to a finite sequence of linear programs. Advantages and disadvantages of this approach with respect to existing ones are discussed.
Then, such a decomposition method is generalized to the case of a quadratic objective function and applied to the optimal design of a pipeline freely resting on the seabed. The decomposition procedure is animated in terms of the engineering meaning.

454

Generating Facets in the Fourier-Motzkin Elimination Method

I.G. Rosenberg, Universite de Montreal, Montreal, Canada

The following problem arose in the study of certain combinatorial objects (intersection patterns, two symbol balanced arrays and regular selfcomplementary graphs). Given an integer $n \times (k+\ell)$ matrix A describe the set
$S_A = \{y \in \mathbb{N}^\ell : A(x,y)^T \geq 0$ for some $x \in \mathbb{N}^k\}$ (where $\mathbb{N}$ is the set of nonnegative integers). Thus we are confronted here not with a standard optimization problem but with a harder task of determining all values of parameters $y \in \mathbb{N}^\ell$ such that $A(x,y)^T \geq 0$ is solvable in nonnegative integers. First we relax the problem by dropping the integrality constraint. The Fourier-Motzkin elimination is then applicable but, as it is well known, the number of inequalities becomes usually prohibitive after performing a few iterations. Although a considerable number of these may be indispensable, many inequalities obtained are likely to be redundant. Our main contribution is a method which at each iteration eliminates the superfluous inequalities and thus keeps but the facets of the corresponding polyhedron. The method is based on the monitoring of the "ancestors" among the original inequalities. We have to consider only the inequalities possessing a fairly restricted number of ancestors. To such an inequality we assign a matrix whose rank provides a necessary and sufficient condition for the irredundancy of the inequality. Although admittedly this procedure may be quite slow, it provides a method for constructing the relaxation of S_A. We conclude by modifying the algorithm for the discrete case using Hilbert-Pressburger branching suggested by H.P. Williams (J. Combinat. Theory (A) 21, 118-123 (1976)).

455

A Penalty Function Approach to Integer and Mixed Integer Programming

P. O'Neill, University of Waterloo, Waterloo, Canada

An exact penalty function is presented which, for a sufficiently small value of the penalty parameter, has a global minimum coinciding with the (globally) optimal solution of a constrained integer or mixed integer programming problem. The constrained problem is required to have a continuous real-valued objective function. The penalty function is piecewise continuous and differentiable. Ways of using this function to solve integer and mixed integer programming problems are discussed.

456

A Diophantine Method for Integer Programming

G.J. Turbay-Bernal, Drake University, Des Moines, Ia., U.S.A.

A double description approach is used in conjunction with classical results of Hermite and Smith in the field of integral matrices to develop a representation theorem for all integer points in a regular polyhedral cone. The

representation theorem provides an effective tool for generating arbitrary integer points in the cone described by the inequalities corresponding to the constraints active at the linear programming optimal solution. Using the Farkas Lemma and the representation theorem the concept of "minimal set" is introduced as the integar programming, counterpart of the concept of optimal vertex solution in linear programming.

457

A Nonconvex Problem in Optimal Location Theory: Location with Forbidden Regions

L. Cooper, Southern Methodist University, Dallas, Tx., U.S.A.
I.N. Katz, Washington University, Washington, D.C., U.S.A.

The problem of optimally locating a single source using an arbitrary ℓ_p norm, in the presence of one or more circular forbidden regions is posed and characterized mathematically. It is shown to be a nonconvex mathematical program. The optimal "paths" between the source and the destinations are characterized by solving a variational problem. An analysis is presented which results in a computational algorithm for obtaining these distances, as well as the optimal location of the source. Numerical results are also presented. Generalizations are also presented for other shapes for the forbidden regions.

458

Maximizing Submodular Set Functions: Formulations and Analysis of Algorithms

G.L. Nemhauser, Cornell University, Ithaca, N.Y., U.S.A.
L.A. Wolsey, University of Louvain, Louvain, Belgium

We consider integer programming formulations of problems that involve the maximization of submodular functions. A location problem and an integer quadratic program are well-known special cases. We give a constraint generation algorithm and a branch-and-bound algorithm that uses linear programming relaxations. These algorithms are familiar ones except for their particular selections of starting constraints, subproblems and partitioning rules. The algorithms use greedy heuristics to produce feasible solutions, which, in turn, are used to generate upper bounds. The novel features of the algorithms are the performance guarantees they provide on the ratio of lower to upper bounds on the optimal value.

459

Pairwise Separable Programming

E.V. Denardo, G. Huberman, U.G. Rothblum, Yale University, New Haven, Ct., U.S.A.

As an illustration of our model, consider a bus line. Suppose a certain number n of bus stops is placed to minimize the total distance walked by the population. One might suspect that the marginal benefit of the nth bus stop is nonincreasing in n. One might also suspect that if one bus stop is removed and the remainder are relocated by reoptimizing, each bus stop shifts toward the one removed, but not farther that the original location of the adjacent bus stop. We provide conditions under which these results hold. We demonstrate them by applying lattice programming.

460

Efficient Algorithms for Locating Centers on Tree Networks

R. Chandrasekaran, A. Tamir, Northwestern University, Evanston, Il., U.S.A.

We study discrete as well as continuous problems of locating centers on tree networks using the minmax distance criterion. Polynomially bounded algorithms are presented. The discrete models are analyzed and solved by a unified approach, applying properties of rigid circuit graphs. We also present a general duality result, relating these models where centers can be established anywhere on the tree, to the problem of locating centers so as to maximize the distance between the nearest pairs of centers.

461

The k-Median Problem: Worst-Case and Probabilistic Analysis

C.H. Papadimitriou, Massachusetts Institute of Technology, Cambridge, Ma., U.S.A.
D.S. Hochbaum, Carnegie-Mellon University, Pittsburgh, Pa., U.S.A.

The K-median problem is the following: given n points on the Euclidean plane, select K of them (the medians) so that the sum of the distances from each point to the closest median is minimum. We show that this problem is NP-complete when K, as a function of n, grows as cn^{ε}, for all $0<\varepsilon<1$. It is therefore unlikely that efficient algorithms exist for the exact solution of the K-median problem for this range of values of K.
In previous work on this problem Fisher and Hochbaum developed a polynomial-time "aggregation" heuristic which, with probability 1-o-(1), achieves solutions of relative error o(1), for K = O(logn), when the points are uniformly distributed in the unit square. We present two new heuristics for this problem. One is a partition heuristic similar to Karp's algorithm for the Euclidean traveling salesman problem, and works for $K = \alpha n$, some $0<\alpha<1$. The other is based on the "hexagonal cell" solution of the corresponding continuous location problem, and works for essentially all the intermediate growths of K. Both algorithms are polynomial-time bounded, and produce solutions of arbitrarily small relative error with probability arbitrarily close to 1, as n increases.

462

Traffic Equilibrium and Variational Inequalities

S. Dafermos, Brown University, Providence, R.I., U.S.A.

We discuss certain non-cooperative game-type equilibrium problems that cannot be reduced to convex optimization and

thus cannot be solved by mathematical programming methods. One important case arises in the general traffic equilibrium network model where the travel cost on each link of the transportation network may depend on the flow on this as well as other links of the network. The model has been designed in order to handle situations where there is interaction between traffic on different links (e.g. two-way streets, intersections) or between different modes of transportation on the same link. The travel demands may be fixed or elastic. For this model, we use the techniques of the theory of variational inequalities to establish existence of a traffic equilibrium pattern, to design an algorithm for the construction of this pattern and to derive estimates on the speed of convergence of the algorithm. We also consider by the same technique the case of continuous distribution of traffic.

463

Equilibria for Network Planning: A Survey

T.L. Magnanti, M.I.T., Cambridge, Ma., U.S.A.

Equilibrium conditions, reflecting physical laws or behavioral assumptions about users, arise in a variety of settings involving networks--transportation, communication, water resource planning, and spacially separated economic markets. We review equilibrium models for selected of these applications, discuss existence and uniqueness of solutions, and summarize mathematical programming based algorithms for computing equilibria.

464

Continuum Approximation to Dense Networks and Its Application to the Analysis of Urban Road Networks

A. Taguchi, M. Iri, University of Tokyo, Tokyo, Japan

A continuum approximation is proposed for problems of flows in large and dense networks, where each point of the continuum is characterized by "capacity" and/or "distance/cost" which are convex sets in the tangent space. The problems of flows in continua corresponding to various problems of flows in networks are formulated in variational forms, of which mathematical properties are investigated and for which numerical algorithms are derived by means of the finite-element discretization technique. A practical procedure, based upon concepts from integral geometry, for constructing the continuum which approximates the original network is also proposed.
The effectiveness of the continuum approximation is tested against flow problems on urban raod networks of moderate size, to get satisfactory results. The continuum approximation has many advantages over the ordinary network model; e.g. (i) It is easier to build from the practical standpoint of data gathering; (ii) Its solution helps us to intuitively understand the global characteristics of road networks; (iii) The amount of computation does not depend on the size of the original network.

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Synthesizing Network Equilibrium and Design Models for Location of Urban Activities

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Recent applications of nonlinear programming methods have solved the problem of assigning person trips to an urban transportation network such that the network equilibrium condition of equal journey costs is met. Extensions of these models have incorporated the problems of destination choice and modal choice into this equilibrium modelling framework. These models can in turn be reinterpreted as urban location models.
In this paper, the implications of synthesizing these developments with two design problems are examined. The first is to determine the location of urban activities so as to maximize a benefit function. The second is to determine the capacity of each link so as to minimize travel costs subject to a budget constraint. The paper reviews these developments and identifies limitations and requirements for solution of the synthesized models.

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A New Paradox in Wardrop's User Equilibrium Assignment: How an Increase in Automobile Travel Demand Will Produce a Decrease in Global Travel Costs

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The sensitivity of Wardrop's user equilibrium assignment solution to changes in automobile travel demand is investigated. It is shown that in a congested transportation network for which link costs are non-decreasing functions of the link flows, an increase in the input flows may lead to a decrease in some origin to destination travel costs. Furthermore this may also produce a decrease in the global travel costs. Another consequence of this characteristic is that in a bimodal equilibrium assignment origin to destination transit travel costs may be reduced by increasing automobile travel.
The implications of these properties for a large scale transportation network are examined by considering the effects of variations in input flows on the City of Winnipeg network.

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Mathematical Optimization Versus Practical Performance: a Case Study Based on the Maximum Entropy Criterion in Medical Image Reconstruction

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Practical problems are often translated into problems of mathematical optimization by a process of abstraction, simplification and assumption making which leaves one to wonder whether solving the mathematical optimization problem will help us in any way with our original practical problem. In many fields this is very difficult to evaluate.
In this talk we pick a case study motivated by a problem in diagnostic medicine. A cross-section of the human body can be represented by a two dimensional array X of numbers, each describing the average value of some physical parameter in a small area. Using the attenuation of appropriate transmitted radiation through the body, systems of

linear equations can be set up, with the components of X as the unknowns. It has been argued that the element of choice in the feasible set should be the one which maximizes the "entropy"

$$-\sum_{i,j} x_{ij} \ell n \, x_{ij},$$

since the pictorial appearance of such an optimizer has no features not enforced on it by the data. We examine the validity of this argument by evaluating the performance of medical personnel on images representing cross-sections of the human head reconstructed according to the maximum entropy criterion as compared to other methods.

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Digital Image Restoration Using Quadratic Programming

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The problem of digital image restoration is considered by obtaining an approximate solution to the Fredholm integral equation of the first kind in two variables. The system of linear equations resulting from the discretization of the Fredholm integral equation is converted to a consistent underdetermined system of linear equations. The problem is then solved as a quadratic programming problem with bounded variables, where the square of the unknown vector is minimized. In this method minimum computer storage is needed. A computer simulated example using a spatially separable point spread function is given.

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Simulative Evaluation of Learning System Identification Algorithms

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The Group Method of Data Handling (GMDH) is an algorithm which attempts to generate the underlying functional relationship between the vector of independent variables and the dependent variable for a given set of input-output observations of a system. Since its introduction, the GMDH has been implemented employing various heuristics involving preprocessing of the data, partitioning of the data into a training set and a checking set, varying the form of the polynomial to be used in each node of the network and various intermediate variable selection criteria. In addition, many revised GMDH algorithms have also been introduced.
In order to evaluate the performance of each such algorithm and its associated heuristics quantitatively, a generalized evaluation procedure is developed in which error, functional dependency, data dependency and instability of the algorithm are defined and computed. Quantitative comparisons of performance among the original GMDH, variations of the original GMDH, and the authors' improved GMDH algorithms are made. Relative performance measures among algorithms obtained by the procedure are verified through the application of algorithms to example system identification problems.

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Optimization in Data Analysis

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In this communication, we still consider "The Central Relation Problem", defined in the (other) communication : "Optimization in Data Analysis".
But this time we will consider a completely different method.
It is a general heuristic approach allowing us to obtain very rapidly a good approximate solution for problems of large size.
This kind of heuristic has certain remarkable combinatorial and mathematical properties, in particular concerning the local optimality conditions.
The description of the heuristic and of its mathematical properties will be the purpose of the communication.

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Optimization in Data Analysis (Heuristic Methods)

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In this communication, we present optimization techniques to solve one of the main problem of Data Analysis: "The Central Relation Problem". This problem arises in Ordinal as well as in Cluster Data Analysis.
It is the problem of the aggregation of several individual binary relations, defined on n objects, by a particular collective binary relation.
For example:
-In Ordinal Data Analysis the obtainment of a collective ranking deduced from several individual rankings or preference relations; this is the well known "collective choice problem" or "preferences aggregation problem", first mentioned by Condorcet and related to Arrow and Kemeny's works, for instance....
-In Cluster Data Analysis, the obtainment of an optimal (central) collective partition (equivalence relation) deduced from several individual partitions or similarity relations; this other problem of Data Analysis is a much discussed problem in France (e.g. J.P. Benzecri) and in U.S.S.R. (e.g. S.Mirkin).
For these types of problems, there were no effective algorithms until now allowing the exact solving of real problems in a reasonable amount of time.
The communication includes the three following parts:
1) Position of the problem:
The problem is to approximate or to aggregate m binary relations, defined on n objects, by a particular binary relation.
The particular relation we are looking for, could be wether a quasiorder or an order relation wether an equivalence relation. This part will include the definition, the modelisation as well as some theoretical properties of the problem.
2) Solving methods:
We will propose a general linear model; the ranking problems as well as the clustering problems being particular cases.
We will present the mathematical properties of the obtained models; they are large scale linear programs with special structures.
The associated algorithms are efficient when n (the number of objects to rank or to classify) is less or equal to 80, for larger sizes we have developed original heuristics.
We have built an optimization system in PL/1 under CMS, including the IBM MPSX/370 system as a sub-system.
3) Presentation of practical problems.
The use of the algorithms we have presented in (2), has allowed us to obtain the exact solution, in a reasonable

amount of time, of more than a hundred real problems;
some of the applications we have solved, can be found in
the book: "Optimisation en Analyse Ordinale des Donnees",
Masson Paris 1979, that we have written.
Most of these problems have been proposed to us by french
firms and organisms such as INSEE (French National Institu-
te of Statistics) or CRIC (Communication Research Centre
of the Sorbonne), for instance.
We will discuss some of these examples.

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