A return of the MPS Symposium to Europe after nine years brought a record 799 registrants and 656 scheduled presentations during five busy days at the University of Amsterdam, August 5-9. The meeting, headed by an organizing committee of J. K. Lenstra, A. H. G. Rinnooy Kan and A. Schrijver, featured an outstanding scientific program, awarding of the Society prizes, and a lively social program against the backdrop of the infinitely varied life of the city.

Opening ceremonies included plenary talks by J. Tindbergen, first recipient of the Nobel Prize in Economics, and W. R. Pulleyblank who summarized progress and trends in mathematical programming. The George B. Dantzig Prize for original research with a major impact on mathematical programming went to Martin Grötschel and Arkady Nemirovsky. Three Delbert Ray Fulkerson Prizes for outstanding papers in discrete mathematics were awarded to Alfred Lehman; Martin Dyer, Alan Frieze and Ravi Kannan; and Nikolai Mnev. The Beale/Orchard-Hays Prize for excellence in computational mathematical programming was given to Irvin Lustig, Roy Marsten and David Shanno. Reports by the various prize committees are in this issue of OPTIMA.

1991 Prize Recipients

Grötschel and Nemirovsky share 1991 Dantzig Prize

Martin Grötschel, Technische Universität Berlin and Zuse Institut, Berlin and Arkady S. Nemirovsky, CEMI, USSR Academy of Sciences, Moscow were awarded the 1991 Dantzig Prize by the MPS and SIAM during opening ceremonies at the MPS Symposium.
The MPS business meeting on Wednesday featured the awarding of the A. W. Tucker Prize for outstanding student paper to Michel X. Goemans of MIT. Leslie Hall of Princeton and Mark Hartman of the University of North Carolina were Tucker prize finalists. Details on the work of all finalists follow.

Also at the business meeting, MPS Chairman George Nemhauser announced that the 15th Symposium will be held at the University of Michigan in Ann Arbor in August, 1994. John R. Birge of the Department of Industrial and Operations Engineering will be program chairman.

The awarding of two prize winners and the selection of these specific individuals was particularly attractive to the prize committee for several reasons. First, by giving two awards, two outstanding researchers whose contributions go far beyond their own individual contributions could be recognized; they have been role models to other researchers and have precipitated entire streams of research within the optimization community. Second, with these two selections, contributions from both the discrete and continuous domains of mathematical programming could be recognized. Third, these awards would be the first time that the Dantzig Prize has recognized work conducted by a researcher from continental Europe and from the Eastern bloc.

The individual prize citations are given below:

"Martin Grötschel has made numerous and substantive contributions to the theoretical, algorithmic, and computational aspects of combinatorial optimization. As one of the principal architects of the field of polyhedral combinatorics, he has played an important intellectual and leadership role in the emergence of polyhedral combinatorics as a field with both deep intellectual content and considerable practical utility. Through a broad and impressive range of investigations that touch upon many aspects of theory, computation, and application, including such topics as the traveling salesman problem, the linear ordering problem, statistical physics, large-scale circuit layout design, network design, and combinatorial implications of the ellipsoid algorithm, Martin Grötschel has been one of the few members of the mathematical programming community who rightly earns the title of 'man for all seasons.'"
“Arkady S. Nemirovsky has been a pioneer in the study of the complexity of continuous optimization problems. His numerous contributions have both defined an entire new field of scientific investigation within mathematical optimization and produced innovative and significant algorithms and methods of analysis. His contributions include the conceptualization and analysis of informational complexity for smooth, convex and stochastic programming problems, the development of new algorithms such as the ellipsoid algorithm and the assessment of the basic arithmetic complexity for broad classes of convex problems. Arkady S. Nemirovsky has been an innovator, conceptualizer, and a brilliant analyst, and as such has been leading contributor to the field of mathematical optimization in the past two decades.”

In the process of reaching its decision, the Committee also recommended that the future Dantzig Prize Committees adopt the view that the Prize be awarded for a body of research that has had a significant impact upon the field of mathematical programming, and not for a single piece of work or a singular contribution. This interpretation is consistent with the credentials held by all the previous recipients of the award.

T. L. Magnanti (Chair)
M. W. Parducci
R. T. Rockafellar

The 1991 Beale/Orchard-Hays Prize

After lively discussions about the relative merits of the many excellent nominees, the Beale/Orchard-Hays Prize Committee is pleased to announce that the 1991 prize has been awarded to Irvin J. Lustig, Princeton University, Roy E. Marsten, Georgia Institute of Technology, and David F. Shanno, Rutgers University, for their paper “Computational experience with a primal-dual interior point method for linear programming,” first published in 1989 as Technical Report SOR 89-17 of the Department of Civil Engineering and Operations Research of Princeton University and subsequently presented at the NY ORSA/TIMS Meeting in October 1989 and published in Linear Algebra and Its Applications, Volume 152, pages 191-222, 1991.

This paper is the culmination of a body of research on computational aspects of primal-dual interior point methods for linear programming, aspects of which were also reported in the following earlier papers:


Since its re-introduction in 1984, the interior point method for linear programming has been the subject of lively debate with respect to its performance relative to the simplex method. While this competition, which has been an invigorating force in the mathematical programming area, fortunately still continues, the thorough description in the prize paper of its algorithmic approaches and the free distribution to research universities of the OB1 code that implements these methods has succeeded in firmly establishing the competitiveness of the interior point method. In addition to summarizing and re-interpreting some of the techniques introduced in the earlier papers for handling feasibility issues and simple bounds, the prize paper presents a method for handling free variables and discusses the role of the barrier parameter and the effects of preprocessing, scaling, and removing variables. Finally, it presents a convincing computational comparison with MINOS 5.3, a widely distributed, state-of-the-art simplex code. In order to limit further stimulation of the interior-point/simplex controversy, we conclude simply with the observation that this comparison certainly presents very good evidence that, for large-scale linear programs, these methods are worthy adversaries.

The Beale/Orchard-Hays Prize Committee

Robert R. Meyer (Chair)
Jorge More
John Tomlin
Laurence Wolsey
1991 PRIZE RECIPIENTS

The 1991
D.R. Fulkerson
Prizes in Discrete
Mathematics

Up to three Fulkerson Prizes are jointly awarded by the Mathematical Programming Society and the American Mathematical Society every three years.

Both Societies appoint a committee that has the task of making recommendations to the Societies. The prizes are given for single papers, not series of papers or books. The term “discrete mathematics” is intended to include graph theory, networks, mathematical programming, applied combinatorics, and related subjects.

The 1991 Fulkerson Prize Committee consisted of Louis J. Billera, Martin Grötschel (chairman) and Paul Seymour. The committee recommended three papers for the award. MPS and AMS accepted the recommendations. The laudations that were submitted to MPS and AMS and that describe the scientific achievements of the prize-winning papers are listed below.

First Award

Alfred Lehman is Professor of Mathematics at the University of Toronto, Toronto, Ontario, Canada.

Among the prize-winning papers, Lehman’s paper is undoubtedly closest to the heart of Ray Fulkerson’s research. It is a fundamental contribution to combinatorial optimization, one that Fulkerson would have greatly admired. It brings our understanding of clutters with the width-length inequality almost to the same level as that of perfect matrices.

Let us put Lehman’s result in perspective.

One of the core problems in combinatorial optimization is to characterize the linear programs that have an integer optimal solution. This problem seems to be out of reach, at least for the time being. A more specialized question is: What are the matrices $A$ such that the linear program $\max c^T x \text{ s.t. } Ax \leq b$ has an integral optimum solution for all vectors $c$ and $b$? This leads to the theory of totally unimodular matrices, that was developed by Hoffman and Kruskal (1956) and many others. These matrices are now fully understood through the work of Seymour (1980).

Another line of attack on the general problem seeks for a characterization of those 0/1-matrices $A$ for which the so-called packing problem $\max c^T x \text{ s.t. } Ax \leq 1, x \geq 0$ has an integral optimum solution for all vectors $c \geq 0$. This area of research has resulted in the theory of antiblocking polyhedra developed by Fulkerson (1971, 1972) and the theory of perfect graphs and perfect matrices. Alfred Lehman in a seminal preprint of 1965, which was published in Lehman (1979), characterized the antiblocking pairs of polyhedra by a so-called width-length inequality. Further outstanding results are due to L. Lovász (1972) who solved the famous weak perfect graph conjecture and to M. Padberg (1974) who proved beautiful regularity properties of the so-called almost perfect matrices or equivalently the minimally imperfect graphs.

The related covering problem is to characterize those 0/1-matrices $A$ for which $\min c^T x \text{ s.t. } Ax \geq 1, x \geq 0$ has an integral optimum solution for all $c \geq 0$. This is the problem the paper we talk about addresses.

For the study of this problem, Fulkerson (1970, 1971) introduced the theory of blocking polyhedra. Cornuèjols and Novik (1989) call the matrices for which this integrality property holds ideal, and Seymour (1990) refers to the clutters that can be derived from the rows of ideal matrices as clutters with the max-flow min-cut property.

As in the antiblocking case, Alfred Lehman provided in his preprint of 1965 a width-length inequality that characterizes ideal matrices and clutters with the max-flow min-cut property, respectively. It is natural to ask for an excluded minor characterization of ideal matrices, i. e., a characterization of almost ideal matrices. This is the blocking analogue of the "antiblocking question" of determining minimal imperfect graphs or almost perfect matrices. This blocking analogue seems just as hard as the open antiblocking problem, or even harder, for there is not even a conjecture as to what the answer might be. Almost no progress was made for a long time despite the fact that many prominent researchers of the field worked hard on it. Even proving an analogue of Padberg’s theorem had eluded researchers until Lehman’s breakthrough.

In the paper for which the Fulkerson Prize is awarded, Alfred Lehman made a fundamental and unexpected step towards a decent characterization of almost ideal matrices. Quite a variety of different almost ideal matrices are known. For instance, in the language of clutters, there is the set of edge sets of all odd circuits of $K^e$, the set of all consecutive triples from eight vertices arranged in a circle and several others. Yet Lehman proved that all these almost ideal matrices have certain very remarkable regularities, except for one infinite family, the so-called degenerate projective plane clutters. Lehman’s theorem is a deep blocking analogue of Padberg’s result on almost perfect matrices.

Many people have wondered why Lehman’s 1965 paper was only published in 1979. The committee members recently learned that Lehman was reluctant to publish this paper despite the fact that it contains really outstanding results. He felt that the final touch to the theory was missing. This is what the prize-winning
paper is adding to the theory of blocking polyhedra.
A number of papers have meanwhile appeared that restate Lehman's result in other mathematical languages, polish the approach, provide different proof techniques and view it from different angles. Among these are papers by Cornuéjols and Novik (1989), Padberg (1990), and Seymour (1990).

References:

Second Award
Martin Dyer is from the School of Computer Studies of the University of Leeds, U.K.; Alan Frieze is from the Department of Mathematics, and Ravi Kannan is from the Department of Computer Science, of the Carnegie-Mellon University, Pittsburgh, Pennsylvania, USA.

The paper is concerned with the following problem: we are given a convex body in n-dimensional Euclidean space, and we wish to find its volume. Earlier work of Grötschel, Lovász and Schrijver gave an algorithm, with running time polynomial in n, that estimates the volume to within a factor that is exponential in n; and results of Elekes and of Bárány and Füredi showed that if the body is presented by means of a “membership oracle”, which is usual, then no algorithm with polynomial running time can estimate volume within a polynomial factor. However, these results are concerned with algorithms in the course of which no random decisions are made. The prize-winning paper of Dyer, Frieze and Kannan shows that, in remarkable contrast, if we permit the algorithm to incorporate flipping coins, then one can estimate volume with high precision; more precisely, for any arbitrarily small \( \epsilon > 0 \), one can generate a random variable, with the property that its ratio to the true volume is between 1 - \( \epsilon \) and 1 + \( \epsilon \) with probability at least \( \frac{1}{4} \). The algorithm has running time bounded by a polynomial in n and \( 1/\epsilon \). This is the first instance of the use of a random algorithm providing a super-polynomial improvement in accuracy.

The method is, roughly, to surround the body with a nicer (that is, algorithmically more tractable) convex body, to generate a uniformly distributed random point inside the larger body and to count how often it falls into the smaller one. This provides an estimate of the ratio of the two volumes. There are a number of difficulties with this approach, arising from the fact that if we ask the larger body to be “really” nice, the ratio of the volumes becomes exponential, and we cannot estimate it in polynomial time by Monte Carlo methods. The paper overcomes this by using a series of bodies, each inside the next; but the intermediate bodies are no nicer than the original, and this produces the second difficulty–how can one generate a random point, uniformly distributed over a convex body? Solving this in a satisfactory way is the main achievement of the paper. The technique used is to approximate the body by its intersection with a sufficiently dense cubic lattice and to take a random walk on these lattice points. This converges to a uniform distribution, but it is important that it does so in polynomial time, and proving the latter is complicated. However, it would suffice to prove that the second eigenvalue of the “Laplacean” of the walk is well separated from the first eigenvalue, and for this the paper uses a recent result of Sinclair and Jerrum, which relates the separation of these eigenvalues to connectivity and expansion properties of the underlying graph. (The result of Sinclair and Jerrum is itself an extension of a result of Alon.) So it suffices to establish that the underlying graph indeed has these expansion properties, but unfortunately that is not true. To make it true the paper adjoins to the domain of the random walk a few more neighbouring points from outside the body, to smooth it off, and then the expansion property is shown to hold for the enlarged domain by applying isoperimetric inequalities from geometry.

This paper has had a major impact on numerical integration, statistical sampling and other fields. Also, there has been a great deal of subsequent work on modifying the
techniques of Dyer, Frieze and Kannan to reduce the running time of the algorithm. In particular, papers by Lovász-Simonovits, Applegate-Kannan, and Dyer-Frieze have brought down the running time to roughly \( n^3 \), which promises practical applicability in the near future.

References:


Third Award


Nikolai Mnev is a Senior Researcher at the Institute of Socio-Economic Problems of the Academy of Sciences of the USSR in Leningrad. The work in this paper constitutes Mnev's 1986 Ph.D. thesis, written under A. M. Vershik in Leningrad [Mn86] and was first published as a research announcement in [Mn85].

The results of this paper represent a landmark in discrete geometry. They relate directly to the theory of oriented matroids, a subject partly invented in the Ph.D. thesis [Bl74] of R. G. Bland, a student of Ray Fulkerson. One of Bland's aims was to provide a combinatorial framework for the treatment of linear programming. This led, for example, to his discovery of what has come to be known as Bland's pivot rule to avoid cycling in the simplex method. While Bland's work showed how far the combinatorial approach to linear programming might be carried, this paper of Mnev points to the inherent limitations of any approach that ignores the data to any essential degree.

A precursor to the result of Mnev is a long-standing conjecture in discrete geometry, first formulated by Ringel [Ri56]. Though Ringel stated it in a form concerning lines in the plane, we give the version in terms of points.
1991 PRIZE RECIPIENTS

Isotopy Conjecture: Given two labeled sets of n points in general position in the plane (i.e., no three on a line), such that corresponding triples of points have the same orientation (clockwise or counterclockwise), then it is possible to continuously move one set of points into the other so that all the intermediate configurations are in general position (and so also maintain orientations).

The Isotopy Conjecture was considered almost obvious by many, although serious attempts to prove it went on for many years without much success. It can be put in a form that appears more relevant to mathematical programming. Suppose A x ≤ b and A' x ≤ b' describe combinatorially equivalent convex polytopes (i.e., having the same facial structure). Then these two data sets are indistinguishable from the point of view of any purely combinatorial approach to linear programming. The conjecture would imply that one could continuously change the data A,b into the data A',b' preserving the combinatorial type of the resulting polytope throughout.

The results of Mnev show that this conjecture is as false as it could be. To understand what is true, consider the case of a configuration of n points in the plane $p_i = (x_i, y_i)$, i=1,...,n, which we consider to be represented by a single point $(x_0, y_0, y_1, ..., y_n)$ in $\mathbb{R}^n$. The combinatorial type, or oriented matroid, of the configuration is the set of orientations of the various triples $p_ip_jp_k$ and the representation space of a given combinatorial type is the set of all points in $\mathbb{R}^n$ having this combinatorial type. The Isotopy Conjecture thus states that the representation space of any combinatorial type is connected. Since a representation space is determined by certain determinants being positive, negative or zero, it is a semialgebraic set (i.e., the solution set of a system of polynomial equations and strict inequalities). What Mnev showed was that given any semialgebraic set $M$, one can construct, for some $n$, a configuration of $n$ points in the plane whose combinatorial type has a representation space that is in a sense equivalent to $M$ (so that, for example, they will have the same homotopy type).

This result is called the Universality Theorem; it says, roughly, that arbitrary semialgebraic sets are no more general than those encoded by combinatorial types of point configurations (oriented matroids) or convex polytopes.

The problem of determining the topological type of the set of polytopes having a given combinatorial type was proposed by Vershik (himself a student of L.V. Kantorovich) to his seminar in Leningrad as early as the mid-1970's. Here again the expectation was that these spaces would be connected. For something as broad as universality to be true was completely unexpected. Technically, the proof is quite an achievement, combining combinatorial geometry, classical projective geometry and algebraic geometry in a very clever way.

The idea is to mimic the computation of a polynomial function by means of constructions, originally due to von Staudt (a student of Gauss), used to coordinatize projective planes. These constructions are based on the usual ruler-and-compass constructions of Euclid. The key is to show that the combinatorial type so defined by a system of polynomials has a representation space which is equivalent (in a well-defined sense) to the semialgebraic set cut out by this system.

Thus representation spaces of point configurations—and, by a straightforward equivalence by means of Gale duality, representation spaces of convex polytopes—can be almost arbitrarily complicated topologically, having any number of connected components, holes, etc. This indicates, for example, that algorithms for solving linear programming problems may ignore the actual data at their own risk—two combinatorially equivalent polytopes can be represented by data coming from totally different regions in the representation space. It is possible that this result contains the seeds of a proof that there can be no strongly polynomial algorithm for linear programming.

Given a semialgebraic set $M$, the corresponding combinatorial type can be constructed in time (and has size) that is linear in the complexity of $M$, so it follows that the problem of deciding if a given system of polynomial equations and inequalities has a solution (the so-called decision problem for the Existential Theory of the Reals, referred to as ETR) is linearly reducible to deciding whether a given oriented matroid arises from an actual configuration. Since ETR is known to be NP-hard, it follows that the problem of deciding whether an oriented matroid is realizable by a configuration of points is also NP-hard.

(An independent proof of this is given by Shor in [Sh91], where one can also find a clear exposition of the proof using Mnev's methods.) Finally, it follows that the algorithmic Steinitz problem—determining whether a given poset is actually the face lattice of some convex polytope—is also NP-hard.

Incidentally, after the proof of the Universality Theorem, but before the extent of Mnev's work was fully understood in the West, there was a flurry of activity in the U.S. and Europe leading to a counterexample to isotopy by Jaggi, Mani-Levitska, Sturmfels and White [JMSW89]. See [BS89] for a discussion of this and related work as well as a description of the work of Mnev.
1991 PRIZE RECIPIENTS

Tucker Prize to Goemans; Hall and Hartman Finalists

The A. W. Tucker Prize honors a paper written by a student on any aspect of mathematical programming. The 1991 prize committee, which consisted of Richard W. Cottle (chair), Thomas M. Liebling, Richard A. Tapia and Alan C. Tucker, selected Michel Goemans as the winner with Leslie Hall and Mark Hartman being finalists. All three presented their work in a special session at the 14th Symposium.

A summary of the background and work of the finalists follows:

Michel X. Goemans, "Analysis of Linear Programming Relaxations for a Class of Connectivity Problems."
Born and raised in Brussels, Belgium, Michel X. Goemans received his Diploma in Applied Mathematics from l'Université Catholique de Louvain in Louvain-la-Neuve. In 1987, he commenced doctoral studies in Operations Research at the Massachusetts Institute of Technology. During the summer of 1989, Mr. Goemans was a member of Dr. David Johnson's group at AT&T Bell Laboratories in Murray Hill, New Jersey. He received his Ph.D. in 1990 under the supervision of Professor Dimitris J. Bertsimas. Upon graduation, he joined the teaching staff in the Department of Mathematics at MIT. A paper by Mr. Goemans describing the "parsimonious property" was awarded second prize in the 1990 George E. Nicholson student paper competition sponsored by the Operations Research Society of America.

Mr. Goemans' paper belongs to the body of literature concerning worst-case and probabilistic analysis of (heuristics for) combinatorial optimization problems. The goal of this thesis is to evaluate analytically linear programming relaxations for a class of connectivity problems including the survivable network design problem, the k-edge-connected problem, the TSP, the k-TSP and the Steiner tree problem. The central theme in this study is the aforementioned "parsimonious property" which in rough terms says that if the cost function satisfies the triangle inequality, there exists an optimal solution to a classical LP relaxation for which the degree of each vertex is the smallest it can possibly be. The work has three major sections, corresponding to the investigation of structural properties, worst-case analysis, and probabilistic analysis, respectively.

In the first part, Mr. Goemans specifies the combinatorial optimization problems to be discussed and the corresponding linear programming relaxation. He derives his result on the parsimonious property and develops a number of its consequences. He obtains the Held-Karp lower bound for the traveling salesman problem. He shows that the LP relaxation bounds corresponding to the Steiner tree problem, the k-edge-connected network problem, and the Steiner k-edge-connected network problem can be computed in the manner of Held and Karp (using Lagrangian relaxation and solving minimum spanning tree problems as subproblems).

In the second part, Mr. Goemans uses the parsimonious property to carry out worst-case analyses of the duality gap corresponding to the LP relaxations. He introduces two heuristics for the survivable network design problem and gives bounds that depend on the actual connectivity requirements. He shows that the value of the LP relaxation of
the Steiner tree problem is within twice the value of the minimum spanning tree heuristic and that analogous results for several generalizations of the Steiner tree problem can be obtained. Other contributions in this part of the work include a new relaxation (of the Held-Karp type) for the k-person traveling salesman problem and a demonstration that an existing heuristic for this problem gives a value that is within \( \frac{3}{4} \) times the value of the new relaxation.

The third part contains probabilistic analysis of the duality gap for the LP relaxations. Concentrating on the Euclidean model, he generalizes a theorem of Steele on the asymptotic behavior of Euclidean functionals. The generalization is particularly convenient for the analysis at hand. Mr. Goemans shows that the duality gap (for the various problem types) is almost surely a constant; he provides theoretical and empirical bounds on these constants. The analysis leads to the conclusion that the undirected LP relaxation for the Steiner tree problem is “fairly loose.”

Mr. Goemans’ parsimonious property has far-reaching consequences in the complexity analysis of heuristics in the worst case and probabilistic sense. His very well written paper combines new ideas with others that had been around for some time and thereby unifies problems whose kinship was suspected but never spelled out.

Leslie Ann Hall, “Two Topics in Discrete Optimization: The Polyhedral Structure of Capacitated Trees and Approximation Algorithms for Scheduling.”

Leslie Ann Hall attended college at Yale University where she earned a B.S. in Mathematics in 1982. After a year working in New York City and a year on a Fulbright Scholarship studying mathematics at the Technische Hochschule Darmstadt in Germany, she entered the doctoral program at M.I.T. Operations Research Center. In 1989, she received her Ph.D. and since has been an Assistant Professor in the Department of Civil Engineering and Operations Research at Princeton University. Ms. Hall’s thesis consists of two distinct parts. Professor Thomas Magnanti was the advisor for the first part and Professor David Schmoys was the advisor for the second part.

The first part presents a description of the polyhedral structure of the capacitated minimal spanning tree problem. This variant of the minimal spanning tree problem has a designated root and the requirement that each subtree of the root (a subtree with only one edge incident to the root) contains at most \( k \) nodes. This capacitated problem is essentially the same as the identical-customer vehicle routing problem. As part of her investigations, Ms. Hall showed that the intersection of a well-known and much-studied integer polyhedra (the spanning tree polyhedron and the matching polyhedron) is itself an integer polyhedron. Demonstrating great insight, she also found a number of novel types of facets for describing the capacitated minimal spanning tree polyhedron.

The second part of the thesis concerns a general framework for developing polynomial-time approximation schemes for a variety of important \( NP \)-hard scheduling problems. Heretofore, such approximation schemes have been ad hoc and generally applied to only the simplest (least realistic) scheduling models. She called her method the “outline-scheme” approach. Her “outline” captures critical information about an instance of a scheduling problem and allows an optimal or near-optimal solution to be computed in polynomial time.

In this well-written doctoral dissertation, Ms. Hall shows versatility, treating two disjoint subjects. In the first, she effectively uses simple proof techniques to characterize a non-trivial polyhedron from combinatorial optimization and also finds non-trivial facets of others. In the second part, she develops interesting heuristics for scheduling problems. The manuscript pairs charming modesty with scientific rigor.

Mark E. Hartmann, “Cutting Planes and the Complexity of the Integer Hull.”

Mark E. Hartmann was born in Redwood City, California, and grew up in Salt Lake City, Utah. In 1985, he received his undergraduate and Master’s degrees in Mathematical Sciences at The Johns Hopkins University. His doctoral studies were undertaken at Cornell University from which he received the Ph.D. in 1989. Mr. Hartmann’s Ph.D. thesis was supervised by William Cook with whom he studied for a year at the Institut für Ökonometrie und Operations Research in Bonn. He completed the writing of the dissertation while teaching probability and statistics at Johns Hopkins. He is currently a Postdoctoral Fellow and an Assistant Professor at the University of North Carolina at Chapel Hill.

This work belongs to the area of polyhedral combinatorics. A significant part of it is concerned with establishing lower bounds on the Chvátal ranks of polyhedra. (The Chvátal rank of a polytope is the number of times a certain “closure” operator needs to be applied to obtain its integer hull, i.e., the convex hull of the integer points in the polytope.) The Chvátal rank gives a measure of the “tightness” of the problem formulation.

In the first part of this paper, Mr. Hartmann describes two general methods for proving lower bounds on the Chvátal rank of a polytope. The first is a type of integral transformation that enables one to transfer lower bounds from an embedded polytope for which it can be easier to prove a lower bound. This method gives lower bounds for common relaxations of the set covering, set partitioning and knapsack problems—all based on Chvátal’s bound for the stable set problem. The second is a geometric method of “defending” the integer hull by finding a sequence of point sets such that if all the points in a given set are contained in a polytope, then all points in the next set survive the closure operation applied to the polytope. This method generalizes the one used by Chvátal in obtaining lower bounds.
for the stable set problem. It is used for obtaining lower bounds on the Chvátal rank of various combinatorial optimization problems, thereby settling, in a unified manner, conjectures of Chvátal on the traveling salesman problem, Barahona, Grötschel and Mahjoub on the bipartite subgraph problem, and Jünger on the acyclic subdigraph problem.

The second part of the paper addresses the study of integer points in an arbitrary rational polytope. It gives an upper bound on the number of vertices of the integer hull of the polytope in terms of the dimension, the number of inequalities and maximum size of an inequality in a system of inequalities used to describe the polytope. The proof involves dissecting the polytope into smaller polytopes, called reflecting sets, and then showing that the number of reflecting sets which contain vertices of the integer hull is at most the number of reflecting sets “near the surface” of the polytope.

The decomposition of the polytope into reflecting sets can be made constructive, thereby yielding a generalization of Lenstra’s celebrated result that the integer linear programming problem can be solved in polynomial time when the dimension is fixed. Indeed, it is even possible to obtain a list of the vertices of the integer hull in polynomial time.

Mr. Hartmann’s writing style is remarkably full of beauty, power and precision. The results are not entirely unexpected, since many had been conjectured before, but they are treated in a unified manner. The paper is notable for its brilliant, transparent—hence convincing—proofs.

—RICHARD W. COXILE

E.M.L. Beale Trust

This Trust commemorates Evelyn Martin Lansdowne Beale, FRS (1928-1985) who was the second chairman of the Mathematical Programming Society. During his career as a Scientific Officer at the Admiralty Research Laboratory, a Group Leader at CEIR (Corporation for Economic and Industrial Research), the Scientific Adviser at Scicon (Scientific Control Systems Ltd.), and a Professor of Mathematics at Imperial College, London, he was motivated by the needs of commerce and industry, he was highly proficient at building mathematical models of real-world problems for investigation by computer calculations, he led the development of the UMPiRe and SCICONIC mathematical programming systems, he invented many of the algorithms for linear, quadratic and integer programming that are fundamental to such systems, and he presented and published many learned papers that are major contributions to basic research. This career is a brilliant example of sustained work that not only served industry but also was of great importance to the academic development of statistics, operational research and mathematical programming. He showed clearly that experience in the business world can be of immense value to academic studies and that participation and profound understanding at the frontiers of academic research are often vital to the successful solution of real commercial problems. His professional knowledge was of particular benefit to his colleagues and to students at Imperial College, where as a Visiting Professor he taught actively and guided research over a period of 18 years until shortly before he died.

The purpose of the Trust reflects those achievements. In order to make an impact, it aims at a relatively narrow range of activities that would benefit substantially from moderate financial support. Indeed the Trust Funds will be devoted to the assistance of students and their studies in the Mathematics Department of Imperial College. Martin attached great importance to his duties in this department, and he gave inspiration, encouragement and guidance to many of its students, so the Trust is a very suitable memorial to the academic side of his career.

The Trust is registered with Charity Commission. Section 5 of the Trust Deed gives the following details of the activities that may be supported:
The Trustees may pay or apply the income of the Trust Fund for the purposes of the advancement of education and learning in the Department of Mathematics ("the Department") of the Imperial College of Science and Technology of Prince Consort Road, London SW7 ("the College") and in particular for the assistance and support of students and their studies in the Department in such manner as the Trustees shall in their absolute discretion think fit and in particular (and without prejudice to the generality of foregoing) the Trustees may pay or apply such income:

(a) for the provision of bursaries to allow or facilitate the admission to the Department of students who in the opinion of the Trustees deserve or are in need of financial support

(b) for the provision of funding for the continuation by students of the Department of their studies beyond the period for which funding is normally provided for them by local or central government or by other bodies or authorities (whether public or private)

(c) for the payment of travel and living expense incurred or to be incurred by students of the Department in attending conferences

(d) for the payment of travel and living expenses incurred or to be incurred by students of the Department in connection with:
   (i) visits to academic, industrial, scientific or other establishments
   (ii) visits enabling students of the Department to collaborate with researchers outside the College
   (iii) visits enabling students of the Department to accompany their supervisors while such supervisors may be on sabbatical leave

(e) for the payment of travel and living and other expenses incurred or to be incurred by students of other universities or educational or other establishments while visiting the College

(f) towards the improvement of facilities for students within the Department

(g) for the avoidance of doubt in this clause the expression "students" shall mean both undergraduate students and graduate students of the Department.

It is hoped that many members of the Mathematical Programming Society will show their appreciation of Martin's career and friendliness by making a donation to this memorial. The Trustees are Mrs. E. M. L. Beale (Windover House, Treyarnon Bay, Padstow PL2885, England), Prof. M. J. D. Powell (DAMTP, Silver Street, Cambridge CB3 9EW, England) and Prof J. T. Stuart (Department of Mathematics, Imperial College, London SW7 2BZ, England).

If you wish to make a contribution, please send a cheque (made payable to the "E.M.L. Beale Trust") to one of the Trustees. All contributions will be acknowledged. We would be very grateful for your support.

-M. J. D. Powell

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Research in Parallel Processing

The interest in parallel processing is rapidly increasing, but information on the research groups working in the field with respect to subjects and parallel equipment makes up a distributed database with no well-defined search methods.

As a first step towards changing this situation, COAL intends to compile a list of sites working with parallel processing in optimization. Hence if you work with parallel optimization in practice, please send a short description containing information on the hardware available, the projects being carried out (project titles only) and the names (and if possible e-mail addresses) of the people involved in the research.

The description can be sent by mail or e-mail using the following addresses. News on the project including how to get access to the compiled information will appear in the next issue of OPTIMA and the COAL Newsletter.

Jens Clausen
DIKU
UNIVERSITETSPARKEN 1
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DENMARK
e-mail:clausen@diku.dk
OPTIMIZATION DAYS 1991
Montréal, Quebec
CANADA
May 4, 5 and 6, 1992

Co-sponsored by:
Center de recherche sur les transports (C.R.T.)
Université de Montréal
École des Hautes Études Commerciales
École Polytechnique de Montréal

Groupe d’Études et de Recherche en Analyse des Decisions (GERAD)
École des Hautes Études Commerciales
École Polytechnique de Montréal
McGill University

Collège Militaire Royal de Saint-Jean

The scientific meeting "Optimization Days" is organized each year jointly by the above institutions of the Montréal region. The aim of the meeting is to survey current trends of research in optimization methods and their applications and to provide a good opportunity for interaction between various research groups.

All those interested in optimization methods and their present or potential applications are kindly invited to participate. We appeal especially to those who can give talks on new methods of optimization and their applications.

Sessions will consist of invited and contributed talks. Papers presenting original developments as well as those of expository nature will be considered. The languages of the conference will be French and English. Plenary speakers will be:

L. Devroye (Montréal)
B. Gavish (U.S.A.)
P. Jaillet (U.S.A.)
C. Lemaréchal (FRANCE)
C. Revelle (U.S.A.)

In 1992, Optimization Days will be held at École des Hautes Études Commerciales, 5255, avenue Deceles, Montréal.

Two copies of a 100-200 word summary defining clearly the content of the paper, together with a registration form, should be forwarded before December 15, 1991, to:

Michel Gendreau and Patrice Marcotte
Center de recherche sur les transports
Université de Montréal
Case postale 6128, Succ. "A"
Montréal, CANADA, H3C 3J7
Telephone: (514) 343-7575
E-mail: jot92@crt.umontreal.ca
Fax: (514) 343-7121

Registration fee: ($CDN) Before April 15: $100; After: $125 (Students $20)
($USD) Before April 15: $90; After $100 (Students $20)

Authors should also send a copy of their summary via e-mail, if this is possible.

Authors will be notified of the acceptance of their talks by March 15, 1992. Summaries of the talks will be distributed at the conference. For more information, please contact the above.

FOURTH SIAM CONFERENCE ON OPTIMIZATION

Sponsored by:
SIAM Activity Group on Optimization
May 11-13, 1992
Hilton Regency Hotel
Chicago, Illinois

The Fourth SIAM Conference on Optimization will address the most important recent developments in linear, nonlinear, and discrete optimization. The conference will feature recent advances in algorithms and software for the solution of optimization problems. It will also feature important applications of optimization in control, networks, manufacturing, chemical engineering, operations research, and other areas of science and engineering.

The conference will bring together mathematicians, operations researchers, computer scientists, engineers, and software developers. It will provide an excellent opportunity for sharing ideas and problems among specialists and users of optimization in academia, government, and industry.
Conference Themes:
Large-scale optimization
Interior point methods
Algorithms for optimization problems in control
Network optimization methods
Parallel algorithms for optimization problems
Contributed presentations in lecture or poster format are invited in all areas of optimization research and applications.

Minisymposia
A minisymposium may focus on any topic consistent with the conference theme. It consists of four 15-minute presentations, with an additional five minutes for discussion after each presentation. Prospective minisymposium organizers are invited to provide a title, a description (not exceeding 100 words) and a list of speakers and titles of their presentations.

For more information contact:
SIAM
3600 University City Science Center
Philadelphia, PA 19104-2688 U.S.A.
Telephone: (215) 382-9800;
Fax: (215) 386-7999
e-mail: siamconf@wharton.upenn.edu

FIFTH INTERNATIONAL SYMPOSIUM ON DYNAMIC GAMES AND APPLICATIONS
University of Geneva and International Academy of the Environment
Conches-Geneva
SWITZERLAND
July 15-16, 1992
Organizer: Département d’économie commerciale et industrielle, University of Geneva

Sponsor: The International Society of Dynamic Games (ISDG)
The Symposium is the fifth in a series of meetings dedicated to the area of dynamic games and is the official, biannual scientific meeting of the ISDG. The aim of the meeting is to bring together researchers from various disciplines where dynamic game settings are studied and to report the latest developments both in theory and application.

This year, the Symposium is putting particular emphasis on the theme Dynamic Games and Environmental Management Modeling.

The deadline for receipt of title and three copies of a 500-word extended abstract is February 1, 1992.

Early registration fee is US$200 before May 1, 1992.

A collection of papers presented at the Symposium will be published.

For further information, please contact:
A. Haurie, Chairman,
Département d’économie commerciale et industrielle Faculté SES
Université de Genève
2, rue de Candolle
CH-1204, Geneva
SWITZERLAND
Telephone: (int'l) 41 22 705 72 44
Fax: (int'l) 41 22 28 52 13
e-mail: HAURIE@CGEUGE11

The 5th Meeting of the European Chapter on Combinatorial Optimization (ECCO) will take place in Graz, Austria, April 13-15, 1992. The conference will be hosted by the Institute of Mathematics, University of Technology, Graz. For further information, please use the address given below. There will be no conference fee, but participants are expected to cover their own travel, accommodation and living expenses.

Time Schedule:
October 1991: Second announcement and hotel information
January 31, 1992: Deadline for abstracts
March 1992: Last announcement with preliminary program

Address:
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SECOND STOCKHOLM OPTIMIZATION DAYS

The Royal Institute of Technology
Stockholm
SWEDEN
August 12-13, 1991

A second, and much expanded, Stockholm Optimization Days was held on Monday and Tuesday following the MPS Symposium. Some 25 speakers from 11 countries presented papers on a variety of optimization topics including dual methods, decomposition, subgradient optimization, global optimization, interior methods, large-scale optimization, as well as a variety of applications. Copies of the program, including abstracts, may be obtained from the conference organizer, P. O. Lindberg, Division of Optimization and Systems Theory, Department of Mathematics. The conference was sponsored by the Göran Gustafsson Foundation and the Swedish National Board for Technical Development.
Special reduced rate for members of all OR societies affiliated with IFORS (International Federation of Operations Research Societies): US$80 per volume

Annals of Operations Research, ISSN 0254-5330, regular price per volume including postage: Swiss Francs 305.-. Please request extensive prospectus for whole series; vol. 1 - 33, 1984 - 1991. Proposals for new volumes should be addressed to Peter L. Hammer, Editor-in-Chief.

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Z. Füredi, “Turan Type Problems,” RRR 2-91.
J. Kahn, “Recent Results on Some Not-So-Recent Hypergraph Matching and Covering Problems,” RRR 4-91.

J. Miao and A. Ben-Israel, “Minors of the Moore-Penrose Inverse,” RRR 29-91.
Nonsmooth Optimization and Related Topics
Edited by F. H. Clark, V. F. Dem’yanov and F. Giannessi
Plenum Press
ISBN 0-306-42347-1

This comprehensive book forms the proceedings of the International School of Mathematics devoted to Nonsmooth Optimization held in Erice from June 20 to July 1, 1988. It reflects the variety of the field and its wide scope through 27 contributions, briefly noted as follows. The classifications given are for the convenience of the reader; it is the responsibility of the reviewer to introduce such an artificial arrangement. The main themes of the book are: 1. Analysis of nonsmoothness; 2. Optimality conditions; 3. Calculus of Variations; 4. Stability questions; 5. Algorithms; 6. Applications.

1. Analysis of Nonsmoothness

The contribution of F. H. Clarke exemplifies how the classification we adopt is artificial. Motivated by a question of regularity of the solutions of a problem in the calculus of variations, he introduces tools of nonsmooth analysis centered around what is called proximal analysis. The elegant concepts of this theory apply to calculus of normals to an intersection and pertain to the realm of perturbation analysis so that four of the announced sections could host this chapter.

The study of A. D. Ioffe centers around the concept of subdifferential. Again it is motivated by crucial problems in optimization theory (here an example arising from optimal control is treated). It clearly shows the difficulties and the achievements of the concept of subdifferential, especially in the infinite-dimensional case.

The provocative and amusing title of the lecture by V. F. Dem’yanov “Smoothness of nonsmooth functions” can also be taken seriously. In fact most of the efforts of researchers in nonsmooth analysis consist of devising tools which are not too far from the smooth case or obey rules which are close enough to the usual laws. Dem’yanov enlivens his preceding views in accepting functions whose directional derivatives are differences of convex functions.

Generalized derivatives obtained from a tangent cone notion are considered by K. H. Elster and J. Thierfelder. Their approach is axiomatic as were those of A. D. Ioffe (Proc. Ferma’s Days 1985), D. Ward and others. A key point of their contribution lies in their requirement that the recession cone to the tangent cone to a set must contain the recession cone of the set. This ensures that a closed tangent cone to an epigraph is an epigraph.

A. Marino and C. Saccon introduce a class of functionals for which the nonconvexity is controlled. They apply the properties they get to existence and multiplicity results of solutions to variational inequalities of Von Karman type associated with the plate problem with obstacle.

2. Optimality Conditions

The links of nonsmoothness with optimality conditions are strong and well known. It is also the case for generalized convexity conditions: this justifies the refinements of convexity considered in the Chapter treated by E. Castagnoli and P. Mazzoleni.

F. Giannessi, M. Pappalardo and L. Pellegrini chose to attack the question of necessary conditions in mathematical programming problems by considering the image of the mapping (j,k) gathering the objective j and the constraint mapping k.

J.-B. Hiriart-Urruty addresses nonconvex optimization problems, in particular the maximization of a convex function on a convex set and the minimization of a difference of two convex functions. His aims are local and global optimality conditions, which he reaches through the techniques of Convex Analysis, especially using the approximate subdifferential.

B. N. Pshenichny deals with implicit multifunction theorems and approximation of sets.

The problem of controllability studied by J. Warga is a weakening of an openness property. Given a set $X, x_0 \in X$, a convex subset C with nonempty interior intC of a topological vector space Z, the problem consists of finding conditions in order to have for some $v \in R^n, t > 0$ and each $t \in (0, r)$ $f(x_t) + tv \in \text{int} f(X \cap g^{-1}(\text{int C})).$

This problem is intimately linked with higher order optimality conditions in optimal control.

3. Calculus of Variations and Optimal Control

The contribution of E. De Giorgi and L. Ambrosio reflects the vitality of the calculus of variations. Motivated by the study of energy functionals corresponding to mixtures of different fluids as liquid crystals, they introduce new classes of functions of bounded variation type with which classical techniques of calculus of variations can be used. One has to note that among the new features of their approach lies the fact that the functionals are defined in terms of Hausdorff measures.

R. B. Vinter provides a unified treatment of necessary conditions for a class of nonstandard problems in dynamics optimization, in particular optimal multiprocess problems. Again the power of proximal analysis is illustrated.

4. Perturbation and Stability Questions

The central part of the contribution of P. D. Loewen deals with a formula for the generalized gradient of the value function of a perturbed differential inclusion problem.

J. Gauvin gives a synthetical simplified version of the works of Gauvin-Jainin about the performance function of parametrized mathematical programming problems.

R. T. Rockafellar studies the multifunctions whose values are the Kuhn-Tucker points of a general class of parametrized minimization problems, and
he gives conditions to ensure their differentiability. Among the appealing features of his treatment lies the symmetry of the roles played by primal and dual variables.

Saddle functions are considered in the contributions of E. Cavazzuti and N. Pacchiarotti who present closure and convergence concepts and a compactness theorem.

The analogy with fuzzy mathematics in which sets blur to become functions is exploited in the work of S. Dolecki who constructs operators akin to T-functionals and conjugations.

J. Morgan deals with stability questions for two-level optimization problems, focusing her attention to well-posedness properties.

R. Orlandi, O. Petrucci and M. Tosques study the perturbation of curves of maximum slope for a class of nonsmooth and nonconvex functions; they obtain a compactness theorem.

C. Zalinescu opens a chapter for systematic asymptotic convex analysis. In particular, he obtains a formula for the recession function of a marginal function.

5. Algorithms

G. Di Pillo and F. Facchine study a notion of exact penalization for mathematical programming problems which are described by Lipschitzian functions and give conditions for exactness.

The material presented by M. Gaudio and M.F. Monaco is centered on the idea of bundle algorithm. However, quadratic approximations are considered also in view of searching directions for descent methods.

It is a combination of the bundle idea and of the trust region method that J. Zowe undertakes. Besides a study of the trajectory of optimal solutions to the model problem as a function of the radius of the trust region (or rather of a penalty coefficient), he presents convergence results and numerical experiences.

N.Z. Shore reports on subgradient type methods with spiredation in connection with estimates of the value of the dual problem of a mathematical programming problem, in particular quadratic problems.

E. Polak gives an elementary exposition of the principles governing descent algorithms for minimax problems; in particular, he describes a two-phases method of centers related to Huard’s method of centers.

6. Applications

Applications are given in several contributions of the volume. Two chapters are more obviously oriented towards applications. One is the treatment of quasi-variational inequalities given by M. De Luca and A. Maugeri in view of transportation networks. Another one is certainly the overview of the optimization problems pertaining to the flight of an aircraft in a windshear presented by A. Miele and T. Wang. Three problems are studied, in particular the take-off trajectories and the minimization of the maximum deviation of the absolute path inclination from a reference value.

Since we have arrived at the end of this brief account of the topics in this volume, we will skip the other problems treated by the two preceding contributors (in particular the question of abort landing trajectories) to conclude that the variety and the depthness of the subjects treated in this book show that the take-off of nonsmooth optimization is successful and its flight is enjoyable.

—J. MORET

**Ramsey Theory**

**2ND EDITION**

by R.L. Graham, B.L. Rothschild and J.H. Spencer

Wiley, 1990

ISBN 0-471-50046-1

This is the second edition of a successful book first published 10 years ago. As the authors proudly state in the introduction, “The response to the first edition of this volume has been most gratifying. Before its publication this subject matter had been generally regarded as a collection of loosely tied results. Today it is recognized for what it is—a cohesive subdiscipline of Discrete Mathematics. We are particularly pleased with the name given to this subdiscipline: Ramsey Theory!”

The authors updated the first edition by adding just a few remarks here and there and a single (major) new result: Shelah’s proof of Van der Waerden theorem, with detail hierarchy considerations. Thus the authors do not attempt to expand the volume with a fine mosaic of recent development; instead they keep the nice style of the first edition which concentrates on the main stream of the spectacular development (during the 1980s). The interested reader may consult Mathematics of Ramsey Theory (J. Nesetril, V. Rodl. eds.), Springer Verlag, 1990, and forthcoming book by H.J. Prömel and B. Voigt for a more extensive treatment of the recent development.

—J. NESETRIL
The contents and authors of Volume A follow:

—W. Kern

Journals

Vol. 51, No. 2
M. Grötschel and O. Holland, “Solution of Large-Scale Symmetric Travelling Salesman Problems.”
E.K. Yang and J.W. Tolle, “A Class of Methods for Solving Large, Convex Quadratic Programs Subject to Box Constraints.”
H. Tuy, “Normal Conical Algorithm for Concave Minimization over Polytopes.”
F.A. Al-Khayyal, “Necessary and Sufficient Conditions for the Existence of Complementary Solutions and Characterizations of the Matrix Classes Q and Q_0.”
L.S. Zarembo, “Perfect Graphs and Norms.”

Vol. 51, No. 3
P. Alart and B. Lemaire, “Penalization in Non-Classical Convex Programming via Variational Convergence.”
M. Fischetti, “Facets of Two Steiner Arborescence Polyhedra.”
ACHIM BACHEM, having assumed several new responsibilities at Cologne and in the German OR societies, has resigned as Associate/Book Review Editor of OPTIMA. We thank him for 10 years of excellent service to the Society and welcome PROFESSOR ADOLPHUS J.J. TALMAN (Tilburg) who is the new Associate Editor. MARTIN GRÖTSCHEL has moved to Berlin Institute of Technology as professor of mathematics and vice-president of the Konrad-Zuse-Zentrum for Information Technology Berlin. PANOS PARDALOS is visiting the ISE Department, University of Florida. FAIZ AL-KHAYYAL (Georgia Tech) is the new chairman of the MPS Committee on Algorithms (COAL). Deadline for the next OPTIMA is February 1, 1992.

Books for review should be sent to the Book Review Editor, Professor Adolphus J.J. Talman Department of Econometrics Tilburg University P.O. Box 90153 5000 LE Tilburg Netherlands

Journal contents are subject to change by the publisher.

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